

The Legacy of John von Neumann

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The Legacy of John von Neumann

James Glimm, John Impagliazzo,
and Isadore Singer, Editors

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Preface

John von Neumann is one of the giants of modern mathematics. His career was marked by the founding and invention of a number of new fields of mathematics and by fundamental contributions to many others. His interests spanned pure mathematics, basic and applied science, mathematical analysis, the foundation of quantum mechanics, and scientific computing.

A conference, cosponsored by the American Mathematical Society and Hofstra University, was organized to study the legacy of John von Neumann. Experts in the many fields that were initiated and influenced by von Neumann met from 29 May to 4 June 1988, at Hofstra University to examine the ideas of von Neumann and to trace these ideas and their development and evolution to the present day. The present volume records some of these lectures and also includes views of von Neumann as a person, as he was known by people close to him. It documents, in a powerful and coherent fashion, a major current in the scientific and intellectual life of our times and the individual who gave it birth.

Von Neumann believed in the power of mathematical reasoning and he believed in its importance to modern civilization. This belief, expressed through his life work, is the legacy of John von Neumann.

The conference was conceived by Marshall Harvey Stone and was his final scientific activity. It is thus fitting to honor Marshall Stone by dedicating this volume to him.

James Glimm
John Impagliazzo
Isadore Singer

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John von Neumann: A Personal View

MARINA v.N. WHITMAN

I must confess, first of all, to considerable misgivings when I was first invited to participate in this symposium. Obviously, I have no competence to comment on the many specialized areas in which my father made noteworthy contributions, subjects that will be covered in varying degrees of detail by many expert specialists during the course of the conference. And as far as personal recollections are concerned, my uncle Nicholas clearly has an enormous advantage, partly because of his chronological proximity to my father and partly because of the intensive study he has made of my father's background and family environment, some of the fruits of which he has just shared with us.

I couldn't quite see what I had to add to these many varieties of expertise, but I finally agreed to participate in the spirit of Woody Allen's famous phrase that "ninety percent of life is just showing up". I didn't want to pass up the opportunity to make whatever contribution I can to this significant occasion, and I've already made ninety percent of it simply by standing here in front of you.

I will spend a few minutes commenting briefly on what, from my observation point, motivated my father. I think what I have to say on that will be very complementary to what Nicholas has already said. And then I will spend a few more minutes on his own view of his intellectual legacy and his concerns—his fears, in fact—about its longevity.

First, as regards motivation, from the vantage point of someone whose adult relationship with John von Neumann was relatively brief but close: Looking back over what has been said and written during the thirty-plus years since his death, it seems to me that the forces that drove him have often been poorly understood, even badly misunderstood. One purportedly serious evaluation, for example, combined pop sociology and Marxist ideology to conclude that von Neumann was driven by a desire for power and a

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fascination with being close to the rich and powerful.

Now, there is no question that my father enjoyed the good life, liked to live well, and counted a number of celebrities among his friends and colleagues. But I believe that his wide-ranging activities, including his worldly involvements as well as his strictly intellectual pursuits, were motivated by two profound convictions. The first was the overriding responsibility that each of us has to make full use of whatever intellectual capabilities we were endowed with. He had the scientist's passion for learning and discovery for its own sake and the genius's ego-driven concern for the significance and durability of his own contributions. And the second was the critical importance of an environment of political freedom for the pursuit of the first, and for the welfare of mankind in general.

I'm convinced, in fact, that all his involvements with the halls of power were driven by his sense of the fragility of that freedom. By the beginning of the 1930s, if not even earlier, he became convinced that the lights of civilization would be snuffed out all over Europe by the spread of totalitarianism from the right. So he made an unequivocal commitment to his home in the new world and to fight to preserve and reestablish freedom from that new beachhead.

In the 1940s and 1950s, he was equally convinced that the threat to civilization now came from totalitarianism on the left, and his commitment was just as unequivocal to fighting it with whatever weapons lay at hand, scientific and economic as well as military. It was a matter of utter indifference to him, I believe, whether the threat came from the right or from the left. What motivated both his intense involvement in the issues of the day and his uncompromisingly hardline attitude was his belief in the overriding importance of political freedom, his strong sense of its continuing fragility, and his conviction that it was in the United States, and the passionate defense of the United States, that its best hope lay.

Now, a little bit about John von Neumann's legacy, from the vantage point of his daughter and only child: In particular, I will focus on his concerns during the last year or two of his life. Especially toward the end, but even before, he became deeply concerned about the question of his ongoing legacy. He was concerned partly with immortality in the personal or religious sense, but that is a topic for another day.

He was also profoundly concerned, however, with the nature of his legacy in this world, in two respects. One had to do with the durability of his work, his intellectual contributions; he was surprisingly insecure about that. And, interestingly enough, I don't think he was a very accurate prophet regarding what turns the practical applications of his pioneering work would take. For example, he clearly expected that the computer would have its impact primarily on scientific research and military work. He was particularly interested in its role in advancing the accuracy of weather forecasting and, ultimately, climate modification. I don't think progress in this area has been nearly as

far or as fast as he hoped and expected. Similarly, I think he anticipated that the theory of games would have more impact, to borrow John MacDonald's book title, on poker, business, and war than it has turned out to have had so far, at least.

On the other hand, if anyone had ever told him that the company I work for, General Motors, would produce and utilize literally millions of computers every year (each of the roughly eight million vehicles we produce each year contains several, not to mention the ones in our plants and offices), I think he would have been startled. And the notion of adults fulminating against computers as corrupters of youth in the form of video games would have amused and perhaps secretly pleased the playful, childlike aspect of his personality.

In fact, my father foresaw the inadequacy not only of his own forecasts but of such forecasts in general. In the 1955 *Fortune* article my uncle Nicholas quoted a few minutes ago, he said: "All experience shows that technological changes profoundly transform political and social relationships. Experience also shows that these transformations are not a priori predictable, and that most contemporary first guesses concerning them are wrong."

The second focus of John von Neumann's concern about his earthly legacy was, to put it succinctly, me. I was his only offspring and, toward the end of his life, he became acutely conscious that all his eggs were in one basket, genetically speaking (if biological inaccuracy can be forgiven for the sake of metaphor). So he put tremendous pressure on me to perform up to the peak of my abilities, and made clear his displeasure with the path I appeared to be taking. I had married young, right out of college, and he thought that this was a bad beginning, not because he was not quite fond of my husband, but simply because he feared (and it was a reasonable fear, in the 1950s) that a woman who married young was very probably reducing her chances of making a significant intellectual or professional contribution. And, as Nicholas has pointed out, he tended to take a somewhat statistical view of the world.

Statistically he was right, of course, but I like to think that in this particular case he was wrong. I'm no John von Neumann, obviously, but I have had a reasonably successful and highly rewarding career as an academic economist, a presidential adviser, and now as a corporate executive.

And beyond me is the next generation, the grandchildren whose accomplishments he couldn't foresee because he died far too early, before they were born or even contemplated.

I'm deeply sorry that he never got to know the grandson who has translated a six-year-old's dream of "someday finding a cure for cancer" into a career as a molecular biologist doing research on the chemistry of intercellular message transmission. He would have found comforting continuity, I think, in the fact that Malcolm and his colleagues are exploring mysteries of the universe as fundamental as many of those that preoccupied my father, as well as because the work that they are doing is in some sense a fitting sequel to von Neumann's

own explorations, culminating in his posthumously-published *The computer and the brain*, into relationships among mechanical, electrical, chemical, and biological processes.

I'm equally sorry that he could never know the granddaughter who, having avoided scientific subjects throughout her university career, realized after graduation that her newly-focused goal of radically improving health care would require medical training and that that, in turn, would require filling the substantial gap between ninth-grade biology and premedical requirements. She filled the gap and is now a medical student, firm in her belief in the chemical-biological unity of mind and body and determined to fashion a career which will harness her skills in the service of *mens sana in corpore sano* for those whose access to such services is limited by accidents of geography or socioeconomic situation.

John von Neumann would have felt reassured and gratified, I believe, by the choices his progeny and his progeny's progeny have made, to do what he considered most important, that is, to utilize our intellectual capacities right up to their limits, to try to achieve whatever potential we have. He would have been relieved and gratified that the forces of light appear to be holding their own, perhaps even gaining, against the forces of darkness in the sphere of political freedom. He would have been surprised and perhaps amused, as well as disappointed in some areas, at the twists and turns his contributions have taken in affecting our everyday lives. He would have felt reassured to know that we are all still here to talk about such matters in the year of our Lord 1988, given the fears he expressed in the article Nicholas quoted that mankind might not make it to 1980. And he would be tremendously gratified to know that the intellectual leaders gathered here are devoting their Memorial Day weekend, as well as the week that follows, to a discussion of the legacy of John von Neumann.

VICE PRESIDENT AND GROUP EXECUTIVE, GENERAL MOTORS CORPORATION

Remembering John von Neumann

PETER D. LAX

I am just about the youngest member of the generation that had direct scientific contact with John von Neumann. I am here to give an account of some of the things von Neumann said, and some of the things said about him.

When von Neumann showed unusual mathematical talent at age 15 his parents arranged for him to be tutored by Gábor Szegő. Mrs. Szegő often recalled that Szegő came home with tears in his eyes from his first encounter with the young prodigy. The brilliant solutions to the problems posed by Szegő, written on the stationery of his father's bank, can be seen in the von Neumann Archives in Budapest. After Szegő left Budapest, von Neumann's mathematical education was taken over by Michael Fekete; this resulted in a joint publication in 1922 on the Transfinite Diameter, a subject to which Fekete subsequently devoted most of his scientific life.

The work which first brought von Neumann center stage was his theory of selfadjoint operators, about which Marshall Stone spoke at this conference. The following story illustrates well the originality of von Neumann's approach: In the thirties Erhardt Schmidt asked the young Franz Rellich to describe his recent research in Hilbert space theory. Rellich was happy to oblige and started in the now standard fashion: "Let H be a Hilbert space and L a linear operator..." at which Schmidt interrupted him: "Please, young man, say infinite matrix." The success of von Neumann's approach hinged on giving up the notion of operators as infinite matrices and substituting a geometric view. Von Neumann retained a lifelong interest in the theory of selfadjoint operators; I recall how pleased and excited he was in 1953 when he learned of Kato's proof of the selfadjointness of the Schrödinger operator for the helium atom.

Von Neumann combined, in a unique fashion, extreme quickness, very broad interests, and a fearsome technical prowess. The popular saying was:

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“Most mathematicians prove what they can, von Neumann proves what he wants.” Once in a discussion about the rapid growth of mathematics in modern times, von Neumann was heard to remark that whereas thirty years ago a mathematician could grasp all of mathematics, that is impossible today. Someone asked him: “What percentage of all mathematics might a person aspire to understand today?” Von Neumann went into one of his five-second thinking trances, and said: “About 28 percent.”

It is hard to believe today that von Neumann’s great versatility led some pure and puritanical mathematicians to charge that he had merely skimmed the cream off subjects, and left to others the arduous task of getting to the bottom. The story is told of the time when the mathematics faculty of the Institute for Advanced Study was discussing the appointment of Enrico Bombieri. His great contributions to function theory, calculus of variations, minimal surfaces, and algebraic number theory were described; everyone was favorably impressed, except one distinguished professor who commented: “This fellow dabbles in this, dabbles in that; he will end up like von Neumann.”¹

Von Neumann was addicted to thinking, and in particular to thinking about mathematics. Eisenhower surmised that his legendary quickness was enhanced by his having thought before about a great many questions. The obverse side of his compulsive thinking was a boredom with most mathematical discoveries except those that had something very surprising about them.

During and after the Second World War, von Neumann took on many official duties, time-consuming and bearing a heavy burden of responsibility. He did this not only because of a sense of obligation, but also out of a deep desire to leave his mark on the world, not only on the world of mathematics. About 15 years ago Eugene Wigner visited Budapest and received a celebrity treatment, including two long interviews on television. He was asked if it were true that in the late forties and early fifties the scientific policy of the U.S. Government was set by von Neumann. Wigner replied with his characteristic precision: “That is not quite so; but after von Neumann analyzed a problem, it was clear what had to be done.”

His official obligations left less time for scientific work, if not for the doing of it, for recording it. He was willing to lecture on his work and ideas, but didn’t have the leisure to write them up for publication. He didn’t have much time for preparing them either; but so great was his power of concentration that his lectures were models of good organization and his sentences grammatically resplendent, with a slight tendency toward germanic length. After one of the rare occasions when he couldn’t finish a proof he remarked ruefully: “I knew three different ways of proving this result, but unfortunately I chose a fourth way.”

¹ It was recently pointed out by someone present that the second part of the statement belongs to the *ben trovato* category.

Problems of computing in science, the design of algorithms, the design of computing machines, and the structure of programming languages fascinated von Neumann endlessly. I heard him recall how, early in his forays into programming, he spent much time proving that every program that could be coded in one particular language could also be coded in another particular language. "Only later did it occur to me," he said, "that I was merely reproving a special case of Turing's theorem."

It is fashionable these days to talk about non-von Neumann computers, meaning ones that process in a parallel rather than serial manner. This ever so slightly pejorative designation is unfair to von Neumann; at the time he formalized the concept of a computer, the state of technology pointed to a serial design. There is little doubt that today, when sophisticated chips are abundantly available at low cost, von Neumann would have favored a parallel design. It is a backhanded compliment to his intellectual prestige that those who propose to change the paradigm from serial to parallel behave as if they were committing symbolic patricide.

The tragedy of von Neumann's early death robbed mathematics and the sciences of a natural leader and an eloquent spokesman, and deprived a whole younger generation of beholding the most scintillating intellect of this century.

COURANT INSTITUTE OF MATHEMATICAL SCIENCES, NEW YORK UNIVERSITY

Nonmathematical Reminiscences about Johnny von Neumann

FRANÇOISE ULAM

As I am not a mathematician, it was as a sort of camp follower that I attended a few such gatherings in the past. It is a new experience for me to be asked to participate, so I shall begin these after dinner reminiscences with a few simple vignettes:

I never saw Johnny other than properly dressed in a business suit and tie, no matter what the occasion. I have at home, and I expect most of you have seen, the photo of Johnny and Klari at the Grand Canyon, posing in a group of riders, before the descent to the bottom. Johnny is the last one, hatless but fully dressed in coat and tie, on the only mule facing the wrong way!

For a nonathletic, nonoutdoorsy person he could surprise you sometimes, as when he joined the Lake Peak hike Edward described this afternoon—that was a great summer favorite of the Fermis and Bethes. That memorable day I noticed he wore his usual city clothes!

More seriously, in Los Alamos he also worked! I can still see him in the coffee room of T-division covering a blackboard with the first stirrings of flow-diagram coding, while casting unconscious sidelong glances at every feminine pair of legs that passed by! His lines and arrows, instructions and addresses boggled my nontechnical mind, so I chose to stay with one of the old mechanical machines on which Stan and Everett's simplified H-bomb calculations were made as others prepared the more detailed Von Neumann-Evans trial runs while they waited for one of the two Maniacs Edward described to become operational.

Speaking of the Maniac, my ten-year-old granddaughter was amazed when I told her that the ancestors of her pocket calculator used to fill a room, and that the programs it contains are derived from those first created by a friend of her grandfather—the very programs just mentioned, that were born of the need to calculate whether it was possible or not to build an H-bomb, on

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the subject of which I felt Johnny was more in tune with Edward than with Oppenheimer, though he defended the latter eloquently during the so-called Affair.

I first met Johnny in 1941 when Stan introduced me to him, at a mathematical meeting in Chicago shortly after we were married. That morning the mathematicians had been offered, for distraction, a slaughterhouse tour. Of course, Johnny had gone along! (Stan and I more cowardly passed this up.) I say "of course" because it was not long before I became aware that he seemed to be always willing to go wherever the action was! He joined us after the tour, pale and shaken, and admitted that it had been a rather gory trip.

There were but few other times when I saw him visibly upset and not his usual composed self. One, when he came to us immediately upon returning from the Alamogordo bomb test. Another, on what was to be his last visit to Los Alamos. He already knew the nature of his illness, and did not look well at all. But at our house for lunch, he would not tell Stan what was wrong. He wanted, I think, to spare him.

At our first meeting I had been struck by the intensity of his dark, dark eyes and his cheerful rotundity. I told Stan his appearance reminded me of Riquet à la Houppe, a short, roly-poly knight in a French fairy tale. Stan must have repeated this to him for in a following math letter he took time to remark: "My respectful homage to Françoise... God knows that my similarity to Riquet à la Houppe... is rigorously exterior. I am a miserable sinner, and I never rescue maidens... But her high and utterly undeserved consideration thrills me to the core of my being." I was amused by the impeccably polite way in which he was making fun of me! and properly impressed that his encyclopedic knowledge included the chubby Perrault knight.

Our acquaintance began in earnest during his and Klari's visits to Los Alamos. It is in that context that I got to know him, more often than not in association with Klari, who became a close friend. As we lived a stone's throw from their quarters at the Lodge it was easy for them to walk over after work, for a meal, or a drink, or parties on our lawn, or other entertainment their visits always generated. For in addition to being a hard worker, Johnny seemed to be a *bon vivant* par excellence, always ready, for instance, for a wild drive to neighboring Spanish cafés in search of the peppery enchiladas he loved. Stan said they probably reminded him of Hungarian goulash!

I used to marvel how anyone could be so sociable and still do creative work, until I realized that you, mathematicians, live a double life. Even when you are here, with us, on the earth, your eyes have a look that shows you are cerebrating! "Thinking!" you call it. I suppose that Johnny's mind was always tuned to the far reaches of abstract cogitations and it did not matter whether he was at work or play.

Play was when on Christmas Eves he and Klari would come to help us put together our daughter's unassembled toys. Can you see us four on the floor, struggling with the parts of a dollhouse? Johnny and Stan could never figure out how to insert tab A into slot B!

Play was when we went with friends to the banks of the Rio Grande for candle-lit dinners at Edith Warner's adobe house, which had no running water or electricity! This Easterner in mocassins cooked on a wood stove, and her meals were served by her long-braided Indian friend, Tilano. Johnny appreciated, as we did, the contrast of their timeless Indian life with the 20th century reality "on the hill".

One year we were invited to the von Neumanns in Princeton for a family celebration. Johnny and Klari's mothers were there, Michael and Nikki, too, and a number of Princeton friends. Mrs. von Neumann, Sr. seemed definitely the respected matriarch, and Johnny, the head of the clan and a devoted family man. The same man who could solicitously inquire about Claire's measles in a letter full of math.

This brings me to what I want to mention next, namely Johnny's correspondence with Stan. After Johnny died, Stan asked me to gather all the Johnny letters I could find in his files. As he never threw a letter away there were quite a few, many in Johnny's clear long hand on an assortment of ocean liners, hotels, or university stationery that attest to his peripatetic existence. They span most of his American life from 1935 to his death in 1957.

Needless to say, they are first and foremost mathematical but they are also a wonderful echo of their non-ivory-tower, non-absent-minded-professor conversations. Rational, more intellectual than affective, laced with touches of irony, they alternate math with gossip, politics, and family news. I have a feeling, as when they talked, that this constant switching of subjects provided them with a necessary release from the tension of abstract thought.

I'd like to give you a few samples of the tone and tenor of their more mundane parts. They show Johnny better than I can.

The earliest prewar ones are full of gaiety and parties: "Do come and stay as long as possible... the Robertsons are yelling for caviar!" "Come in time to make the Harvard-Princeton game... I have already tickets for both of us." Can you imagine Stan and Johnny at a football game? Do you suppose they talked mathematics and exchanged unprintable jokes the whole time?

Then as the clouds of war break over Europe and finally engulf the world, the tone, the preoccupations, change: "I have been made a member of the War Department's 'Scientific Advisory Committee to the Ballistics Research Laboratory' at Aberdeen... I have been put on the 'War Preparedness Committee' of the AMS+AMA too. The functions of all these sets, and sets of sets are not very well defined as yet, but I suppose that they will be when 'The Day' comes around..." (of the U.S. entry into the war).