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by

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Preface

OVER the past two decades, we have witnessed a rapid development of solid-state technology with its apparently unending proliferation of new devices. In order to cope with this situation, a steady stream of new theory, being general and independent of devices, has emerged. One of the most significant developments is the introduction of scattering techniques to network theory. The purpose of this book is to present a unified and detailed account of this theory and its applications to the design of broadband matching networks and amplifiers. It was written primarily as a late text in network theory as well as a reference for practicing engineers who wish to learn how the modern network theory can be applied to the design of many practical circuits. The background required is the usual undergraduate basic courses in networks as well as the ability to handle matrices and functions of a complex variable.

In the book, I have attempted to extract the essence of the theory and to present those topics that are of fundamental importance and that will transcend the advent of new devices and design tools. The guiding light throughout the book has been mathematical precision. Thus, all the assertions are rigorously proved; many of these proof are believed to be new and novel. I have tried to give a balancec treatment between the mathematical aspects and the physical postulates which motivate the work, and to present the materia. in a concise manner, using discussions and examples to illustrate the concepts and principles involved. The book also contains some of the personal contributions of the author that are not available elsewhere in the literature.

The scope of this book should be quite clear from a glance at the table of contents. Chapter 1 introduces many fundamental concepts related to linear, time-invariant n-port networks, defines *passivity* in terms of the universally encountered physical quantities *time* and *energy*, and reviews briefly the general characterizations of an n-port network. Its time-domain passivity conditions are then translated into the equivalent frequency-domain passivity criteria, which are to be employed to obtain the fundamental limitations on its behavior and utility. Thus, this chapter, as the title implies, may be taken as the foundation for any subsequent network study as well as for the material treated in the remainder of the book.

Chapter 2 gives a fairly complete exposition of the scattering matrix associated with an n-port network, starting from a one-port network and using the concepts from transmission-line theory. Fundamental properties of the scattering matrix and its relation to the power transmission among the ports are then derived. The results are indispensable in developing the theory of broadband matching to be treated in the last two chapters.

In seeking fundamental limitations on network or device behavior, performance criteria are often overly idealistic and are not physically realizable. To avoid this difficulty, Chapter 3 considers the approximation problem along with a discussion of the approximating functions. It is shown that the ideal low-pass brick-wall type of gain response can be approximated by three popular rational function approximation schemes: the maximally-flat (Butterworth) response, the equiripple (Chebyshev) response, and the elliptic (Cauer-parameter) response. This is followed by presenting the corresponding ladder network realizations which are attractive from an engineering viewpoint in that they are unbalanced and contain no coupling coils. Explicit formulas for element values of these ladder networks with Butterworth or Chebyshev gain characteristic are given, which reduce the design of these networks to simple arithmetic. Confining attention to the low-pass gain characteristic is not to be deemed restrictive as it may appear. This is demonstrated by considering frequency transformations that permit low-pass characteristic to be converted to a high-pass, band-pass, or band-elimination characteristic.

Using the results developed in the first three chapters, Chapter 4 treats Youla's theory of broadband matching in detail, illustrating every phase of the theory with fully worked out examples. In particular, the fundamental gain-bandwidth limitations for Bode's parallel RC load and Darlington's type-C load are established in their full generality. The extension of Youla's theory to active load impedance is taken up in Chapter 5. It is demonstrated that with suitable manipulations of the scattering parameters, the theory can be applied to the design of negative-resistance amplifiers. This is especially significant in view of the continuing development of new one-port active devices such as the tunnel diode. Many readers will find the perusal of this chapter to be a gratifying and stimulating experience.

In selecting the level of presentation, considerable attention has been given to the fact that many readers may be encountering these topics for the first time. Thus basic introductory material has been included. For example, since many readers are not familiar with the subject of elliptic functions, in Chapter 3 on Approximation and Ladder Realization, an entire section is devoted to the discussion of elliptic functions and some of their fundamental properties that are needed in subsequent analysis. In fact, the section on elliptic response has never been so concisely and systematically treated elsewhere.

The text has grown out of a graduate course entitled "Linear Network Theory" organized at Ohio University. Over the period of years, the material has naturally evolved and up-dated into a shape quite different from the original. However, the basic objective of establishing the fundamentals in this area has remained unchanged throughout. There is little difficulty in fitting the book into a one-semester, or two-quarter course in linear network theory and design. It can be used equally well as a text in advanced network synthesis. For example, as an advanced text in modern network synthesis, Chapters 2, 4 and 5 plus some sections of Chapter 3 would serve for this purpose. Some of the later chapters are also suitable as topics for advanced seminars.

A special feature of the book is that results of direct practical value are included. They are design curves and tables for networks having Butterworth, Chebyshev or elliptic response. These results are extremely useful in that many of the design procedures may be reduced to simple arithmetic and that they find great use in the conduct of research. For example, it is often necessary to check one's hypothesis by specific examples; here they are ready at hand.

A variety of problems has been given at the end of each chapter, some of which are routine applications of results derived in the text. Others, however, require considerable extension of the text material. In all, there are 271 problems.

Much of the material in the book was developed from my research during the past few years. It is a pleasure to acknowledge publicly the research support of the Ohio University Baker Fund Awards Committee. Thanks are also due to many friends and colleagues who reviewed various portions of my manuscript and gave useful suggestions: among them are Professor M. E. Van Valkenburg of University of Illinois, Professor L. O. Chua of University of California at Berkeley, Professor S. P. Chan of University of Santa Clara, and Professor B. J. Leon of Purdue University. I am also indebted to many graduate students who have made valuable contributions to the improvement of this book. Special thanks are due to Mr. S. W. Leung who plotted some of the gain curves in Chapter 4, and to my doctoral students Dr. S. Chandra who gave the complete book a careful reading and Major T. Chaisrakeo who assisted me in computing the elliptic response as well as in many other ways. Finally, I wish to thank my wife and children for their patience and understanding to whom this book is dedicated.

Athens, Ohio WAI-KAI CHEN

Contents

CHAPTER 1

Foundations of Network Theory

AN *electrical network* is a structure composed of a finite number of interconnected *elements* with a set of *ports* or *accessible terminal pairs* at which voltages and currents may be measured and the transfer of electromagnetic energy into or out of the structure can be made. The elements are idealizations of actual physical devices such as resistors, capacitors, inductors, transformers and generators; and obey the established laws of physics relating various physical quantities such as current, voltage and so forth. Fundamental to the concept of a *port* is the assumption that the instantaneous current entering one terminal of the port is always equal to the instantaneous current leaving the other terminal of the port. A network with n such accessible ports is called an *n-port network* or simply an *n-port*, as depicted in Fig. 1.1. In this chapter, we introduce many fundamental concepts related to linear, time-invariant n-port networks. We first define *passivity* in terms of the universally encountered physical quantities *time* and *energy* and review the general characterizations of an n-port network. We then translate the time-domain passivity conditions into the equivalent frequency-domain passivity criteria, which are to be employed to obtain the fundamental limitations on its behavior and utility.

Since in this book we deal exclusively with linear, lumped and time-invariant n-port networks, the adjectives "linear", "lumped" and "time-invariant" are to be omitted in the discussion unless they are used for emphasis. Much of the discussion and results obtained in the first two chapters are sufficiently general to be applicable to general linear systems.

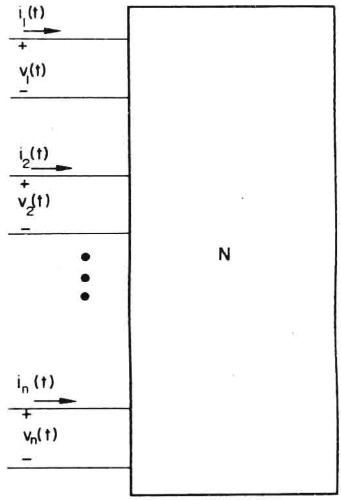

FIG. 1.1. The general symbolic representation of an n-port network N.

1. Basic network postulates

From the historical evolution of network theory, the physical nature of a network can best be described by a set of postulates, which make the theory as simple and as powerful as possible.

Referring to the general symbolic representation of an n-port network N of Fig. 1.1, in which the port voltages $v_k(t)$ and currents $i_k(t)$ can be conveniently represented by the *port-voltage* and *port-current vectors*,

$$\boldsymbol{v}(t) = [v_1(t), v_2(t), \ldots, v_n(t)]', \tag{1.1a}$$

$$\boldsymbol{i}(t) = [i_1(t), i_2(t), \ldots, i_n(t)]', \tag{1.1b}$$

respectively, where the prime denotes the matrix transpose. We say that the two n-vectors $\boldsymbol{v}(t)$ and $\boldsymbol{i}(t)$ constitute an *admissible signal pair,* written as $[\boldsymbol{v}(t), \boldsymbol{i}(t)]$, for the n-port network N. We shall generally be concerned with n-port networks that satisfy the following constraints on $\boldsymbol{v}(t)$ and $\boldsymbol{i}(t)$.

1.1. Real-time function postulate

It simply states that if the excitation signals of an n-port are real functions of time, the response signals must also be real functions of time.

Although there is certainly no such thing as a nonreal signal in the real, physical world, it is important to bear in mind that in network theory we often work with signals that are functions of a complex variable, since the use of these signals has become a convenient artifice in the study of networks. For example, in the steady-state analysis of a one-port whose impedance is $z(s)$, it is customary to employ a voltage excitation $V(j\omega)$. Then according to the postulate, if the voltage signal has the form

$$v(t) = \text{Re } V(j\omega)e^{j\omega t} = |V(j\omega)| \cos(\omega t + \theta), \tag{1.2}$$

where $V(j\omega) = |V(j\omega)|e^{j\theta}$ and Re means the *real part of*, the response current signal must also be a real function of time. In fact, following the usual conventions, the steady-state current is given by

$$i(t) = \text{Re}\left[\frac{V(j\omega)}{z(j\omega)}e^{j\omega t}\right] = \left|\frac{V(j\omega)}{z(j\omega)}\right| \cos(\omega t + \theta - \phi), \tag{1.3}$$

where $z(j\omega) = |z(j\omega)|e^{j\phi}$.

We remark that the complex variable $s = \sigma + j\omega$ is often referred to as the *complex frequency*. With this designation, if we refer simply to frequency, it is not clear whether we mean s or ω. To emphasize the distinction, people often say *real frequency* to mean ω, which is the *imaginary* part of s. The real part σ of s, misleading as it may be, is called the *imaginary frequency*, and was in general use before 1930. Another convention is to name ω *radian frequency* and σ *neper frequency*, thus avoiding the near metaphysical names. But, no matter what we call them, the two components of frequency add together to give complex frequency. For the present, we shall use the term real frequency for ω. When we speak of the real-frequency axis, we mean the $j\omega$-axis of the complex frequency plane.

1.2. Time-invariance postulate

Intuitively, an n-port network N is considered time-invariant if a given excitation produces the same response no matter when it is applied. Formally, we say that an n-port N is *time-invariant* if for every admissible signal pair $[v_1(t), i_1(t)]$ and for every real finite constant τ, there is an admissible signal pair $[v_2(t), i_2(t)]$ such that

$$v_1(t) = v_2(t + \tau), \tag{1.4a}$$

$$i_1(t) = i_2(t + \tau). \tag{1.4b}$$

An n-port that is not time-invariant is called *time-varying*. In other words, an n-port is time-invariant if its terminal behavior is invariant to a shift in the time origin. Thus, if the parameters of an n-port, which is devoid of any initial conditions, are constant then the n-port is time-invariant. The converse, however, is not necessarily true. It is quite easy to conceive of an n-port with time-varying physical elements which exhibits a port behavior that is time-invariant. Figure 1.2 shows a one-port composed of a series connection of

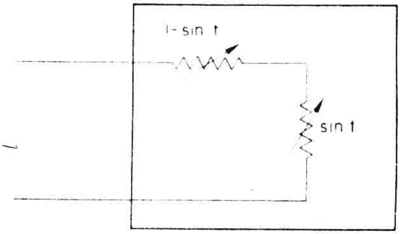

FIG. 1.2. A one-port network with time-varying physical elements which exhibits a port behavior that is time-invariant.

two time-varying resistors, whose input impedance is one ohm. According to the above definition, this one-port is considered to be time-invariant. Suppose, however, that another two-port is formed from this one-port, as shown in Fig. 1.3. This new two-port becomes time-varying. Also, in general, n-ports with initial stored energies that affect port behavior must be considered to be time-varying from the port behavior standpoint.

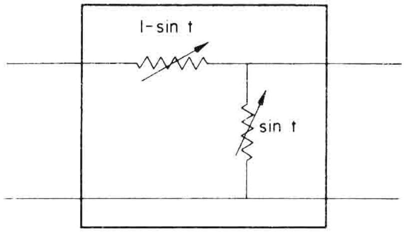

FIG. 1.3. A time-varying two-port network.

1.3. Linearity postulate

Generally speaking, a linear n-port is one in which the response is proportional to the excitation. More precisely, an n-port is said to be *linear* if for all admissible signal pairs

$$[v_1(t), i_1(t)] \quad \text{and} \quad [v_2(t), i_2(t)] \tag{1.5a}$$

and for all real finite constants c_1 and c_2, then

$$[c_1 v_1(t) + c_2 v_2(t), c_1 i_1(t) + c_2 i_2(t)] \tag{1.5b}$$

is an admissible signal pair. In other words, a linear n-port obeys the principle of superposition, and its admissible signal pairs comprise a linear space. Quite often, an n-port is called *nonlinear* if it is not linear. However, we must bear in mind that almost all nonlinear analysis techniques include linear case in their domain of applicability as well. Thus, care must be taken to assure the proper interpretation of the term "nonlinear".

Consider the one-port of Fig. 1.4, in which the capacitor is initially

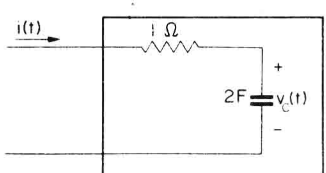

FIG. 1.4. A nonlinear one-port network in which the capacitor is initially charged to a voltage $V_0 \neq 0$.