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Preface

Mathematical methods has been a dominant research path in computational vision leading to a number of areas like filtering, segmentation, motion analysis and stereo reconstruction. Within such a branch visual perception tasks can either be addressed through the introduction of application-driven geometric flows or through the minimization of problem-driven cost functions where their lowest potential corresponds to image understanding.

The 3rd IEEE Workshop on Variational, Geometric and Level Set Methods focused on these novel mathematical techniques and their applications to computer vision problems. To this end, from a substantial number of submissions, 30 high-quality papers were selected after a fully blind review process covering a large spectrum of computer-aided visual understanding of the environment.

The papers are organized into four thematic areas: (i) Image Filtering and Reconstruction, (ii) Segmentation and Grouping, (iii) Registration and Motion Analysis and (iiii) 3D and Reconstruction. In the first area solutions to image enhancement, inpainting and compression are presented, while more advanced applications like model-free and model-based segmentation are presented in the segmentation area. Registration of curves and images as well as multi-frame segmentation and tracking are part of the motion understanding track, while introducing computational processes in manifolds, shape from shading, calibration and stereo reconstruction are part of the 3D track.

We hope that the material presented in the proceedings exceeds your expectations and will influence your research directions in the future. We would like to acknowledge the support of the Imaging and Visualization Department of Siemens Corporate Research for sponsoring the Best Student Paper Award.

Nikos Paragios
Olivier Faugeras
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A Study of Non-smooth Convex Flow Decomposition

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Abstract. We present a mathematical and computational feasibility study of the variational convex decomposition of 2D vector fields into coherent structures and additively superposed flow textures. Such decompositions are of interest for the analysis of image sequences in experimental fluid dynamics and for highly non-rigid image flows in computer vision.

Our work extends current research on image decomposition into structural and textural parts in a twofold way. Firstly, based on Gauss' integral theorem, we decompose flows into three components related to the flow's divergence, curl, and the boundary flow. To this end, we use proper operator discretizations that yield exact analogs of the basic continuous relations of vector analysis. Secondly, we decompose simultaneously both the divergence and the curl component into respective structural and textural parts. We show that the variational problem to achieve this decomposition together with necessary compatibility constraints can be reliably solved using a single convex second-order conic program.

1 Introduction

The representation, estimation, and analysis of non-rigid motions is relevant to many scenarios in computer vision, medical imaging, remote sensing, and experimental fluid dynamics. In the latter case, for example, sophisticated measurement techniques including pulsed laser light sheets, modern CCD cameras and dedicated hardware, enable the recording of high-resolution image sequences that reveal the evolution of spatial structures of unsteady flows [1].

In this context, two issues are particularly important. Firstly, the design and investigation of variational approaches to motion estimation that are well-posed through regularization but do not penalize relevant flow structures are of interest. A corresponding line of research concerns the use of higher-order regularizers as investigated, for example, in [2,3,4]. Secondly, representation of motions by components that capture different physical aspects are important for most areas of application mentioned above. Referring again to experimental fluid dynamics, for example, the extraction of coherent flow structures which are immersed into additional motion components at different spatial scales [5], poses a challenge for image sequence analysis.

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The decomposition of images has become an interesting and active area of research quite recently. Based on the seminal paper [6] introducing total variation based image denoising, and on the use of norms that are suited for representing oscillating patterns [7], a range of novel variational and computational approaches have been suggested for decomposing images of general scenes into basic components related to geometry, texture, and noise; e.g., [8,9,10,11].

In the present paper, we focus on function decomposition from the viewpoint of non-rigid variational motion analysis, and based on our recent work [12]. Specifically, we consider Meyer's [7] variational model

$$\min \mathrm{TV}(f^s)\,, \quad \text{s.t.} \quad f^s + f^t = f\,,\ \|f^t\|_G \leq \delta \tag{1}$$

as a representative approach to the decomposition of a function f into its basic structural and textural parts f^s, f^t, and study the feasibility of an extension to the decomposition of motion vector fields. Our objective is the *simultaneous* decomposition of a vector field into physically relevant components related to its divergence and curl, *and* the decomposition of these components into parts with intrinsic variations at different scales.

In section 2, we introduce the discrete representation of vector fields by its basic components related to divergence, curl, and boundary values. Based on an accurate discretization employing various finite-dimensional spaces and corresponding operators, a variational model for the simultaneous decomposition of these components is proposed in section 3. From the computational point of view, we prefer to reformulate our variational problem as a convex conic program in subsection 4 because all compatibility constraints defining our decomposition can be included at once. While conic programming has found widespread applications in all branches of computational science, it has only recently been suggested for the decomposition of scalar-valued image functions [13]. Numerical experiments demonstrate the feasibility of our approach in section 5.

2 Vector Field Representation

2.1 Flow Discretization

For discretizing the relevant differential operators we apply the *mimetic finite difference method* introduced by Hyman and Shashkov in [14]. This method preserves the integral identities satisfied by the continuous differential operators by appropriately defining their discrete analogues simultaneously with respect to two grids which we call primal and dual grid. Then we define

H_P: space of scalar fields on vertices,
H_V: space of scalar field on cells,
H_S: space of vector fields defined normal to sides,
H_E: space of vector fields defined tangential to sides,

and H_P^o, H_S^o, H_E^o as their restricted versions of inner scalar/vector fields, see Fig. 1. Likewise, we consider the restriced spaces H_P^o, H_S^o, H_E^o also as naturally embedded in H_P, H_S, H_E with zero boundaries. While H_P and H_V are equipped with the usual Euclidian norm, the norms on H_S and H_E include boundary weights, see appendix. The discrete versions of the first order operators ∇, div and curl with respect to the primal and dual grid are given by

$$\mathbb{G} : H_P \to H_E,\ \mathbb{D}iv\ : H_S \to H_V,\ \mathbb{C}url\ : H_E \to H_V,$$
$$\overline{\mathbb{G}} : H_V \to H_S,\ \overline{\mathbb{D}iv}\ : H_E^o \to H_P^o,\ \overline{\mathbb{C}url}\ : H_S^o \to H_P^o.$$

Reshaping the scalar/vector fields columnwise into vectors of appropriate lengths, our first-order operators act on the corresponding vector spaces as the matrices specified in the appendix.

Finally, for discretizing $\mathrm{n} \cdot u|_{\partial\Omega}$, we introduce the boundary operator $\mathbb{B}_{\mathrm{n}}$: $H_S \to \partial H_S := H_S \backslash H_S^o$, which restricts the vector field to the vectors at the grid's boundary multiplied by the outer normal vectors. For the matrix form of the operator, we refer to the appendix.

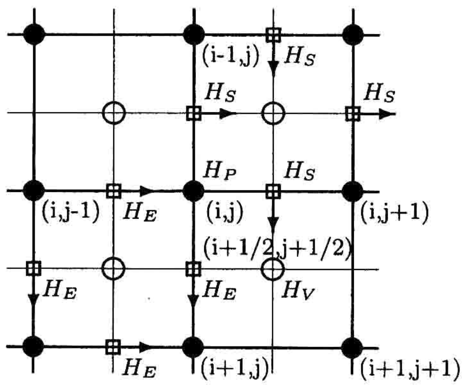

Fig. 1. Spaces H_P, H_V, H_S and H_E

2.2 Flow Representation

For the flow vectors $u \in H_S$, we see by definition of $\mathbb{D}iv$ and $\mathbb{B}_{\mathrm{n}}$ that

$$\mathbf{1}_{\dim H_V}^{\mathrm{T}}\ \mathbb{D}iv\ u = \mathbf{1}_{\dim \partial H_S}^{\mathrm{T}}\ \mathbb{B}_{\mathrm{n}} u, \tag{2}$$

where $\mathbf{1}_n$ denotes the vector consisting of n ones. This is just the discrete version of the *Gaussian Integral Theorem* $\int_\Omega \operatorname{div} u \, \mathrm{d}x = \int_{\partial\Omega} n \cdot u \, \mathrm{d}l$. Conversely, we say that $\rho \in H_V$ and $\nu \in \partial H_S$ fulfill the *compatibility condition* if

$$\mathbf{1}_{\dim H_V}^{\mathrm{T}}\ \rho = \mathbf{1}_{\dim \partial H_S}^{\mathrm{T}}\ \nu \tag{3}$$

Besides the flow representation $u \in H_S$, we will apply a second flow representation. To this end, consider the operator $A : H_S \to H_V \oplus H_P^o \oplus \partial H_S$ given in matrix form by

$$A := \begin{pmatrix} \mathbb{D}iv \\ \overline{\mathbb{C}url} \\ \mathbb{B}_{\mathrm{n}} \end{pmatrix} \in \mathbb{R}^{\dim H_S + 1, \dim H_S}\ , \tag{4}$$

where the $\overline{\mathbb{C}url}$ operator is naturally extended to the whole space H_S here. The operator A has full rank $\dim H_S$. Moreover, we see by (2) that $(\rho, \omega, \nu)^{\mathrm{T}}$ is in the image of A iff ρ and ν fulfill the compatibility condition (3). In this case u can be obtained from given $(\rho, \omega, \nu)^{\mathrm{T}}$ by $u = A^{\dagger}(\rho, \omega, \nu)^{\mathrm{T}}$, where $A^{\dagger} = (A^{\mathrm{T}}A)^{-1}A^{\mathrm{T}}$ denotes the pseudoinverse of A.

Proposition 1. *There exists a one-to-one correspondence between the spaces H_S and*

$$V_S := \{(\rho, \omega, \nu)^T : \mathbf{1}^T_{\dim H_V}\, \rho = \mathbf{1}^T_{\dim \partial H_S}\, \nu\} \ ,$$

where $\rho = \mathbb{D}iv\ u$, $\omega = \overline{\mathbb{C}url}\ u$, $\nu = \mathbb{B}_{\mathrm{n}} u$, and conversely $u = A^{\dagger}(\rho, \omega, \nu)^T$.

3 Variational Approaches

3.1 Flow Decomposition

In this section, we want to decompose flow vectors $u \in H_S$, resp., $(\rho, \omega, \nu)^{\mathrm{T}} \in V_S$ in a meaningful way. To this end, let c_ρ denote the mean of the divergence of u and c_ω the mean of the curl of u, i.e.,

$$c_\rho := \mathbf{1}^{\mathrm{T}}_{\dim H_V}\, \rho \;/\; \dim\, H_V \;=\; \mathbf{1}^{\mathrm{T}}_{\dim H_V}\, \mathbb{D}iv\ u \;/\; \dim\, H_V \ , \tag{5}$$

$$c_\omega := \mathbf{1}^{\mathrm{T}}_{\dim H^o_P}\, \omega \;/\; \dim\, H^o_P \;=\; \mathbf{1}^{\mathrm{T}}_{\dim H^o_P}\, \mathbb{C}url\ u \;/\; \dim\, H^o_P \ . \tag{6}$$

These are the discrete versions of $|\Omega|^{-1} \int_\Omega \mathrm{div}(u)dx$ and $|\Omega|^{-1} \int_\Omega \mathrm{curl}(u)dx$. Then we can decompose the flow $(\rho, \omega, \nu)^{\mathrm{T}} \in V_S$ as

$$(\rho, \omega, \nu) = (c_\rho, c_\omega, \nu) + (\rho^o, \omega^o, 0), \tag{7}$$

where $\mathbf{1}^{\mathrm{T}}_{\dim H_V} \rho_o = \mathbf{1}^{\mathrm{T}}_{\dim H^o_P} \omega_o = 0$. Obviously, we have that $(c_\rho, c_\omega, \nu)^{\mathrm{T}}$, $(\rho^o, \omega^o, 0)^{\mathrm{T}} \in V_S$ again, so that $u = u^c + u^o$ is the corresponding decomposition of $u \in H_S$, where $u^c := A^{\dagger}(c_\rho, c_\omega, \nu)^{\mathrm{T}}$ and $u^o := A^{\dagger}(\rho^o, \omega^o, 0)^{\mathrm{T}}$. The vector u^c, resp. (c_ρ, c_ω, ν), represents the basic pattern of the non-rigid flow and its boundary behaviour while u^o, resp. $(\rho^o, \omega^o, 0)$, is related to the variant flow pattern. Now we want to further decompose the intrinsic flow variation u^o into a structural part u^s and a texture part u^t, i.e., $u^o = u^s + u^t$. By proposition 1, this corresponds to the decomposition

$$(\rho^o, \omega^o, 0) = (\rho^s, \omega^s, 0) + (\rho^t, \omega^t, 0).$$

In summary, our task consists in the decomposition of a given flow field $u \in H_S$ as

$$u = u^c + u^s + u^t. \tag{8}$$

We can apply A to u which provides us, by using in addition (5) and (6), with $(c_\rho, c_\omega, \nu)^{\mathrm{T}}$ and $(\rho^o, \omega^o, 0)^{\mathrm{T}}$. Then, inspired by Meyer's approach (1), we may compute $(\rho^s, \omega^s, 0)$ and $(\rho^t, \omega^t, 0)$ as solutions of the minimization problem

$$\begin{aligned} &J(\rho^s, \omega^s, \rho^t, \omega^t) = \lambda_d \mathrm{TV}(\rho^s) + \lambda_c \mathrm{TV}(\omega^s), \\ \text{s.t.}\quad &\rho^s + \rho^t = \rho^o, \quad \omega^s + \omega^t = \omega^o, \quad \left\|\rho^t\right\|_G \le \delta_d\, , \quad \left\|\omega^t\right\|_G \le \delta_c, \end{aligned} \tag{9}$$

where the discrete TV functionals and the discrete versions of the G norm are defined in the appendix. This variational approach extends Meyer's model for the decomposition of scalar-valued functions to the *simultaneous* decomposition of vector fields into basic flow patterns. Finally, we may formally obtain u^s and u^t by solving the linear systems $(A^{\mathrm{T}}A)u^s = A^{\mathrm{T}}(\rho^s, \omega^s, 0)^{\mathrm{T}}$ and $(A^{\mathrm{T}}A)u^t = A^{\mathrm{T}}(\rho^t, \omega^t, 0)^{\mathrm{T}}$. However, these systems are very ill-conditioned so that we prefer to compute the components of u directly by minimizing the corresponding functional

$$
\begin{aligned}
& J(u^c, u^s, u^t) = \lambda_d \mathrm{TV}(\mathbb{D}iv\ u^s) + \lambda_c \mathrm{TV}(\overline{\mathbb{C}url}\ u^s) \qquad (10)\\
\text{s.t.}\quad & u^c + u^s + u^t = u, \\
& \overline{\mathbb{G}}\mathbb{D}iv\ u^c = 0, \quad \mathbb{G}\overline{\mathbb{C}url}\ u^c = 0, \quad \mathbf{1}^{\mathrm{T}}_{\dim H_P^o} \overline{\mathbb{C}url}\ u^s = 0, \\
& \mathbb{D}iv\ u^t = \rho^t, \quad \overline{\mathbb{C}url}\ u^t = \omega^t, \quad \|\rho^t\|_G \leq \delta_d\,, \quad \|\omega^t\|_G \leq \delta_c.
\end{aligned}
$$

This approach also fits into our flow estimation model in the next section. We note that the third constraint is related to the decomposition (7). While $\mathbf{1}^{\mathrm{T}}_{\dim H_V} \mathbb{D}iv\ u^o = 0$ is automatically fulfilled by the compatibility condition, we have to take care about $\mathbf{1}^{\mathrm{T}}_{\dim H_P^o} \overline{\mathbb{C}url}\ u^o = 0$. However, by the G norm constraint we have $\overline{\mathbb{C}url}\ u^t = \mathbb{D}iv\ p$ for some p which again, by the compatibility condition, and since $\overline{\mathbb{C}url}$ maps to H_P^o, implies that $\mathbf{1}^{\mathrm{T}}_{\dim H_P^o} \overline{\mathbb{C}url}\ u^t = 0$. As a result, we have only to take u^s into account.

Finally, we point out that as in the scalar-valued case, some variations of the approach (10) are easily conceivable. Referring to [8,10], for instance, the constraint $u^c + u^s + u^t = u$ in (10) could be replaced by a L^2 penalty term. This would imply L^2 penalty terms for each component in the decomposition.

3.2 Optical Flow Estimation Through Flow Decomposition

In this section, we combine the usual optical flow estimation method with the structure-texture flow decomposition (8). For a given image sequence $\{g\} \in H_V$, we want to compute the components u^c with constant divergence and curl, the large-scale patterns u^s of divergence and curl with bounded BV-norms, and the small-scale patterns u^t of divergence and curl with bounded G-norms, by solving

$$
\begin{aligned}
& J(u^{c,s,t}) = \left\|\overline{\mathbb{G}}g \cdot (u^c + u^s + u^t) + g_t\right\|_2^2 + \lambda_d \mathrm{TV}(\mathbb{D}iv\ u^s) + \lambda_c \mathrm{TV}(\overline{\mathbb{C}url}\ u^s) \quad (11)\\
\text{s.t.}\quad & \overline{\mathbb{G}}\mathbb{D}iv\ u^c = 0, \quad \mathbb{G}\overline{\mathbb{C}url}\ u^c = 0, \quad \mathbf{1}^{\mathrm{T}}_{\dim H_P^o} \overline{\mathbb{C}url}\ u^s = 0, \\
& \mathbb{D}iv\ u^t = \rho^t, \quad \overline{\mathbb{C}url}\ u^t = \omega^t, \quad \|\rho^t\|_G \leq \delta_d\,, \quad \|\omega^t\|_G \leq \delta_c.
\end{aligned}
$$

Here g_t denotes the discretization of the time derivative by a forward difference and the inner product is taken with respect to H_S. We refer to (11) as TV–G model. However, for the image areas where $\nabla g = 0$, the data term disappears such that the local constraints through the two G-norm terms lead to unbounded solutions. Hence, the flow estimation by solving problem (11) is not well-posed. Therefore, we propose to replace the TV–G model by a TV–L_2 model where the texture flow patterns u^t have divergence and curl with bounded L_2-norms: