Phase-Lock Basics

Second Edition

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PREFACE TO THE SECOND EDITION

This is the second edition of a book about the basics of phase-locked loops (PLLs). Basics tend to be unchanging, but we can learn more about them and their manner of presentation can be improved and tuned to current practice.

Loop filters driven by current sources are now so common that I have given them a more prominent place, allowing them to lead into the study of filters that use op-amps, rather than the reverse. The discussion of various applications, at the end of Part 1, which is designed to show how the basics can be applied to a wide variety of circuits, as well as to broaden the overview, has been expanded to include the now common use of PLLs for timing adjustment in ICs. This discussion includes the delay-locked loop, which is not a PLL but whose common forms can still be analyzed with the basic techniques that are used for PLLs. Frequency synthesis is another important application to which the basics can be applied, and I have broadened that description by adding material on modern fractional-N and sigma–delta techniques.

The most significant change, however, is in the discussion of the operation of PLLs in the presence of large amounts of noise (including the discernment of what amounts are large). The breakthrough that makes the improvements possible is the wide use of simulations with MATLAB®, simulations that the reader can repeat and extend.

The first edition progressed into ever-increasing noise levels by moving from linear formulas, through approximations, to a collection of experimental or simulated results from the literature. It was difficult to know when the approximations had been taken beyond their limits of applicability, and only a limited amount of data is available from the literature. The second edition uses these available results to verify the simulations but is then not limited to the conditions for which results were available. This changes the process of studying nonlinear performance to such a degree that we now begin with the simulations, learning how the loop operates under whatever conditions are of interest, and then progress to useful approximations whose limits are now clear. The approximations are still useful—a formula has advantages over a collection of data—but they are more useful when we know their limits of applicability.

Simulations have also permitted further investigation and a clearer understanding of the limits of sweep speed in noise.

The new process has led also to a clearer understanding of the theory, including a better understanding of what defines broadband noise and how bandwidth affects the results. Simulations also permit the study of narrowband noise, which is the more fundamental process, of which broadband noise is only an approximation in the limit. A chapter has been added on narrowband noise.

In order to keep the book to a desirable size in spite of the additional information, some material, primarily appendices, has been put online at ftp://ftp.wiley.com/public/sci_tech_med/phase_lock. This sometimes provides a benefit of multicolor graphs, which can be easier to read. The numbers of a section that is online may be preceded by the letter "i" to make that clear. The problems and problem answers have also been placed on the Wiley ftp site.

WILLIAM F. EGAN

Cupertino, California
May 2007

PREFACE TO THE FIRST EDITION

The phase-locked loop (PLL) has become so important in electronics over the past three decades that a working knowledge of its principles is an essential asset for many electrical engineers. This book is here to help you gain that knowledge. It concentrates on achieving an understanding of basic principles that are applicable to a wide range of PLL circuits—old, new, and yet-to-be-invented.

In 1977 I was given the opportunity to teach a graduate course, in the Santa Clara University School of Engineering, on the design of phase-locked frequency synthesizers, an area I had been working in for about fourteen years. The text for that course was my notes, which I later expanded into *Frequency Synthesis by Phase Lock*, published by John Wiley and Sons, Inc. in 1981.

In 1985, Professor Timothy Healy asked if I would teach a two-quarter PLL course, one that he had developed and taught for several years. I have done so since then. I used Blanchard's *Phase-Locked Loops* for a text, but I had to develop problems as well as course notes and I engaged in an ongoing study of original references. By 1991 I had introduced enough new and modified material that I decided to supplant Blanchard with my own text, but I thought I would begin gradually by providing the text for the first quarter only. However, before the end of the quarter, I was surprised (shocked?) to find that Wiley had stopped publishing Blanchard and it would not be available. This resulted in a rather intensive effort that produced the rest of my text in time for the second quarter. I have been using my own text since that time, modifying it each time (every other year) based on student comments and my own re-readings and to introduce new material.

In 1995 I submitted the latest version to Wiley as part of a proposal to publish. As was true with my previous book, the editor had the manuscript reviewed by a knowledgeable professional. In this instance, the reviewer was Professor William Tranter, currently of Virginia Tech, who suggested that I introduce a software component to the book, especially since my biography showed experience in developing computer programs related to PLLs. He recommended MATLAB® for consideration. Since I was then prepared to immediately publish a text that had been carefully refined, I was reluctant to modify it. Nevertheless, I began to experiment with MATLAB and, as I did my enthusiasm grew. Here was an opportunity, using the student edition of MATLAB* on a personal computer, to significantly enhance the learning of PLL fundamentals. It was almost as though the reader would now have an opportunity for hands-on experimentation, being able to modify parameters and observe the effect on loop performance. In addition, MATLAB could provide useful design aids and even open some important areas where analysis is very difficult without a computer.

As a result, I have included MATLAB programs (scripts) and other software in this book in a way that does not detract from the original text, placing them in appendices following the chapters where the corresponding material is discussed. Many of the programs are written explicitly in the book, and they, and others, are available for downloading from a Wiley Internet site. That site also provides other aids in the form of Excel spreadsheets. The MATLAB and Excel files can be read by Macintosh® and IBM®-compatible computers, at least. One other very important benefit of the Internet site: the inevitable errata can be provided and updated.

The software is relatively easy to understand and customize to better suit an engineer's particular analysis or design problem. It adds an additional dimension of instruction by illustrating the use and design of computer-aided engineering (CAE) for PLLs; the simulation programs should be especially instructive in this regard. I have found that new ways to use the software to improve my understanding of the design process continue to occur to me. One must limit the volume of such material in a book, but, when one becomes familiar with the methods by which the software can be used, one is able to apply it as the situation demands.

Phase Lock Basics can be used in many ways by both the professor and the self-directed student. One can concentrate on the development of the theory or on the examples or work the problems, all of which have answers in the back, or can experiment with the MATLAB programs. Of course, a combination will give the most complete learning experience. That experience will also depend on the reader's background. For the graduate engineer, this material not only adds important specific knowledge but it can serve to integrate and solidify previously learned fundamentals, showing how the theory is used in the design of a particular kind of circuit, the PLL. For those lacking some of the desirable background, results, in the forms of equations, graphs, and descriptions of techniques, are still there. And, for you fortunate designers who already possess an understanding of PLLs, the many graphs and the software can help make your work more productive and, I hope, enjoyable.

*The Professional Version of MATLAB, with appropriate toolboxes, may also be used. Those who cannot gain access to either, but can use a program with similar capabilities, may be guided by the text form of the scripts. However, it will be considerably easier to use some version of MATLAB directly.

The text concentrates on the second-order loop. Loops of any greater complexity have too many parameters to permit general discussion. The standard parameters for second-order systems are natural frequency ω_n and damping factor ζ. Commonly, graphs are provided for various kinds of loop responses, in which the x axis is normalized to ω_n and multiple curves are given for various common values of ζ, enabling the reader to denormalize and interpolate to find results appropriate to a particular problem. There is one other necessary parameter, however, which I call α. For practical reasons (too many parameters), when plots of loop responses are given, they must be restricted to a few values of α. Other texts have generally given response curves for type 2 loops, those employing an integrator, and sometimes for loops with low-pass filters. These loops correspond to $\alpha = 1$ and $\alpha = 0$, respectively, but a continuum of useful loop filters exists between those extremes, and I show how transient response curves representing the extreme values of α can be combined to give the response for any value of α. MATLAB can generate responses, in both the time and frequency domain, for any value of α.

Many practical loops are second-order but some are more complex. Often the more complex loops can be approximated as second order so the second-order curves still give valuable information. I show how some of these curves are affected by the introduction of an additional pole, as is common, for example, in frequency synthesis. To illustrate how all the curves are affected by all the possible additional pole frequencies would, again, require too many dimensions to be practical. However, there is a common third-order configuration that can be analyzed with relative ease, and I have provided this analysis and corresponding curves in an appendix to Chapter 10. (I have placed this in an appendix to not detract from the concentration on loops of second order, on which the main development of the theory is based.) One of the great values of the state-space method, and the MATLAB programs that are based on it, is that it can handle higher order loops so, while one learns the general characteristics of PLLs using the more easily handled second-order theory, one can also expand to more complex problems with the help of MATLAB.

Chapter 10 introduces various applications of PLLs and shows how the theory developed in the previous chapters can be applied to them. This is not a substitute for detailed study in particular areas—there are many other things required for a good knowledge of synthesizer design, for example—but does show how the fundamentals apply. Part 2 of the text introduces the effects of noise, often large amounts of noise, on the performance discussed in Part 1. I have found that Part 1 is a good prerequisite for my synthesizer course, reducing the amount of material that has to be absorbed there. The whole book is an even better preparation, since noise concepts are also very important in synthesizer design. However, the noise levels are relatively low in synthesizer design, its elimination being an important goal, whereas communications systems may be designed to operate in the presence of large amounts of noise, such as are treated in Part 2. Thus this book would also be good preparation for a more advanced study of communication systems that depend on PLLs (coherent communications) and must operate in the presence of noise.

It is not uncommon for students, and designers presumably, to make 2π errors in computing loop bandwidth and other parameters, due to the prevalence of both radians and cycles as units of phase (and in units of frequency). By handling units in a certain way, we are able to employ whichever is more convenient and to freely mix them without succumbing to this error. Proper handling of units is also a necessity in development of the equations. This easy movement between units decreases the importance of differentiating between f and ω symbols (although we will attempt to use the more appropriate symbol), especially since we can equate a frequency in one unit to the equivalent in the other.

While the answers provided for all the problems are of obvious help to the reader studying on his or her own, instructors may prefer problems that do not have answers. I suggest that these can be developed by modification of parameters given in the problems. The devoted but uncertain student can then verify his or her problem solving using the parameters in the book before changing to the parameters specified for the homework. The answers in the back of the book should be highly accurate since most have been proved by use. I have taught electrical engineering graduate students using nine two-hour classes plus two periods of exams for each half of the text. However, the software has not been employed in these courses nor has some other new material.

I would like to thank Professor Healy and Santa Clara for the opportunity to teach the Phase-Locked Loops course over the years and the students who have taken it for the contribution they have made, by interaction in the classroom and through their homework as well as by their suggestions and comments, in the development of the material in this book. I hope it will help you to better understand and appreciate the vast and fascinating world of phase-locked loops.

WILLIAM F. EGAN

Cupertino, California
June 1998

SYMBOLS LIST AND GLOSSARY

$\Rightarrow$	imply, become, go to
$\approx$	approximately equal to
$\equiv$	identically equal to, rather than being equal only under some particular condition
$\triangleq$	is defined as
AGC	automatic gain control
AM	amplitude modulation
BM	balanced mixer
B_n	noise bandwidth
C	controlled signal in standard control system terminology; fed-back input to the summing junction
c	cycle
closed-loop transfer function	C/R with the loop operating
damping factor	ζ, a standard parameter of the second-order system, Section 4.2
dB	decibel, 10 $\log_{10}$(power ratio)
dBc	decibels relative to carrier power, i.e., 10 $\log_{10}$(power/carrier power)
dBc/Hz	ratio of sideband power density, per hertz bandwidth, to total signal power; decibels relative to "carrier" per hertz noise bandwidth
dBm	decibels relative to 1 mW
DBM	doubly balanced mixer
dBr	decibels relative to 1 rad^2

dBr/Hz	phase noise power spectral density in decibels relative to 1 rad^2 per hertz bandwidth
dBV	decibels relative to 1 volt
DDS	direct digital synthesizer
DLL	delay-locked loop
DPLL	digital phase-locked loop
E	error signal in standard control system terminology; output from the summer
error	output from summer in standard control system terminology
ExOR	exclusive-OR function, equal to one if and only if the two inputs differ
$E[y]$	expected value of y
$E[y^2(\,)]$	expected value of y^2
$\overline{F}$	mean frequency or skip rate (in Hz)
frequency power spectral density	$S_f(\omega_m)$, mean square frequency σ_f^2 in a narrow frequency band divided by the bandwidth. The band is narrow enough that the value of $S_f(\omega_m)$ does not change appreciably when the bandwidth decreases
$F(s)$	frequency sensitive part of filter transfer function, which is $K_{LF}F(s)$
f_x	see ω_x for any x
F_x	see Ω_x. for any x
G	transfer function
$\|G\|$	gain (absolute value of G)
gain margin	reciprocal of open-loop gain at frequency at which open-loop phase has 180° excess phase. The gain increase necessary for oscillation of the loop
G_F	forward transfer function of the loop
G_R	reverse transfer function of the loop (1 except in Chapter 10)
H	closed-loop transfer function, C/R in Fig. 2.5
hold-in	range $\pm\Omega_H$ of mistuning Ω over which lock can be maintained as the loop is tuned (Fig. 8.2)
IC	integrated circuit
ICO	current-controlled oscillator
$\mathbb{Im}$	imaginary part of
integrator-and-lead	filter of form $k(\omega_z + s)/s$
I_v	modified Bessel function (first kind) of order v
J_v	Bessel function (first kind) of order v
$K = K_p K_{\mathrm{LF}} K_v$	s × [open-loop gain at DC] (∞ for a type 2 loop)
$K' = K'_p K_{\mathrm{LF}} K_v$	maximum value of K (as the phase changes)
K_{LF}	DC gain of the loop filer (∞ in a type 2 loop)
K_p	phase-detector gain, change in voltage divided by change in phase
K'_p	maximum K_p (maximum as phase changes)
K_{pd}	K_p for coherent detector

K_v	VCO gain, change in VCO frequency divided by change in tuning voltage
L	single-sideband power spectral density (noise) relative to carrier
$\mathscr{L}$	single-sideband power spectral density under small modulation index. When due to phase modulation, always equals $S_\varphi/2$
$\lim_{a\to b}$	limit as a approaches b
low-pass filter	filter with transfer function of the form $K_{\text{LF}}/(1+s/\omega_p)$
L_φ	single-sideband power spectral density due to phase noise
m	modulation index, peak phase deviation
mistuning	$\pm\Omega$: difference between the input (reference) frequency and the VCO center frequency (Fig. 8.2)
n'	phase equivalent to voltage at PD output due to additive noise
natural frequency	ω_n: a standard parameter of the second-order system
NCO	numerically controlled oscillator
N_i	input noise power to a limiter
N_0	one-sided noise power spectral density
OA	output accumulator
one-sided spectrum	representation of power spectral density in which all power is shown at positive frequencies
op-amp	operational amplifier
open-loop transfer function	transfer function around the loop as if it were broken to allow the response of the opened loop to be measured
P	signal power
P_c	carrier power; main signal power
PD	phase detector
phase margin	additional phase lag to cause the loop to oscillate, to have -360° at the frequency at which open-loop gain is unity
phase power spectral density	$S_\varphi(\omega_m)$, mean square phase σ_φ^2 in a narrow frequency band divided by the bandwidth. The band is narrow enough that the value of $S_\varphi(\omega_m)$ does not change when the bandwidth decreases
PLL	phase-locked loop
PM	phase modulation
P_o	total output power from a limiter
power spectral density	power per unit frequency, the limit (at a given frequency) of the ratio of power to bandwidth as bandwidth approaches zero
PPSD	phase power spectral density (which see)
PSD	power spectral density (which see)
pull-in	process of acquiring lock. Range $\pm\Omega_{PI}$ of mistunings Ω over which lock can be acquired (Fig. 8.2)

$P_y(f')$	power spectral density of the parameter y at the frequency f'
Q	quality factor, (energy stored)/(energy dissipated in a cycle), a measure of the sharpness of a resonance curve
r	ratio of the video-equivalent noise bandwidth of the RF filter (half of the actual noise bandwidth) to the noise bandwidth of the loop. Also Eqs. (8.61), (10.A.6)
R	normalized sweep rate, $\|d\Omega/dt\|\text{rad}/\omega_n^2$ (see Table 9.1)
R	reference signal in standard control system terminology; independent input to the summer
rad	radian
rad^2/Hz	a measure of PPSD, the density that produces a phase variance of δ rad^2 in a narrow bandwidth δ Hz
$\mathbb{R}e$	real part of
reference	input to standard control system
r_T	relative threshold, ratio of threshold voltage to maximum signal available from a detector (Section 9.5.2)
s	the Laplace variable, $\sigma + j\omega$
S	demodulated signal power
SBM	singly balanced mixer
sec	second
seize	range $\pm\Omega_s$ of mistuning Ω over which the loop will acquire lock without skipping a cycle (Fig. 8.2)
$S_f(\omega_m)$	frequency power spectral density (which see) at frequency ω_m
S_i	input signal power to a limiter
S/N	signal-to-noise (ratio), sometimes specifically at the output rather than input, also called ρ
spectrum analyzer	an instrument that displays power vs. frequency by measuring power passing through a filter whose center frequency is swept in synchronism with the abscissa of the display
$S_y(f')$	power spectral density of y at frequency f'
S_φ	phase power spectral density {which see}
$S_{\varphi n}(\omega_m)$	phase power spectral density of noise at frequency ω_m
$S_{\varphi s}(\omega_m)$	phase power spectral density of signal at frequency ω_m
S_ω	frequency power spectral density (which see)
T_m	mean time to first cycle skip
T_{PI}	pull-in time
two-sided spectrum	representation of power spectral density in which half of the power is shown at positive frequencies and half is shown at negative frequencies
u_1	phase-detector output voltage or current
u_2	VCO tuning voltage or current

v — subscripts indicate responses to noise at VCO input

V_T — threshold voltage

V_v — VCO signal amplitude

W — RF bandwidth (one-sided)

α — parameter in the numerator of the second-order response, Eq. (6.4). 0 for lag filter; 1 for integrator and lead, otherwise

$$\alpha \equiv 1 - \frac{\omega_n}{2\zeta K} \equiv \frac{1}{1 + \omega_z/K} \equiv \frac{1}{1 + \omega_z\omega_p/\omega_n^2} \quad (6.5)$$

γ — radius of gyration of a noise band, Eq. (15.41)

γ_d — factor in the S/N of frequency discriminators that differs for the various kinds (Section 15.6)

$\Delta\varphi$ — $\varphi_{in} - \varphi_{out}$

$\Delta\omega$ — $\omega_{out} - \omega_{in} = -\omega_e$

ζ — damping factor, a standard parameter of the second-order system

η — suppression factor or efficiency, multiplies K_p to produce effective PD gain. Subscripts:

- A AGC, Eq. (16.5)
- e phase error, Eq. (18.20)
- L limiter, Eq. (16.14)
- m modulation, Eq. (18.16)
- n noise, Eq. (18.12)
- p PD efficiency as a multiplier, Eq. (13.12)

ρ — signal-to-noise ratio (S/N), sometimes specifically at the output rather than input

ρ_{L0} — signal-to-noise ratio in a bandwidth equal to the loop noise bandwidth under linear conditions, $1/\sigma_{\varphi 0}^2$

σ_φ^2 — mean square phase deviation, variance of phase

σ_ω^2 — mean square frequency deviation, variance of frequency

$\sigma_{\omega s}^2$ — variance of the signal frequency

τ_1 — ($1/\omega_p$) time constant of the loop pole

τ_2 — ($1/\omega_z$) time constant of the loop zero

φ_0 — output phase response to additive input noise, linear response assumed (loop parameters are not changed by the noise)

φ_e — phase error, change in $\Delta\varphi$(often from the center of the PD range)

$\phi_i(t)$ — modulation-related input phase

φ_{in} — loop input phase (at phase detector)

φ_n — output phase response to additive input noise, nonlinear response assumed (loop parameters are changed by the noise)

$\phi_o(t)$ — modulation-related output phase