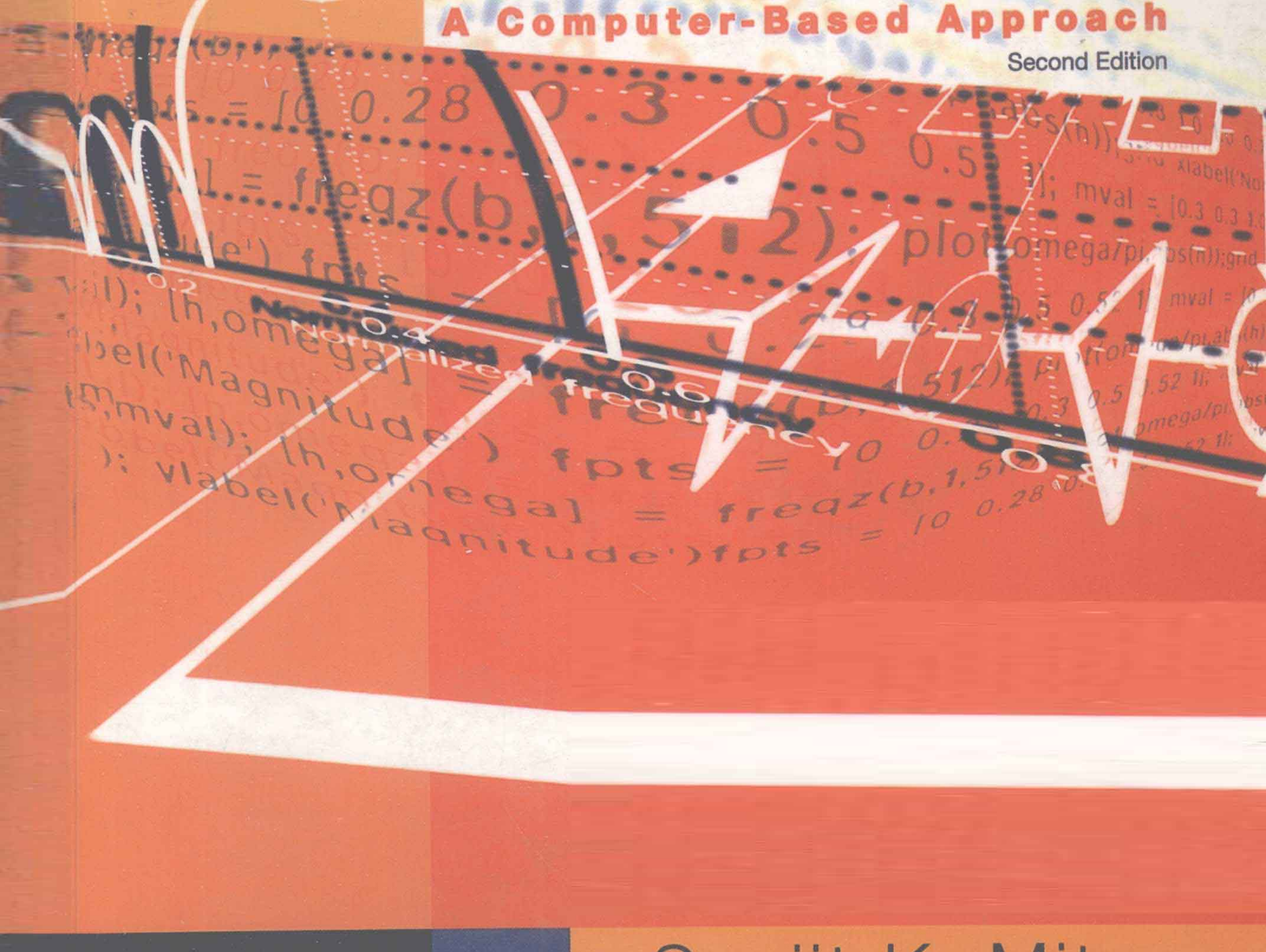
Digital Signal Processing

A Computer-Based Approach

Second Edition



Sanjit K. Mitra

INTERNATIONAL EDITION

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A Computer-Based Approach

Second Edition

Sanjit K. Mitra

Department of Electrical and Computer Engineering
University of California, Santa Barbara

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DIGITAL SIGNAL PROCESSING

International Edition 2002

10 09 08 07 06 05 04 03

20 09 08 07 06 05 04 03

CTF SLP

Library of Congress Control Number: 2001092070

ISBN 0-07-252261-5

When ordering this title, use ISBN 0-07-122607-9

Printed in Singapore

www.mhhe.com

McGraw-Hill Series in Electrical and Computer Engineering

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About the Author

Sanjit K. Mitra received his M.S. and Ph.D. in electrical engineering from the University of California, Berkeley, and an Honorary Doctorate of Technology from Tampere University of Technology in Finland. After holding the position of assistant professor at Cornell University until 1965 and working at AT&T Bell Laboratories, Holmdel, New Jersey, until 1967, he joined the faculty of the University of California at Davis. Dr. Mitra then transferred to the Santa Barbara campus in 1977, where he served as department chairman from 1979 to 1982 and is now a Professor of Electrical and Computer Engineering. Dr. Mitra has published more than 500 journal and conference papers, and 11 books, and holds 5 patents. He served as President of the IEEE Circuits and Systems Society in 1986 and is currently a member of the editorial boards for four journals: *Multidimensional Systems and Signal Processing; Signal Processing; Journal of the Franklin Institute*; and *Automatika*. Dr. Mitra has received many distinguished industry and academic awards, including the 1973 F. E. Terman Award, the 1985 AT&T Foundation Award of the American Society of Engineering Education, the 1989 Education Award of the IEEE Circuits and Systems Society, the 1989 Distinguished Senior U.S. Scientist Award from the Alexander von Humboldt Foundation of Germany, the 1996 Technical Achievement Award of the IEEE Signal Processing Society, the 1999 Mac Van Valkenburg Society Award and the CAS Golden Jubilee Medal of the IEEE Circuits & System Society, and the IEEE Millennium Medal in 2000. He is an Academician of the Academy of Finland. Dr. Mitra is a Fellow of the IEEE, AAAS, and SPIE and is a member of EURASIP and the ASEE.

Preface

The field of digital signal processing (DSP) has seen explosive growth during the past three decades, as phenomenal advances both in research and application have been made. Fueling this growth have been the advances in digital computer technology and software development. Almost every electrical and computer engineering department in this country and abroad now offers one or more courses in digital signal processing, with the first course usually being offered at the senior level. This book is intended for a two-semester course on digital signal processing for seniors or first-year graduate students. It is also written at a level suitable for self-study by the practicing engineer or scientist.

Even though the first edition of this book was published barely two years ago, based on the feedback received from professors who adopted this book for their courses and many readers, it was clear that a new edition was needed to incorporate the suggested changes to the contents. A number of new topics have been included in the second edition. Likewise, a number of topics that are interesting but not practically useful have been removed because of size limitations. It was also felt that more worked-out examples were needed to explain new and difficult concepts.

The new topics included in the second edition are: calculation of total solution, zero-input response, zero-state response, and impulse response of finite-dimensional discrete-time systems (Sections 2.6.1–2.6.3), correlation of signals and its applications (Section 2.7), inverse systems (Section 4.9), system identification (Section 4.10), matched filter and its application (Section 4.14), sampling of bandpass signals (Section 5.3), design of highpass, bandpass, and bandstop analog filters (Section 5.5), effect of sample-and-hold operation (Section 5.11), design of highpass, bandpass, and bandstop IIR digital filters (Section 7.4), design of FIR digital filters with least-mean-square error (Section 7.8), constrained least-square design of FIR digital filters (Section 7.9), perfect reconstruction two-channel FIR filter banks (Section 10.9), cosine-modulated L-channel filter banks (Section 10.11), spectral analysis of random signals (Section 11.4), and sparse antenna array design (Section 11.14). The topics that have been removed from the first edition are as follows: state-space representation of LTI discrete-time systems from Chapter 2, signal flow-graph representation and state-space structures from Chapter 6, impulse invariance method of IIR filter design and FIR filter design based on the frequency-sampling approach from Chapter 7, reduction of product round-off errors from state-space structures from Chapter 9, and voice privacy system from Chapter 11. The fractional sampling rate conversion using the Lagrange interpolation has been moved to Chapter 10. Materials in each chapter are now organized more logically.

A key feature of this book is the extensive use of MATLAB® -based[1] examples that illustrate the program's powerful capability to solve signal processing problems. The book uses a three-stage pedagogical structure designed to take full advantage of MATLAB and to avoid the pitfalls of a "cookbook" approach to problem solving. First, each chapter begins by developing the essential theory and algorithms. Second, the material is illustrated with examples solved by hand calculation. And third, solutions are derived using MATLAB. From the beginning, MATLAB codes are provided with enough details to permit the students to repeat the examples on their computers. In addition to conventional theoretical problems requiring analytical solutions, each chapter also includes a large number of problems requiring solution via MATLAB. This book requires a minimal knowledge of MATLAB. I believe students learn the intricacies of problem solving with MATLAB faster by using tested, complete programs, and then writing simple programs to solve specific problems that are included at the ends of Chapters 2 to 11.

Because computer verification enhances the understanding of the underlying theories and, as in the first edition, a large library of worked-out MATLAB programs are included in the second edition. The original MATLAB programs of the first edition have been updated to run on the newer versions of MATLAB and the *Signal Processing Toolbox*. In addition, new MATLAB programs and code fragments have been added in this edition. The reader can run these programs to verify the results included in the book. Altogether there are 90 MATLAB programs in the text that have been tested under version 5.3 of MATLAB and version 4.2 of the *Signal Processing Toolbox*. Some of the programs listed in this book are not necessarily the fastest with regard to their execution speeds, nor are they the shortest. They have been written for maximum clarity without detailed explanations.

A second attractive feature of this book is the inclusion of 231 simple but practical examples that expose the reader to real-life signal processing problems which has been made possible by the use of computers in solving practical design problems. This book also covers many topics of current interest not normally found in an upper-division text. Additional topics are also introduced to the reader through problems at the end of each chapter. Finally, the book concludes with a chapter that focuses on several important, practical applications of digital signal processing. These applications are easy to follow and do not require knowledge of other advanced-level courses.

The prerequisite for this book is a junior-level course in linear continuous-time and discrete-time systems, which is usually required in most universities. A minimal review of linear systems and transforms is provided in the text, and basic materials from linear system theory are included, with important materials summarized in tables. This approach permits the inclusion of more advanced materials without significantly increasing the length of the book.

The book is divided into 11 chapters. Chapter 1 presents an introduction to the field of signal processing and provides an overview of signals and signal processing methods. Chapter 2 discusses the time-domain representations of discrete-time signals and discrete-time systems as sequences of numbers and describes classes of such signals and systems commonly encountered. Several basic discrete-time signals that play important roles in the time-domain characterization of arbitrary discrete-time signals and discrete-time systems are then introduced. Next, a number of basic operations to generate other sequences from one or more sequences are described. A combination of these operations is also used in developing a discrete-time system. The problem of representing a continuous-time signal by a discrete-time sequence is examined for a simple case. Finally, the time-domain characterization of discrete-time random signals is discussed.

Chapter 3 is devoted to the transform-domain representations of a discrete-time sequence. Specifically discussed are the discrete-time Fourier transform (DTFT), the discrete Fourier transform (DFT), and the z-transform. Properties of each of these transforms are reviewed and a few simple applications outlined. The chapter ends with a discussion of the transform-domain representation of a random signal.

This book concentrates almost exclusively on the linear time-invariant discrete-time systems, and

[1]Matlab is a registered trademark of The MathWorks, Inc., 24 Prime Park Way, Natick, MA 01760-1500, Phone: 508-647-7000, **http://www.mathworks.com**.

Chapter 4 discusses their transform-domain representations. Specific properties of such transform-domain representations are investigated, and several simple applications are considered.

Chapter 5 is concerned primarily with the discrete-time processing of continuous-time signals. The conditions for discrete-time representation of a bandlimited continuous-time signal under ideal sampling and its exact recovery from the sampled version are first derived. Several interface circuits are used for the discrete-time processing of continuous-time signals. Two of these circuits are the anti-aliasing filter and the reconstruction filter, which are analog lowpass filters. As a result, a brief review of the basic theory behind some commonly used analog filter design methods is included, and their use is illustrated with MATLAB. Other interface circuits discussed in this chapter are the sample-and-hold circuit, the analog-to-digital converter, and the digital-to-analog converter.

A structural representation using interconnected basic building blocks is the first step in the hardware or software implementation of an LTI digital filter. The structural representation provides the relations between some pertinent internal variables with the input and the output, which in turn provides the keys to the implementation. There are various forms of the structural representation of a digital filter, and two such representations are reviewed in Chapter 6, followed by a discussion of some popular schemes for the realization of real causal IIR and FIR digital filters. In addition, it describes a method for the realization of IIR digital filter structures that can be used for the generation of a pair of orthogonal sinusoidal sequences.

Chapter 7 considers the digital filter design problem. First, it discusses the issues associated with the filter design problem. Then it describes the most popular approach to IIR filter design, based on the conversion of a prototype analog transfer function to a digital transfer function. The spectral transformation of one type of IIR transfer function into another type is discussed. Then a very simple approach to FIR filter design is described. Finally, the chapter reviews computer-aided design of both IIR and FIR digital filters. The use of MATLAB in digital filter design is illustrated.

Chapter 8 is concerned with the implementation aspects of DSP algorithms. Two major issues concerning implementation are discussed first. The software implementations of digital filtering and DFT algorithms on a computer using MATLAB are reviewed to illustrate the main points. This is followed by a discussion of various schemes for the representation of number and signal variables on digital machines, which is basic to the development of methods for the analysis of finite wordlength effects considered in Chapter 9. Algorithms used to implement addition and multiplication, the two key arithmetic operations in digital signal processing, are reviewed next, along with operations developed to handle overflow. Finally, the chapter outlines two general methods for the design and implementation of tunable digital filters, followed by a discussion of algorithms for the approximation of certain special functions.

Chapter 9 is devoted to analysis of the effects of the various sources of quantization errors; it describes structures that are less sensitive to these effects. Included here are discussions on the effect of coefficient quantization.

Chapter 10 discusses multirate discrete-time systems with unequal sampling rates at various parts. The chapter includes a review of the basic concepts and properties of sampling rate alteration, design of decimation and interpolation digital filters, and multirate filter bank design.

The final chapter, Chapter 11, reviews a few simple practical applications of digital signal processing to provide a glimpse of its potential.

The materials in this book have been used in a two-quarter course sequence on digital signal processing at the University of California, Santa Barbara, and have been extensively tested in the classroom for over 10 years. Basically, Chapters 2 through 6 form the basis of an upper-division course, while Chapters 7 through 10 form the basis of a graduate-level course.

Many topics included in this text can be omitted from class discussion, depending on the coverage of other courses in the curriculum. Because a senior-level course on random signals and systems is required of all electrical and computer engineering majors in most universities, materials in Sections 2.7, 3.10, and 4.9 can be excluded from an upper-division course on digital signal processing. However, these topics are important in the analysis of wordlength effects discussed in Chapter 9, and readers not familiar with this subject are encouraged to review these sections before reading Chapter 9. Likewise, Section 8.4 on number representation and Section 8.5 on arithmetic operations can similarly be omitted from discussion since most students taking a digital signal processing course usually take a course on digital hardware design.

This text contains 231 examples, 90 MATLAB programs, 684 problems, and 186 MATLAB exercises.

Every attempt has been made to ensure the accuracy of all materials in this book, including the MATLAB programs. I would, however, appreciate readers bringing to my attention any errors that may appear in the printed version for reasons beyond my control and that of the publisher. These errors and any other comments can be communicated to me by e-mail addressed to: **mitra@ece.ucsb.edu**.

Finally, I have been particularly fortunate to have had the opportunity to work with the outstanding students who were in my research group during my teaching career, which spans over 35 years. I have benefited immensely, and continue to do so, both professionally and personally, from my friendship and association with them, and to them I dedicate this book.

Sanjit K. Mitra

Acknowledgements

The preliminary versions of the complete manuscript for the first edition were reviewed by Dr. Hrvojc Babic of the University of Zagreb, Croatia; Dr. James F. Kaiser of Duke University; Dr. Wolfgang F. G. Mecklenbräuker of the Technical University of Vienna, Austria; and Dr. P. P. Vaidyanathan of the California Institute of Technology. A later version was reviewed by Dr. Roberto H. Bambmerger of Microsoft; Dr. Charles Boumann of Purdue University; Dr. Kevin Buckley of the University of Minnesota; Dr. John A. Flemming of the Texas A & M University; Dr. Jerry D. Gibson of the Southern Methodist University; Dr. John Gowdy of Clemson University; Drs. James Harris and Mahmood Nahvi of the California Polytechnic University, San Louis Obispo; Dr. Yih-Chyun Jenq of Portland State University; Dr. Troung Q. Ngyuen of Boston University; and Dr. Andreas Spanias of Arizona State University. Various parts of the manuscript were reviewed by Dr. C. Sidney Burrus of Rice University; Dr. Richard V. Cox of the AT&T Laboratories; Dr. Ian Galton of the University of California, San Diego; Dr. Nikil S. Jayant of the Georgia Institute of Technology; Dr. Tor Ramstad of the Norwegian University of Science and Technology, Trondheim, Norway; Dr. B. Ananth Shenoi of Wright State University; Dr. Hans W. Schüssler of the University of Erlangen-Nuremberg, Germany; Dr. Richard Schreier of Analog Devices and Dr. Gabor C. Temes of Oregon State University.

Reviews for the second edition were provided by Dr. Winser E. Alexander of North Carolina State University; Dr. Sohail A. Dianat of the Rochester Institute of Technology; Dr. Suhash Dutta Roy of the Indian Institute of Technology, New Delhi; Dr. David C. Farden of North Dakota State University; Dr. Abdulnasir Y. Hossein of Sultan Qaboos University, Sultanate of Omman; Dr. James F. Kaiser of Duke University; Dr. Ramakrishna Kakarala of the Agilent Laboratories; Dr. Wolfgang F. G. Mecklenbräuker of the Technical University of Vienna, Austria; Dr. Antonio Ortega of the University of Southern California; Dr. Stanley J. Reeves of Auburn University; Dr. George Symos of the University of Maryland, College Park; and Dr. Gregory A. Wornell of the Massachusetts Institute of Technology. Various parts of the manuscript for the second edition were reviewed by Dr. Dimitris Anastassiou of Columbia University; Dr. Rajendra K. Arora of the Florida State University; Dr. Ramdas Kumaresan of the University of Rhode Island; Dr. Upamanyu Madhow of the University of California, Santa Barbara; Dr. Urbashi Mitra of the University of Southern California; Dr. Randolph Moses of Ohio State University; Dr. Ivan Selesnick of Polytechnic University, Brooklyn, New York; and Dr. Gabor C. Temes of Oregon State University.

I thank all of them for their valuable comments, which have improved the book tremendously.

Many of my former and present research students reviewed various portions of the manuscript of both editions and tested a number of the MATLAB programs. In particular, I would like to thank Drs. Charles D. Creusere, Rajeev Gandhi, Michael Lightstone, Ing-Song Lin, Luca Lucchese, Debargha Mukherjee, Norbert Strobel, and Stefan Thurnhofer, and Messrs. Serkan Hatipoglu, Zhihai He, Eric Leipnik, Michael Moore, and Mylene Queiroz de Farias. I am also indebted to all former students in my ECE 158 and ECE 258A classes at the University of California, Santa Barbara, for their feedback over the years, which helped refine the book.

I thank Goutam K. Mitra and Alicia Rodriguez for the cover design of the book. Finally, I thank Patricia Monohon for her assistance in the preparation of the LaTeX files of the second edition.

Supplements

All MATLAB programs included in this book are available via anonymous file transfer protocol (FTP) from the Internet site **iplserv.ece.ucsb.edu** in the directory **/pub/mitra/Book_2e**.

A solutions manual prepared by Rajeev Gandhi, Serkan Hatipoglu, Zhihai He, Luca Lucchese, Michael Moore, and Mylene Queiroz de Farias and containing the solutions to all problems and MATLAB exercises is available to instructors from the publisher.

A companion book *Digital Signal Processing Laboratory Using MATLAB* by the author is also available from McGraw-Hill.

Contents

1 Signals and Signal Processing

Signals play an important role in our daily life. Examples of signals that we encounter frequently are speech, music, picture, and video signals. A signal is a function of independent variables such as time, distance, position, temperature, and pressure. For example, speech and music signals represent air pressure as a function of time at a point in space. A black-and-white picture is a representation of light intensity as a function of two spatial coordinates. The video signal in television consists of a sequence of images, called frames, and is a function of three variables: two spatial coordinates and time.

Most signals we encounter are generated by natural means. However, a signal can also be generated synthetically or by computer simulation. A signal carries information, and the objective of signal processing is to extract useful information carried by the signal. The method of information extraction depends on the type of signal and the nature of the information being carried by the signal. Thus, roughly speaking, signal processing is concerned with the mathematical representation of the signal and the algorithmic operation carried out on it to extract the information present. The representation of the signal can be in terms of basis functions in the domain of the original independent variable(s) or it can be in terms of basis functions in a transformed domain. Likewise, the information extraction process may be carried out in the original domain of the signal or in a transformed domain. This book is concerned with discrete-time representation of signals and their discrete-time processing.

This chapter provides an overview of signals and signal processing methods. The mathematical characterization of the signal is first discussed along with a classification of signals. Next, some typical signals are discussed in detail and the type of information carried by them is described. Then a review of some commonly used signal processing operations is provided and illustrated through examples. Advantages and disadvantages of digital processing of signals are then discussed. Finally, a brief review of some typical signal processing applications is included.

1.1 Characterization and Classification of Signals

Depending on the nature of the independent variables and the value of the function defining the signal, various types of signals can be defined. For example, independent variables can be continuous or discrete. Likewise, the signal can either be a continuous or a discrete function of the independent variables. Moreover, the signal can be either a real-valued function or a complex-valued function.

A signal can be generated by a single source or by multiple sources. In the former case, it is a scalar signal and in the latter case it is a vector signal, often called a multichannel signal.

A one-dimensional (1-D) signal is a function of a single independent variable. A two-dimensional (2-D) signal is a function of two independent variables. A multidimensional (M-D) signal is a function of more than one variable. The speech signal is an example of a 1-D signal where the independent variable is time. An image signal, such as a photograph, is an example of a 2-D signal where the two independent variables are the two spatial variables. Each frame of a black-and-white video signal is a 2-D image signal that is a function of two discrete spatial variables, with each frame occurring sequentially at discrete instants of time. Hence, the black-and-white video signal can be considered an example of a three-dimensional (3-D) signal where the three independent variables are the two spatial variables and time. A color video signal is a three-channel signal composed of three 3-D signals representing the three primary colors: red, green, and blue (RGB). For transmission purposes, the RGB television signal is transformed into another type of three-channel signal composed of a luminance component and two chrominance components.

The value of the signal at a specific value(s) of the independent variable(s) is called its *amplitude*. The variation of the amplitude as a function of the independent variable(s) is called its *waveform*.

For a 1-D signal, the independent variable is usually labeled as *time*. If the independent variable is continuous, the signal is called a *continuous-time signal*. If the independent variable is discrete, the signal is called a *discrete-time signal*. A continuous-time signal is defined at every instant of time. On the other hand, a discrete-time signal is defined at discrete instants of time, and hence, it is a sequence of numbers.

A continuous-time signal with a continuous amplitude is usually called an *analog signal*. A speech signal is an example of an analog signal. Analog signals are commonly encountered in our daily life and are usually generated by natural means. A discrete-time signal with discrete-valued amplitudes represented by a finite number of digits is referred to as a *digital signal*. An example of a digital signal is the digitized music signal stored in a CD-ROM disk. A discrete-time signal with continuous-valued amplitudes is called a *sampled-data signal*. This last type of signal occurs in switched-capacitor (SC) circuits. A digital signal is thus a quantized sampled-data signal. Finally, a continuous-time signal with discrete-valued amplitudes has been referred to as a *quantized boxcar signal*[Ste93]. Figure 1.1 illustrates the four types of signals.

The functional dependence of a signal in its mathematical representation is often explicitly shown. For a continuous-time 1-D signal, the continuous independent variable is usually denoted by t, whereas for a discrete-time 1-D signal, the discrete independent variable is usually denoted by n. For example, $u(t)$ represents a continuous-time 1-D signal and $\{v[n]\}$ represents a discrete-time 1-D signal. Each member, $v[n]$, of a discrete-time signal is called a *sample*. In many applications, a discrete-time signal is generated from a parent continuous-time signal by sampling the latter at uniform intervals of time. If the discrete instants of time at which a discrete-time signal is defined are uniformly spaced, the independent discrete variable n can be normalized to assume integer values.

In the case of a continuous-time 2-D signal, the two independent variables are the spatial coordinates, which are usually denoted by x and y. For example, the intensity of a black-and-white image can be expressed as $u(x, y)$. On the other hand, a digitized image is a 2-D discrete-time signal, and its two independent variables are discretized spatial variables often denoted by m and n. Hence, a digitized image can be represented as $v[m, n]$. Likewise, a black-and-white video sequence is a 3-D signal and can be represented as $u(x, y, t)$ where x and y denote the two spatial variables and t denotes the temporal variable time. A color video signal is a vector signal composed of three signals representing the three primary colors: red, green, and blue:

$$\mathbf{u}(x, y, t) = \begin{bmatrix} r(x, y, t) \\ g(x, y, t) \\ b(x, y, t) \end{bmatrix}.$$

There is another classification of signals that depends on the certainty by which the signal can be uniquely described. A signal that can be uniquely determined by a well-defined process such as a mathematical expression or rule, or table look-up, is called a *deterministic signal*. A signal that is generated in a random fashion and cannot be predicted ahead of time is called a *random signal*. In this text we are primarily concerned with the processing of discrete-time deterministic signals. However, since practical discrete-time systems employ finite wordlengths for the storing of signals and the implementation of the signal processing algorithms, it is necessary to develop tools for the analysis of finite wordlength effects on the performance of discrete-time systems. To this end, it has been found convenient to represent certain pertinent signals as random signals and employ statistical techniques for their analysis.

Some typical signal processing operations are reviewed in the following section.