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Statistics and Econometric Models

VOLUME TWO

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Translated by Quang Vuong

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Testing, Confidence Regions, Model Selection, and Asymptotic Theory

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CHAPTER 14

Introduction to Tests of Hypotheses

14.1 Testing Theory and Modelling

The purpose of the methods described in the first volume is to specify and estimate, on the basis of the data, a model of which the validity is not questioned. On the contrary, in the theory of hypothesis testing, which is the general topic of the second volume the validity of the model is now challenged.

For instance, one may wonder whether the specified model is not too "large," i.e., whether a submodel defined by a subset of the family $\mathcal{P}$ of possible probability distributions is not preferable. This is the basis of *significance tests.* Conversely, one may wonder whether the specified model is not too restrictive, i.e., whether the true distribution that has generated the observations actually belongs to $\mathcal{P}$. In the latter case, one frequently talks about *specification tests.* As a matter of fact, we shall not make a distinction between these two kinds of testing situations, for the approach that is generally considered in specification testing is to nest $\mathcal{P}$ in a larger family and to examine whether $\mathcal{P}$ is an acceptable restriction of this larger family. Hence the second problem reduces to the first problem. There exists, however, another approach to the problem of specification testing. This will be discussed in Chapter 22 when studying *nonnested hypotheses tests.*

Over the last fifty years, the statistical methods of the theory of hypothesis testing have considerably developed under the impulse of statisticians such as J. Neyman, E. Lehman and A. Wald. As for the theory of statistics in general, this development has its source in the increasing role of probabilistic modelling as a scientific tool. Another reason, however, which may be more fundamental and specific to the theory of hypothesis testing, is the progressive disappearence of the idea that a model can be validated with certainty on the basis of the data. Such an idea, which was frequently held during the nineteen century, was gradually forsaken especially at the beginning of the twentieth century when physicists started to question the theory of classical mechanics considered up to then as the definitive theory. The relative value of a model in a collection of competing models then proved to be a valuable concept. As a natural consequence, the theory of hypothesis testing, whose main purpose is to arbitrate among models, received an increasing interest.

14.2 Hypotheses

A testing problem is defined by a statistical model $(\mathcal{Y}, \mathcal{P})$ and by a partitioning of the family $\mathcal{P}$ into two subfamilies $\mathcal{P}_0$ and $\mathcal{P}_1 = {}^c\mathcal{P}_0$. These two subfamilies define respectively two *hypotheses* about the true distribution P_0 generating the observations, namely

$$H_0 : P_0 \in \mathcal{P}_0$$

and

$$H_1 : P_0 \in \mathcal{P}_1.$$

It is frequently convenient to identify H_0 with $\mathcal{P}_0$ and H_1 with $\mathcal{P}_1$. Although the two hypotheses H_0 and H_1 play a symmetric role in this section, they are given hereafter two different names: H_0 is called the *null hypothesis* while H_1 is called the *alternative hypothesis.* The union of H_0 and H_1 defines the hypothesis $H : P_0 \in \mathcal{P} = \mathcal{P}_0 \cup \mathcal{P}_1$, which is called the *general* or *maintained hypothesis.*

Definition 14.1: *A hypothesis is called simple if it contains a unique probability distribution. It is called composite otherwise.*

In a parametric model $(\mathcal{Y}, \{P_\theta, \theta \in \Theta\})$ the hypotheses H_0 and H_1 are defined, in general, by two subsets Θ_0 and $\Theta_1 = {}^c\Theta_0$ of Θ. When the model is identified, such a definition of the null and alternative hypotheses is identical to that based on a partition of $\mathcal{P}$ into $\mathcal{P}_0$ and $\mathcal{P}_1$. This is because the mapping that associates P_θ to θ is a one-to-one and onto mapping, i.e., a bijective mapping from Θ to $\mathcal{P}$. When the model is not identified, however, a difficulty arises. Specifically, there may exist values for the parameter $\theta_0 \in \Theta_0$ and $\theta_1 \in \Theta_1$ leading to the same probability distribution, i.e., such that $P_{\theta_0} = P_{\theta_1}$. In this case the subsets $\mathcal{P}_0 = \{P_\theta, \theta \in \Theta_0\}$ and $\mathcal{P}_1 = \{P_\theta, \theta \in \Theta_1\}$ are no longer disjoint.

Definition 14.2: *A testing problem defined by* Θ_0 *and* $\Theta_1 = {}^c\Theta_0$ *is identified if* P_{θ_0} *is different from* P_{θ_1} *for every* $\theta_0 \in \Theta_0$ *and* $\theta_1 \in \Theta_1$.

It is obvious that a model is identified if and only if every testing problem is identified. As Example 14.4 illustrates, however, *some* testing problems can be identified even though the model is not identified.

14.3 Examples

Example 14.1: A machine produces steel balls whose diameters are independently and identically distributed as $N(\theta, \sigma_0^2)$. It is assumed that the accuracy σ_0^2 of the machine is a characteristic known to the investigator and that the mean diameter θ of the produced steel balls is a parameter that can be chosen. One observes n diameters $Y_1, \ldots, Y_n$ and one wishes to test whether the tuning of the machine corresponds to the posted value θ_0.

In this example the statistical model is

$$\left(\mathbb{R}^n, \left\{(N(\theta, \sigma_0^2))^{\otimes n}, \theta \in \mathbb{R}^+\right\}\right).$$

The null hypothesis of good tuning is H_0: $\theta = \theta_0$ and the alternative hypothesis is H_1: $\theta \neq \theta_0$. The null hypothesis is simple while the alternative hypothesis is composite.

Example 14.2: It is assumed that the production Q_t of a given commodity at time t, $t = 1, \ldots, T$, can be modelled by the Cobb–Douglas production function

$$\log Q_t = a + b \log N_t + c \log K_t + u_t,$$

where N_t denotes the quantity of labor input and K_t denotes the quantity of capital input. It is assumed that the random disturbances u_t, $t = 1, \ldots, T$, are independently and identically distributed as $N(0, \sigma^2)$.

The model is parametric. If $\log Q_t$, $t = 1, \ldots, T$, are viewed as the observations, the model is

$$\left(\mathbb{R}^T, \left\{\bigotimes_{t=1}^{T} N(a + b \log N_t + c \log K_t, \sigma^2), (a, b, c, \sigma^2) \in \mathbb{R}^3 \times \mathbb{R}^+\right\}\right).$$

One may want to test the hypothesis H_0 of constant returns, i.e., the property that multiplying labor input and capital input by a same factor leads to multiplying production by this factor. Such a hypothesis is identical to the condition that the Cobb–Douglas production function is homogenous of degree one. In terms of the parameters, this translates into the condition $b + c = 1$. It is clear that the two hypotheses H_0 and H_1 are composite. For instance, H_0 is given by

$$\Theta_0 = \left\{(a, b, c, \sigma^2) \in I\!R^3 \times I\!R^+, b + c = 1\right\}.$$

One may also want to question the hypothesis of independence among the u_t's. A method for dealing with such a "specification" testing situation is to nest the preceding model into a larger model where the disturbances u_t's satisfy the first-order autoregressive process

$$u_t = \rho u_{t-1} + \varepsilon_t, \qquad |\rho| < 1$$

and the ε_t's are independently and identically distributed as $N(0, \sigma_\varepsilon^2)$. The null hypothesis of independence of the u_t's is then characterized by the condition $\rho = 0$.

Example 14.3: A consumption survey provides observations on health expenditures and incomes of n households, (C_i, R_i), $i = 1, \ldots, n$. It is assumed that the pairs (C_i, R_i) are independently and identically distributed with unknown density $f(c_i, r_i)$ with respect to the Lebesgue measure λ_2^+ on $I\!R^{+2}$. The statistical model is defined by the family

$$\mathcal{P} = \left\{\prod_{i=1}^{n} f(c_i, r_i) \cdot \lambda_{2n}^+, \ f \text{ arbitrary on } I\!R^{+2}\right\}.$$

Suppose that one wishes to test whether health expenditures are independent of incomes. The null hypothesis corresponds to the subfamily

$$\mathcal{P}_0 = \left\{\prod_{i=1}^{n} g(c_i)h(r_i) \cdot \lambda_{2n}^+, \ g \text{ and } h \text{ arbitrary on } I\!R^+\right\}.$$

The model is nonparametric. The null and alternative hypotheses are composite.

Example 14.4: At time $t = 1, \ldots, T$, the quantity exchanged Q_t and the price p_t of an agricultural product are determined by the demand equation

$$Q_t = \alpha p_t + \beta x_{t-1} + \gamma z_{t-1} + \delta + u_t,$$

and the supply equation

$$Q_t = a p_t^* + b + v_t,$$

where x_{t-1} and z_{t-1} are some variables treated as nonstochastic, u_t and v_t are zero-mean random errors uncorrelated contemporaneously and over time. The variable p_t^* denotes the producers' expectation at time $t-1$ of price at time t. In addition, it is assumed that this expectation is a function of past exogenous variables and is given by

$$p_t^* = \phi_1 x_{t-1} + \phi_2 z_{t-1} + \phi_3.$$

Using this expression in the supply equation, one obtains

$$\begin{cases} Q_t = \alpha p_t + \beta x_{t-1} + \gamma z_{t-1} + \delta + u_t, \\ Q_t = \beta_1 x_{t-1} + \beta_2 z_{t-1} + \beta_3 + \nu_t, \end{cases}$$

where

$$\begin{aligned} \beta_1 &= a\phi_1, \\ \beta_2 &= a\phi_2, \\ \beta_3 &= b + a\phi_3. \end{aligned}$$

The parameters a, b, ϕ_1, ϕ_2, and ϕ_3 are not first-order identified although the parameters β_1, β_2, and β_3 are.

One wishes to test whether price expectations are "rational," i.e., whether they coincide with the optimal predictions $p_t^* = E_{t-1}p_t$, where $E_{t-1}p_t$ is the conditional expectation of p_t given the variables known at time $t-1$. Taking first the conditional expectation of the demand and supply equations and then, the difference between the resulting equations, one obtains

$$p_t^* = E_{t-1}p_t = \frac{\beta}{a-\alpha} x_{t-1} + \frac{\gamma}{a-\alpha} z_{t-1} + \frac{\delta - b}{a-\alpha}.$$

Thus the null hypothesis of "rational" expectations can be written as

$$H_0 : \phi_1 = \frac{\beta}{a-\alpha}, \quad \phi_2 = \frac{\gamma}{a-\alpha}, \quad \phi_3 = \frac{\delta - b}{a-\alpha}.$$

The hypothesis H_0 is not identified since ϕ_1, ϕ_2, and ϕ_3 are not identified.

The null hypothesis H_0, however, implies

$$H_0^* : \frac{\phi_1}{\phi_2} = \frac{\beta}{\gamma},$$

i.e.

$$H_0^* : \frac{\beta_1}{\beta_2} = \frac{\beta}{\gamma}.$$