Alphonse J. Jackowski
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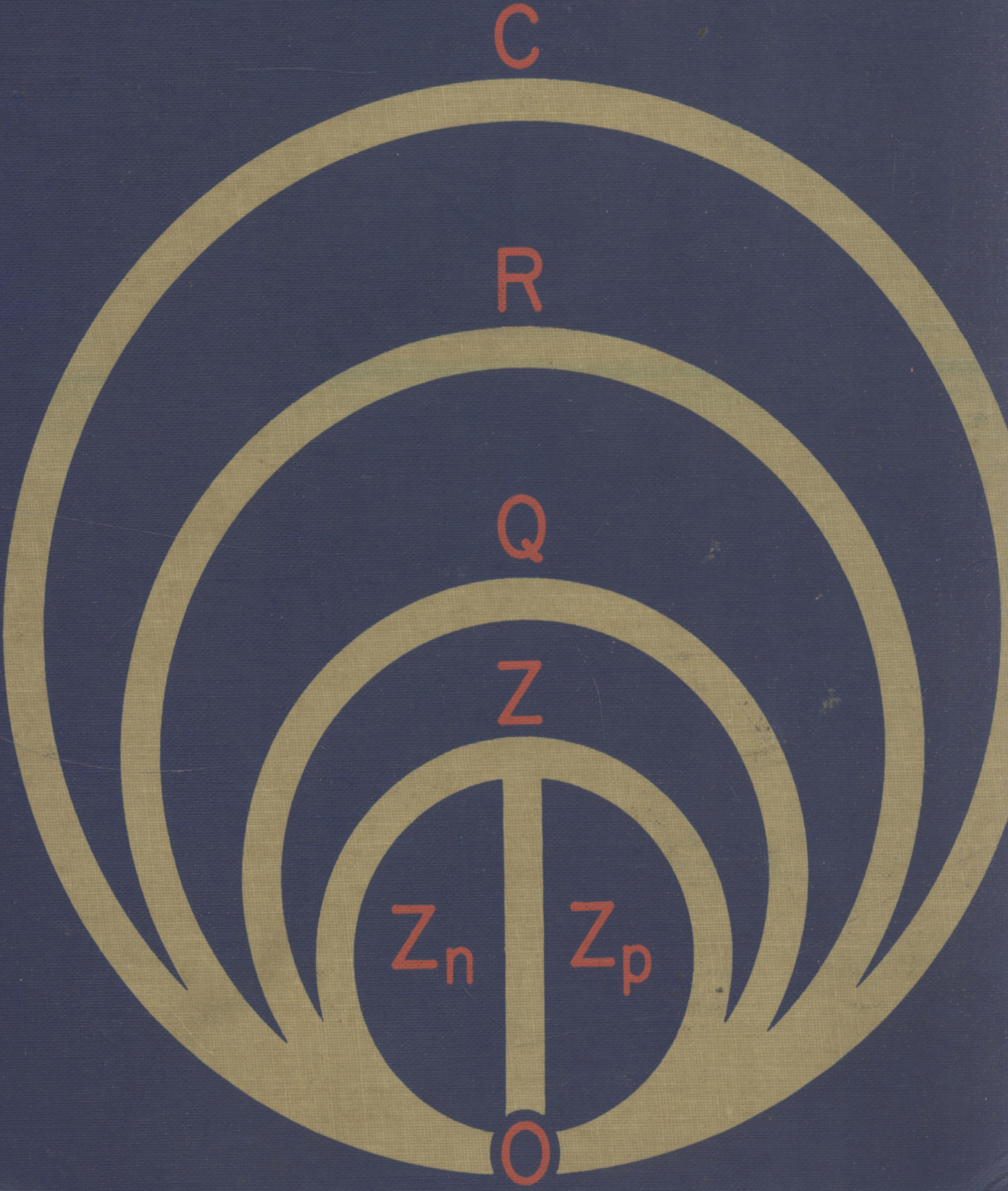
fundamentals of MODERN MATHEMATICS



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PRENTICE-HALL, INC., ENGLEWOOD CLIFFS, NEW JERSEY

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MODERN MATHEMATICS

Jackowski/Sbrega

Current printing (last digit):

10 9 8 7 6 5 4 3 2 1

13-341172-9

Library of Congress Catalog Card Number 75-95705

Printed in the United States of America

PRENTICE-HALL INTERNATIONAL, INC., *London*
PRENTICE-HALL OF AUSTRALIA, PTY. LTD., *Sydney*
PRENTICE-HALL OF CANADA, LTD., *Toronto*
PRENTICE-HALL OF INDIA PRIVATE LTD., *New Delhi*
PRENTICE-HALL OF JAPAN, INC., *Tokyo*

fundamentals of

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Affectionately dedicated
to our wives, Kitty and Rita,
without whose patience and understanding
this book would not have been possible

preface

This text is designed for a two-semester course in the fundamental concepts of mathematics. It is strictly a content and not a methods text. Its purpose is two-fold: (1) to provide prospective teachers with the mathematical preparation necessary to teach the modern elementary and junior high school curricula, and (2) to provide the liberal arts student with a terminal course wherein structure and unifying concepts are emphasized. It is also suitable for inservice training programs for teachers.

The authors have been influenced and guided by the recommendations of the CUPM, the CEEB, the SMSG, and other agencies concerned with curriculum improvement. This text is a compromise of these recommendations. A conscious effort has been made to keep the text readable from the point of view of a student with two years of high school mathematics preparation. Understanding is stressed throughout and is enhanced by a balance of intuition and rigor. The structural concepts of group, ring, integral domain, and field are introduced in a natural way and only after simple and familiar examples have been discussed in detail.

A distinguishing feature is the insertion of numerous and frequent sets of exercises. This permits immediate application of newly acquired concepts and reinforces learning. Optional sections and exercises are indicated by an asterisk and may be used for enrichment at the discretion of the instructor.

Chapter 1 introduces the student to the nature of mathematics and the essentials of a deductive system. Chapter 2 presents the elements of set theory used throughout the remainder of the text. Chapter 3 discusses the nature of a mathematical proof and illustrates the various methods of proof and disproof in detail. Chapter 4 is a cursory and intuitive preview of number systems which prepares the student for the detailed treatment of the properties of number systems in the ensuing chapters. Beginning with the set of natural numbers, the successive extensions to the integers, the rational numbers, and the real numbers are motivated by employing only the closure axiom with respect to the four arithmetic operations. The extension of the set of real numbers to the set of complex numbers is based on the need for algebraic closure. In Chapter 5 the properties of the natural numbers are treated. The concept of a mathematical (algebraic) system is introduced and is used to discuss the axioms for the set of natural numbers under addition and multiplication. This is followed by a discussion of the order properties. The principle of finite induction is presented as an optional topic. Chapter 6 treats the properties of the set of integers. The concepts of a group, a ring, and an integral domain are introduced in a familiar setting here. Examples of finite groups and topics from elementary number theory are included. The section concerning the congruence of integers modulo n provides an opportunity to enhance the understanding of the structural concepts presented at the beginning of the chapter. Chapter 7 presents a development of the field of rational numbers using the integral domain of integers as a basis. A discussion of finite fields is included. Chapter 8 consists of an intuitive treatment of the field of real numbers and Chapter 9 completes the discussion of the field of complex numbers begun in Chapter 4. Chapter 10 consists of a detailed treatment of numeration systems. It contains a review of the Hindu-Arabic system and an introduction to systems involving base two, base five, and base twelve. Chapter 11 is a careful development of the foundations of Euclidean geometry. A sequence of theorems recommended by the Commission on Mathematics of the CEEB is presented in Chapter 12. This serves as an efficient preparation for the coordinate geometry of Chapter 13. Chapter 14 introduces the concepts of precision and accuracy and develops the use of approximate numbers and the mensuration formulas for the common geometric figures of two and three dimensions. In the final chapter, which treats relations and functions, the emphasis is on fundamental properties rather than on manipulative techniques.

A preliminary edition of this text was class-tested for two years at Westfield State College, Westfield, Massachusetts.

The authors wish to thank President Leonard J. Savignano and Dean Edward S. Townsend of Westfield State College for their administrative support and encouragement. We are particularly indebted to our colleagues

in the department of mathematics at Westfield who have read parts of the manuscript, have made valuable suggestions, and have class-tested the preliminary edition.

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1
introduction

1.1
The Nature of Mathematics

Mathematics has many aspects and its generality and usefulness are apparent in many forms of human endeavor. The scientist uses the language of mathematics to formulate his theories in concise form. The engineer finds mathematics an essential tool for design and construction. The manufacturer turns to mathematics to regulate and to control the quality of his products. Furthermore, since the advent of the computer, we find mathematics making a tremendous impact on such diverse fields as biology, business, economics, medicine, and even music. Likewise, we find mathematics playing an ever-increasing role in the more mundane phases of everyday life.

Although mathematics is often divided into two main categories, applied mathematics and pure mathematics, mathematics itself is essentially abstract and its power and generality are inherent in its abstract nature. The kind of mathematics we have discussed thus far is applied mathematics, and one involved in such work is called an applied mathematician. On the other hand, the pure mathematician studies mathematics in its own right and finds great aesthetic appeal in its logical structure and abstract systems.