

Probability Models for Economic Decisions



Publisher: Curt Hinrichs
Assistant Editor: Ann Day
Editorial Assistant: Fiona Chong
Fechnology Project Manager: Burke Taft
Marketing Manager: Tom Ziolkowski

Advertising Project Manager: Nathaniel Bergson-Michelson Project Manager, Editorial Production: Kelsey McGee

Print/Media Buyer: *Doreen Suruki* Permissions Editor: *Stephanie Lee*

© 2005 Thomson Brooks/Cole, a part of The Thomson Corporation. Thomson, the Star logo, and Brooks/Cole are rademarks used herein under license.

ALL RIGHTS RESERVED. No part of this work covered by the copyright hereon may be reproduced or used in any form or by any means—graphic, electronic, or mechanical, including photocopying, recording, taping, Web distribution, or infornation storage and retrieval systems, or in any other manner—without the written permission of the publisher.

Printed in the United States of America 1 2 3 4 5 6 7 08 07 06 05 04

For more information about our products, contact us at: Thomson Learning Academic Resource Center 1-800-423-0563

For permission to use material from this text or product, submit a request online at http://www.thomsonrights.com.

Any additional questions about permissions can be

submitted by email to thomsonrights@Thomson.com.

Library of Congress Control Number: 2004109025 ISBN 0-534-42381-7 Production Service: Matrix Productions/Aaron Downey

Copy Editor: Pam Rockwell

Compositor: International Typesetting and

Composition

Cover and Interior Designer: Kaelin Chappell

Cover and Chapter Opener Image: Rick Doyle/CORBIS

Cover Printing: Coral Graphic Services
Printing and Binding: Malloy Incorporated

Brooks/Cole Thomson Learning 10 Davis Drive Belmont, CA 94002 USA

Asia

Thomson Learning 5 Shenton Way #01-01 UIC Building Singapore 068808

Australia/New Zealand

Thomson Learning 102 Dodds Street Southbank, Victoria 3006 Australia

Canada

Nelson 1120 Birchmount Road Toronto, Ontario M1K 5G4 Canada

Europe/Middle East/Africa

Thomson Learning High Holborn House 50/51 Bedford Row London WC1R 4LR United Kingdom

Duxbury Titles of Related Interest

Albright, VBA for Modelers

Albright & Winston, Spreadsheet Modeling and Applications

Albright, Winston, & Zappe, Data Analysis and Decision Making

Berger & Maurer, Experimental Design

Bowerman & O'Connell, Linear Statistical Models

Bowerman, O'Connell, & Koehler, Forecasting, Time Series and Regression

Clemen & Reilly, Making Hard Decisions with Decision Tools

Davis, Business Research for Decision Making

Devore & Farnum, Applied Statistics for Engineers and Scientists

Dielman, Applied Regression Analysis

Durrett, Probability: Theory and Examples

Fourer, Gay, and Kernighan, AMPL: A Modeling Language for

Mathematical Programming

Hildebrand, Ott, & Gray, Basic Statistical Ideas for Managers

Hoerl & Snee, Statistical Thinking: Improving Business Performance

Keller & Warrack, Statistics for Management and Economics

Kirkwood, Strategic Decision Making

Kuehl, Design of Experiments: Statistical Principles for Research Design and Analysis

Lapin & Whisler, Quantitative Decision Making

Lattin, Carroll, & Green, Analyzing Multivariate Data

Lohr, Sampling: Design and Analysis

Lunneborg, Data Analysis by Resampling

Minh, Applied Probability Models

Neuwirth & Arganbright, Mathematical Modeling with Microsoft®Excel

SAS Institute, Inc., JMP-IN: Statistical Discovery Software

Savage, Decision Making with Insight

Schaeffer, Mendenhall, & Ott, Elementary Survey Sampling

Seila, Ceric, & Tadikamalla, Applied Simulation Modeling

Shapiro, Modeling the Supply Chain

Stewart, Introduction to Applied Econometrics

Weiers, Introduction to Business Statistics

Winston, Introduction to Probability Models

Winston, Operations Research: Applications and Algorithms

Winston & Albright, Practical Management Science

Winston & Venkataramanan, Introduction to Mathematical Programming

To order copies, contact your local bookstore or call 1-800-354-9706. For more information, go to www.duxbury.com

Probability Models for Economic Decisions

Roger B. Myerson

University of Chicago



25038

0.44803

BROOKS/COLE

Australia ■ Canada ■ Mexico ■ Singapore ■ Spain United Kingdom ■ United States To my parents, who have continued learning about the world and working to improve it, even in their 60th year of marriage.

About the Author



Roger B. Myerson is the W. C. Norby Professor of Economics at the University of Chicago. He previously taught at the Kellogg School of Management at Northwestern University (1976–2001). His teaching interests include decision analysis, probability modeling, game theory, mathematical optimization theory, social choice and formal political theory, and economics of information. Dr. Myerson has received a number of professional awards, including Guggenheim Fellow (1983–1984), Sloan Foundation Fellow (1984–1986), Fellow of the Econometric Society (elected 1983), Fellow of the American Academy of Arts and Sciences (elected 1993), and an honorary doctorate from the University of Basel (2002). His research interests include game theory, economics of information, and analysis of voting systems.

Preface

his book is an introduction to the use of probability models for analyzing risks and economic decisions. Some prior study of probability may be desirable but is not assumed here, as the basic ideas of probability are introduced in the book itself. Throughout, the focus is on showing the reader how to use probability in complex, realistic situations. All the analytical work in this book is done in Microsoft Excel® spreadsheets, because the spreadsheet medium enables students to handle much more complex problems than they could handle using traditional ways of representing mathematical formulas. As a result of this emphasis on spreadsheet modeling, many readers may also learn sophisticated spreadsheet skills from this book. But the main goal is to make the practical power of probability analysis accessible to students at the undergraduate or MBA level.

The ultimate goal of any quantitative analysis course should be to teach students the art of making quantitative models that can give practical insights into real decision problems. But students usually have difficulty with complex models that involve more than two variables, and so quantitative methods are often taught only in the context of simple applications that lack any realistic complexity. The typical result is that even good students who show a mastery of mathematical concepts on their final exams often cannot see how to apply these concepts in the real world.

This problem was particularly frustrating to me during the 1980s as I did theoretical research in information economics while I taught probability to MBA students at the Northwestern University's Kellogg School of Management. New advances in information economics were teaching economic theorists that analysis of uncertainty has vital importance for understanding the competitive behavior of economic agents, and probability theory is the foundation for all analysis of uncertainty. But it seemed difficult or impossible to communicate this practical importance of probability analysis to my MBA students, even though their career goal was to become the competitively successful economic agents that we theorists

were studying. So in the late 1980s, I began searching for radically new ways to teach probability analysis so that students, after a 10-week MBA-level course, should be able to apply probability analysis to really interesting economic problems and cases. An important motivation in this search was my perception that the powerful new spreadsheet programs had great unexplored potential to transform the way that quantitative analysis is taught. This book is the fulfillment of this long process of experimentation and pedagogical development.

It has now become commonplace for probability and statistics books to do calculations in Excel spreadsheets. In most cases, however, these books have simply taken the old material that professors used to teach on blackboards and moved it into a spreadsheet. Only rarely have authors asked harder questions about how the new spreadsheet medium should change the content of an introductory quantitative analysis course. But if we really want to maximize the practical value of the skills that a student can take away from the course, then such questions need to be asked. Analytical methods that seemed too difficult or complex for an audience of applied business and economics students when we worked on paper and blackboards now may be straightforward for the same students to master in spreadsheets. On the other hand, there is less need for students to memorize basic computational formulas now that computer programs with built-in functions and help screens are so universally available.

The most important advantage of teaching spreadsheets is that they make multivariable models much easier for students to visually understand. Most students lose the big picture when they are asked to think about more than two variables that are represented by letters like \boldsymbol{x} and $\boldsymbol{\alpha}$ on a blackboard, but the same students can visually understand a spreadsheet model with many variables that are represented as cells on a spreadsheet page. Working in spreadsheets brings down the barrier that prevents most students from becoming sophisticated quantitative modelers. So throughout this book, I have tried to show how the methods of probability analysis can be applied to examples of a realistic complexity.

The emphasis on Monte Carlo randomized simulation of probability models grew gradually in the development of this book. At first I emphasized other methods of computing probabilities and expected values, but I realized that other computational methods generally require special assumptions that limit the scope of applications that we can consider. Monte Carlo simulation is the most versatile general framework for modeling any kind of situation that involves uncertainty, and it has the advantage of letting us really see the uncertainty in our models, as our unknown quantities change at every recalculation. With Monte Carlo simulation, we have a general framework that can be used to illustrate and motivate any advanced concept in probability and decision analysis. We can even give new and effective illustrations of conventional statistical topics such as confidence intervals (see Section 2.6) and regression (see Sections 6.7 and 6.8).

As I developed this course, I simultaneously worked to develop my own add-in program (Simtools.xla) to extend Excel's capabilities for probability analysis. I did not use one of the commercially produced add-ins for statistical risk analysis (like Crystal Ball or @Risk) because I wanted to go farther in decision analysis and economic modeling with less alteration of Excel's structure. But I found that there was a synergy between the development of the course and the development of the software. Students would regularly ask me to add a function to simplify a difficult part of the course; and as the new software simplified some topics, it became possible to teach other important topics that had previously seemed too advanced. As a result of this process, there are a number of new functions for facilitating decision analysis that are currently unique to Simtools (see GENLINV, CORAND, and CE, for example) but may eventually be imitated in other simulation add-ins.

I have used this book with MBA and undergraduate students at Northwestern University and the University of Chicago. Most of my students had some prior exposure to probability theory, but they often had difficulty recalling it when it was applied in our course. So this book includes an introductory discussion of all the basic probability concepts that we use. Chapter 1 introduces the basic ideas of probability and conditional probability. Chapter 2 covers discrete random variables, expected values, and standard deviations. In a short digression from probability theory, Chapter 3 introduces risk aversion and utility analysis with constant risk tolerance. Then Chapter 4 introduces continuous random variables, with an emphasis on Normal and Lognormal probability distributions. Chapter 5 covers joint distributions and correlation, and Chapter 6 discusses conditional expectation and regression models.

The later chapters of the book are organized around types of models. Simple decision models with one decision variable are introduced in Chapter 7, with an emphasis on decisions about production quantities (the "newsboy" problem) and decisions about prices (bidding in auctions). Chapter 8 focuses on risk sharing in partnerships and financial markets. Finally, Chapter 9 introduces a wide range of dynamic models for forecasting how quantities may change over time.

There is more than enough material here for a full semester course. In a recent 10-week course where many students needed a review of basic probability, I covered Sections 1.1–1.4, all of Chapter 2, Sections 3.1–3.2, Sections 4.1–4.7, Sections 5.1–5.8, Sections 6.1–6.2 and 6.6, Sections 7.1–7.2 and 7.4–7.6, and Sections 8.1 and 8.3. With students who have a stronger background in probability, I would try to spend less time in Chapters 2 and 4 and devote more time to Chapters 8 and 9. The heart of this book is in Chapter 5 (for relationships among random variables) and Chapter 7 (for decision variables), and these two chapters should get careful treatment in any course. I have also tried to make this book accessible for self-study or professional enrichment outside of the classroom. When

you open up the spreadsheet file for each chapter, you have everything that I would show you in my classroom.

Acknowledgments must begin with Howard Raiffa, who was my teacher and whose impact may be found throughout this book. Sid Deshmukh and Sam Savage have given me good colleagial suggestions and encouragement in this project over many years. It was Donald Jacobs who first told me to use more spreadsheets in my teaching, and Robert Weber first showed me how to use them. Last but not least is my debt to Rebecca Myerson, who carefully read every chapter of this book to find errors in both English and Excel. At the points where my MBA students had typically expressed some confusion, she kept asking questions until I found a way to rewrite it more clearly. I know that not every high school student would have been willing to proofread a long technical book by her father, and I am very grateful to Rebecca for helping to make this a better book.

Chicago, Illinois August 2004

Acknowledgments

he author would like to express his gratitude to the reviewers of this book:

Luiz Brandao, University of Texas at Austin, McCombs School of Business Sudhakar D. Deshmukh, Northwestern University, Kellogg Sheldon Lin, University of Toronto Erol Pekoz, Boston University Sam Savage, Stanford University Patrick Thompson, University of Florida

8/6

www.duxbury.com

www.duxbury.com is the World Wide Web site for Thomson Learning and is your direct source to dozens of online resources.

At www.duxbury.com you can find out about supplements, demonstration software, and student resources. You can also send email to many of our authors and preview new publications and exciting new technologies.

www.duxbury.com Changing the way the world learns®

Brief Contents

	Simulation and Conditional Probability
2	Discrete Random Variables 47
3	Utility Theory with Constant Risk Tolerance 81
4	Continuous Random Variables 107
5	Correlation and Multivariate Normal Random Variables 151
6	Conditional Expectation 199
7	Optimization of Decision Variables 231
8	Risk Sharing and Finance 281
9	Dynamic Models of Growth and Arrivals 325

Contents

	Simulation and Conditional Probability			
	1.0	Getting Started with Simtools in Excel 2		
	1.1	How to Toss Coins in a Spreadsheet 3		
	1.2	A Simulation Model of 20 Sales Calls 7		
	1.3	Analysis Using Excel's Data-Table Command 15		
	1.4	Conditional Independence 18		
	1.5	A Continuous Random Skill Variable from a Triangular Distribution 19		
	1.6	Probability Trees and Bayes' Rule 25		
	1.7	Advanced Spreadsheet Techniques: Constructing a Table with Multiple Inputs 37		
	1.8	Using Models 40		
	1.9	Summary 42 April Appendix C. A.		
		Exercises 43 Mail Control of the Con		
2	Dis	crete Random Variables	47	
	2.1	Unknown Quantities in Decisions Under Uncertainty 49		
	2.2	Charting a Probability Distribution 50		

2.3 Simulating Discrete Random Variables 532.4 Expected Value and Standard Deviation 5

	2.8 Multiple Random Variables 752.9 Summary 77Exercises 78	
3	Utility Theory with Constant Risk Tolerance	81
	3.1 Taking Account of Risk Aversion: Utility Analysis with Probabilities 82	
	3.2 Utility Analysis from Simulation Data 92	
	3.3 The More General Assumption of Linear Risk Tolerance 94	
	3.4 Advanced Technical Note on Utility Theory 96	
	3.5 Advanced Technical Note on Constant Risk Tolerance 101	
	3.6 Summary 105	
	Exercises 105	
4	Continuous Random Variables	107
	4.1 Normal Distributions 108	
	4.2 EXP and LN 113	
	4.3 Lognormal Distributions 116	
	4.4 Generalized Lognormal Distributions 122	
	4.5 Subjective Probability Assessment 124	
	4.6 A Decision Problem with Discrete and Continuous Unknowns	131
	4.7 Certainty Equivalents of Normal Lotteries 135	
	4.8 Other Probability Distributions 137	
	4.9 Summary 145 St. Managed 9.1	
	Exercises 149 EA applying 3	
5	Correlation and Multivariate Normal	
Mile Ministrate	Random Variables	151
	5.1 Joint Distributions of Discrete Random Variables 152	
	5.2 Covariance and Correlation 156	
	analysis I had not the sall of behave 3 - 8.3	

2.5 Estimates from Sample Data

2.7 Decision Criteria

2.6 Accuracy of Sample Estimates

62

66

	5.4	Estimating Correlations from Data 162
	5.5	Making Multivariate Normal Random Variables with CORAND and NORMINV 166
	5.6	Portfolio Analysis with Multivariate Normal Asset Returns 171
	5.7	Excel Solver and Efficient Portfolio Design 176
	5.8	100
	5.9	Using CORAND with Non-Normal Random Variables 186
		More About Linear Functions of Random Variables 190
		Summary 193
		Exercises 195
	Cor	nditional Expectation 199
	6.1	Dependence Among Random Variables 199
	6.2	Estimating Conditional Expectations and Standard Deviations 203
	6.3	The Expected-Posterior Law in a Discrete Example 206
	6.4	Backwards Analysis of Conditional Expectations in Tree Diagrams 211
	6.5	Conditional Expectation Relationships and Correlation 214
	6.6	Uncertainty About a Probability 216
	6.7	Linear Regression Models 220
	6.8	Regression Analysis and Least-Squared Errors 224
	6.9	Summary 227
		Exercises 229
7	Op	timization of Decision Variables 231
	7.1	General Techniques for Using Simulation in Decision Analysis 231
	7.2	Strategic Use of Information 241
	7.3	Decision Trees 246
	7.4	A Simple Bidding Problem 251
	7.5	The Winner's Curse 255
	7.6	Analyzing Competitive Behavior 263
		Summary 271
		Exercises 273

5.3 Linear Functions of Several Random Variables 158

8	Risk	Sharing and Finance	281
	8.1	Optimal Risk Sharing in a Partnership of Individuals with Constant Risk Tolerance 282	
	8.2	Optimality of Linear Rules in the Larger Class of Nonlinear Sharing Rules 291	
	8.3	Risk Sharing Subject to Moral-Hazard Incentive Constraints	295
	8.4	Piecewise-Linear Sharing Rules with Moral Hazard 301	
	8.5	Corporate Decision Making and Asset Pricing in the Stock Market 305	
	8.6	Fundamental Ideas of Arbitrage Pricing Theory 317	
	8.7	Summary 322	
		Exercises 323	
9	Dyr	namic Models of Growth and Arrivals	325
	9.1	Net Present Value 326	
	9.2	Forecasting Models 328	
	9.3	Forecasting Example: Goeing Case 332	
	9.4	Brownian-Motion Growth Models 339	
	9.5	Log-Optimal Investment Strategies 343	
	9.6	Exponential Arrival Models 349	
	9.7	Queuing Models 356	
	9.8	A Simple Inventory Model 362	
	9.9	Project Length and Critical Tasks 366	777-4
	9.10	Summary 370	
		Exercises 372	
	Арр	pendix: Excel Add-Ins for Use with This Book 377	
	Bib	liographic Note 387	
		7.2 Suntigle Use of Information 2.2	

Index

kiv

CONTENTS