

STRENGTH **SECOND**
OF MATERIALS **EDITION**

♦ John N. Cernica

STRENGTH SECOND EDITION OF MATERIALS

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PREFACE TO THE SECOND EDITION

While the basic format, perhaps one feature most responsible for the outstanding success of the first edition, was closely adhered to in this edition, some additions are indeed timely; the inevitable universal adoption of the International System of units (SI) makes the introduction of this item at this time an indispensable added objective.

Most of the example problems have the dual, English and metric, units. Some are treated in the English units with the significant values also given in the metric equivalent; others are treated totally in the metric system. The answers to the problems, given in Appendix C for the even-numbered problems, are likewise in the *dual* system.

The addition of nearly 140 new problems and examples bring the total number in this edition to almost 800; of these about 100 are example problems worked out in detail. This should provide for a rather wide choice of assignments for a long period, for many sections.

Every effort was made to correct some errors detected in the first edition. For this much appreciation is extended to many who brought them to the attention of the author; much appreciation is expressed to those who wish to do so in the future.

The author is most grateful to Dr. R. G. Boggs of U.S. Coast Guard Academy who did a most thorough and exhaustive evaluation of the book, and whose comments were most valuable. Also, thanks is extended to the following persons for their constructive review evaluation and recommendations: Dr. W. J. Lnenicka, Georgia Institute of Technology; Dr. K. Muhl-bauer, University of Missouri-Rolla; Prof. C. M. Antony, University of Syracuse; Dr. W. C. Crisman, Le Tourneau College; Dr. D. H. Suchora, Youngstown State University; Dr. J. M. Dalrymple, Michigan Technological University; Mr. Douglas Kearns, Graduate Student, Youngstown State University; to Miss Mary Ann Stasiak and Mrs. Karen Vanderbilt for the typing of the manuscript.

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PREFACE TO THE FIRST EDITION

The study of strength of materials is perhaps the first opportunity for the student to put to use in a constructive way some of the fundamentals presented to him in calculus, physics, and engineering mechanics. Up to this point he has learned fundamentals for use in the future. He cannot, from these fundamentals alone, tie in the *analysis* of a structure to the *design* of that structure; that is, he cannot intelligently select a material and specify size and other essential properties to meet some specific need. *Strength of Materials* develops the fundamental relationship between applied forces and internal effects in a systematic and basic manner, and thus sets the foundation for the transition to *application*. Furthermore, it permits — and in some instances compels — the student to take up where theory leaves off, and in so doing, provides an avenue for development of sound judgment (we might prefer to call it engineering judgment) and for applying theory in a practical and realistic way.

It is hoped that this book's content, and the fashion used in presenting this content, will form the bridge the student may cross to broader design fields. More specifically, the book has the following objectives:

1. To give the student fundamentals, which once mastered, can serve as tools for him to build with.
2. To emphasize an orderly and systematic format for solutions to problems.
3. To emphasize the need for good judgment in *defining* a problem, making the *assumptions* needed to solve the problem, *selecting* the material, and considering the *design* of the component part of a structure — be it a machine part, a beam for a bridge, or some other item.

To achieve the first objective, the author has taken great care in explaining the basic concepts involved, and in establishing fundamental expressions which, although rigorously derived, would be easily understood and used by the student, with a full knowledge of their physical meaning and of the assumptions and limitations on which their derivations are based. Chapter I covers the concepts of stress and strain in a broad sense. Although an attempt was made to keep the presentation concise, no effort was spared to elaborate on various phases of these basic concepts where needed. The remainder of the book puts these concepts into use, always deriving theoretical expressions from fundamentals.

To attain the second objective, the author has presented numerous example problems in *component form: Given; To Find; Solution*. The separation of these components into explicit forms, and the systematic

procedure followed to get an answer, makes for an efficient and easy-to-understand system for solving problems. Once in the habit of using such a method to solve problems, the student should benefit, not only by decreasing his errors of computation, but also by acquiring a desirable form for presentation to others who must check, approve, or just look, at his work.

The final objective of the text is to emphasize common sense, rather than unquestioned use of available formulas, methods, or specifications.

In presenting this material, the author has drawn upon material presented in one form or another by many authors and many publications, and upon the author's own notes from teaching the subject for many years. The author wishes to acknowledge his indebtedness to Dr. Frank D'Isa and Professor Robert Sorokach for their constructive criticism, and to the Misses Gerri Sfara and Patricia M. Olinik for their typing of the manuscript.

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SYMBOLS AND ABBREVIATIONS

<i>A</i>	area
<i>c</i>	distance from neutral axis to extreme fiber
<i>d</i>	diameter
<i>E</i>	modulus of elasticity
<i>e</i>	eccentricity
<i>F</i>	force
ft	foot or feet
ft-lb	foot-pound
<i>G</i>	shear modulus or modulus of rigidity
<i>g</i>	gravitational acceleration constant
<i>h</i>	height; depth of a beam
hp	horsepower
<i>I</i>	moment of inertia of area
<i>i</i>	radius of gyration
in.	inch or inches
in.-lb	inch-pound
<i>J</i>	polar moment of inertia of area
<i>K</i>	stress concentration factor
<i>k</i>	symbol for p/EI ; spring constant; factor
kN	kilonewtons
kip	kilopound (1000 lb)
ksi	kilopounds (or kips) per square inch
<i>l</i> or <i>L</i>	length
lb	pound
<i>M</i>	bending moment
max	maximum
min	minimum
<i>N</i>	normal force; Newtons
<i>n</i>	cycles; number
<i>P</i>	force; concentrated load
Pa	pascals

p	pressure per unit area
psf	pounds per square foot
psi	pounds per square inch
Q	force; statical moment of area
q	load per unit length; shear flow
R	reaction; radius; resultant force
r	radius; radius of gyration
rad	radian
rpm	revolutions per minute
rps	revolutions per second
s	arc length; distance
T	torque; temperature
t	thickness
U	strain energy
u	strain energy per unit volume
V	shearing force; volume
v	velocity
W	weight; total load
w	load per unit length; weight per unit volume
x, y, z	coordinates
Z	section modulus
α	temperature coefficient of expansion; angle
β	angle
δ	deflection; total elongation
γ	shearing strain
ϵ	tensile or compressive strain
θ	slope of elastic line; angle of twist per unit length
μ	Poisson's ratio
ρ	radius of curvature
σ	normal stress
τ	shearing stress
ϕ	angle of twist; angular coordinate
ω	angular velocity
δ	specific weight

BASE SI UNITS

<u>Multiplication Factor</u>	<u>Prefix</u>	<u>SI Symbol</u>
1 000 000 000	giga	G
1 000 000	mega	M
1 000	kilo	k
0.001	milli	m
0.000 001	micro	μ
0.000 000 001	nano	n

<u>Quantity</u>	<u>Unit</u>	<u>Symbol</u>
length	meter	m
mass	kilogram	kg
force	newton	N
time	second	s

DERIVED SI UNITS

<u>Quantity</u>	<u>Derived SI unit</u>	<u>Name</u>	<u>Symbol</u>
area	square meter	—	m ²
volume	cubic meter	—	m ³
density	kilogram per cubic meter	—	kg/m ³
force	kilogram-meter per second squared	newton	N
moment of force	newton-meter	—	N-m
pressure	newton per meter squared	pascal	Pa
stress	newton per meter squared	pascal	Pa or N/m ²
work, energy	newton-meter	joule	J
power	joule per second	watt	W
kilopound-force (kip)	kilonewtons (kN)		4.45
kilopound-force/sq in.	meganewtons/meter ² (MN/m ²)		6.895
one kilogram force (kgf)	newtons (N)		9.81
pounds per square foot (psf)	newtons per square meter (N/m ²)		47.9
pounds per square inch (psi)	kilonewtons per square meter (kN/m ²)		6.9
inch-pound force (ft-lbf)	newton-meter (N-m)		0.113
foot-pound force (ft-lbf)	newton-meter (N-m)		1.356
horsepower (hp = 550 ft-lbf/sec)	newton-meter/sec (N-m/sec)		745.7

To convert	To	Multiply by
inches (in.)	millimeters (mm)	25.40
inches (in.)	centimeters (cm)	2.540
inches (in.)	meters (m)	0.0254
feet (ft)	meters (m)	0.305
miles (miles)	kilometers (km)	1.61
yards (yd)	metres (m)	0.91
square inches (sq. in.)	square centimeters (cm ²)	6.45
square feet (sq ft)	square meters (m ²)	0.093
square yards (sq yd)	square meters (m ²)	0.836
acres (acre)	square meters (m ²)	4047
square miles (sq miles)	square kilometers (km ²)	2.59
cubic inches (cu in.)	cubic centimeters (cm ³)	16.4
cubic feet (cu ft)	cubic meters (m ³)	0.028
cubic yards (cu yd)	cubic meters (m ³)	0.765
pounds (lb)	kilograms (kg)	0.453
tons (ton)	kilograms (kg)	907.2
kilopound-force (kip)	kilonewtons (kN)	4.45
kilopound-force/sq in.	meganewtons/meter ² (MN/m ²)	6.895
one kilogram force (kgf)	newtons (N)	9.81
pounds per square foot (psf)	newtons per square meter (N/m ²)	47.9
pounds per square inch (psi)	kilonewtons per square	6.9
gallons (gal)	cubic meters (m ³)	0.0038
acre-feet (acre-ft)	cubic meters (m ³)	1233
gallons per minute (gal/min)	cubic meters per minute (m ³ /min)	0.0038
newtons per square meter (N/m ²)	pascals (Pa)	1.00
inch-pound force (ft-lbf)	newton-meter (N-m)	0.113
foot-pound force (ft-lbf)	newton-meter (n-m)	1.356
horsepower (hp = 550 ft-lbf/sec)	newton-meter/sec)	745.7

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Chapter 1 ♦ FORCES AND DEFORMATIONS

1—1 Introduction

Safety and economy, in that order, are perhaps the most important aspects of structural design. A designer must choose a size and type of material strong enough to carry the forces to which it is subjected and rigid enough not to deform excessively, but economical and serviceable enough to dictate its choice over some other material. Sometimes economy—but never safety—is sacrificed for aesthetic values, durability, or low cost of maintenance.

Strength-of-material analysis develops systematically the relationship between externally applied loads and the resulting internal effects, loads, and deformations induced in the body subjected to these loads. This forms a basis for understanding design problems and their solution in a fundamental way, and supplies the background for safe and economical design work. Once a student masters these fundamentals, he can expand and grow; without these fundamentals, he is forever limited.

For example, a carpenter knows from experience that a floor joist does not serve well when it lies with its deeper side on the support; the joist deflects excessively and might perhaps rupture in bending. He draws on the experience of others and his own personal observations, and with this insight he manages to solve this problem by placing the joist with its deeper side vertical. The size of the joist necessary to span a certain distance, and the spacing of these joists, are problems he again solves in a crude way. He has no systematic way of attacking the problem. He could not span other structures of any general type because he lacks fundamental knowledge. He is limited to something he saw or was handed down to him by his predecessors, and he is therefore limited in growth.

The problems encountered in this book generally fall in the category of *design* or *analysis*. If the problem is to select the size and material to build a

machine or a structure to satisfy a certain function under given or assumed conditions, it is a design problem. If the capacity of a given completed structure to carry a certain load is to be investigated, the problem becomes one of analysis.

Seldom, however, is either an *analysis* or a *design* problem clear-cut. Its solution depends on conditions that are *variable*. Almost always, the approach requires reasonable and basic assumptions, utilizing practical considerations and available experimental and theoretical information, before a systematic solution to the problem can be attempted. For example, again assume that a joist supporting a floor is to be *analyzed* for deflection. Load, span, size, and strength of the wood are important factors in the solution of the problem. Although the size can be measured, the strength characteristics are obtained from either testing in the laboratory, or accepting typical test results from others. Furthermore, if plywood sheets are nailed on the top of the joists, should these sheets be assumed effective in reducing the deflection, and, if so, how much? Only a few nails may make the effect of the plywood insignificant, and many nails might mean just the reverse. Likewise, is the load evenly distributed over the span, and is the span the same as the length of the joists, or just the distance between the inside faces of the two walls, or somewhere in between?

In the same manner, in designs for assumed conditions of load, many sizes may fit a certain restriction, such as deflection or strength. But a comprehensive consideration of all the restrictions may eliminate many choices. Thus, the process of selection and elimination to reach a satisfactory design is a process of analysis. The conditions of load and function to be served, as well as the actual selection of suitable components, are idealized in a systematic way, and are proven to be reliable and reasonably basic in insuring satisfactory results.

Much is still to be done in perfecting and refining the approaches to the solution of many problems. But much has been done and much is to be gained by understanding these accomplishments. The presentation of these concepts and the understanding of the basis on which they exist is the undertaking of this book.

1—2 Loads and Reactions

The forces that *act* on a member are referred to as *loads*. The forces *counteracting* the effect of these loads are called *reactions*. Loads and reactions may be generally categorized as external or internal, and static or dynamic. They may be further defined with respect to the area subjected to contact. It is important that definitions of various types be established now in order to eliminate confusion later on.

A *static* load is a force applied gradually and slowly, and not repeated many times.

A *sustained* load is a force acting for a long period of time. The dead weight of a structure is an example of such a load. For some materials under certain conditions of temperature and load, this force has an appreciable effect on the structure, such as permanent deflection of a beam or permanent shortening of a column.

A *repeated* load is a force applied thousands or perhaps millions of times; for example, the forces in various parts of a running engine.

An *impact* load is a force applied within a short period of time, such as a hammer hitting a nail or a weight dropping to the floor. Sometimes this type of load is described as an *energy* load.

An *axial* load is a force or force system whose resultant passes through the centroid of the section on which the force acts.

A *concentrated* load is a force applied at a point.

A *distributed* load is a force or force system spread over an area, either uniformly or nonuniformly.

An *external* load is a force acting on the outside of a structure set in equilibrium. The loads P and the reactions R and H in Figure 1-1 are examples of such loads.

An *internal* load is a force effect within the entity or any component part of a structure set in equilibrium. The forces in members U_1L_1 , U_2L_2 , U_3L_3 in Figure 1-1 are examples of such loads.

To analyze *external* reactions, the loads acting on a body may be replaced by their resultant, frequently a mathematically convenient simplification. *External* reactions are not altered by the application of a load to *any* point along its line of action. *Internal* effects, however, are altered by these

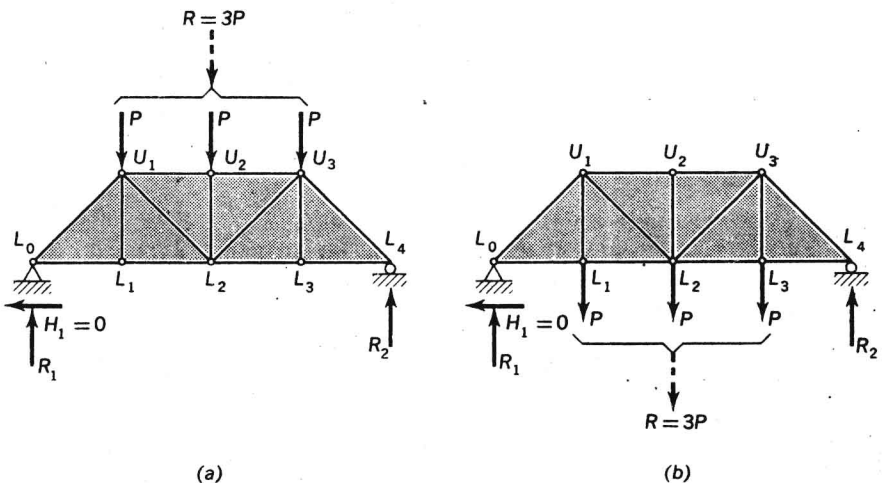


Figure 1-1 A truss symmetrically loaded with three equal loads.

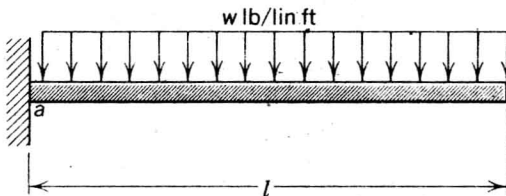
loading substitutions. Figure 1-1 illustrates this. Whether the loads act at points U_1 , U_2 , and U_3 , as shown in Figure 1-1a, or at points L_1 , L_2 , and L_3 , as shown in Figure 1-1b, the reactions are the same; likewise, the reactions would be the same if the loads were replaced by their resultants (represented by the dashed arrows).

Now let us investigate the internal loads, such as the forces in the vertical members. In Figure 1-1a, the forces in the three vertical members, represented by the two capital letters at the joints or connections of these members (a common notation), are $U_1L_1 = 0$, $U_2L_2 = P$, $U_3L_3 = 0$. In Figure 1-1b the forces in these members are different, that is, $U_1L_1 = P$, $U_2L_2 = 0$, and $U_3L_3 = P$. This is an important aspect to keep in mind, because repeatedly problems are solved by first finding the external effects, and then the internal forces.

Example 1-1:

Given: The cantilever beam shown in Figure a.

(a)



Find: The external reactions acting on the beam.

Solution: The uniform load applied to the beam is usually so much larger than the unit weight of the beam that the beam's weight is neglected in the calculations. When and if to neglect the beam's weight is an engineering judgment applied to each situation as it arises. For our beam we shall assume that the weight of the beam is incorporated in w .

We proceed by setting the beam in equilibrium, as shown in Figure b, that is, only the beam *outside* the wall. (A sign convention: Forces acting upward and moments or bending effects creating tension on the bottom surface of the beam are assumed positive, as is explained in Chapter 4. Thus, R being upward, the sign is positive; the negative sign