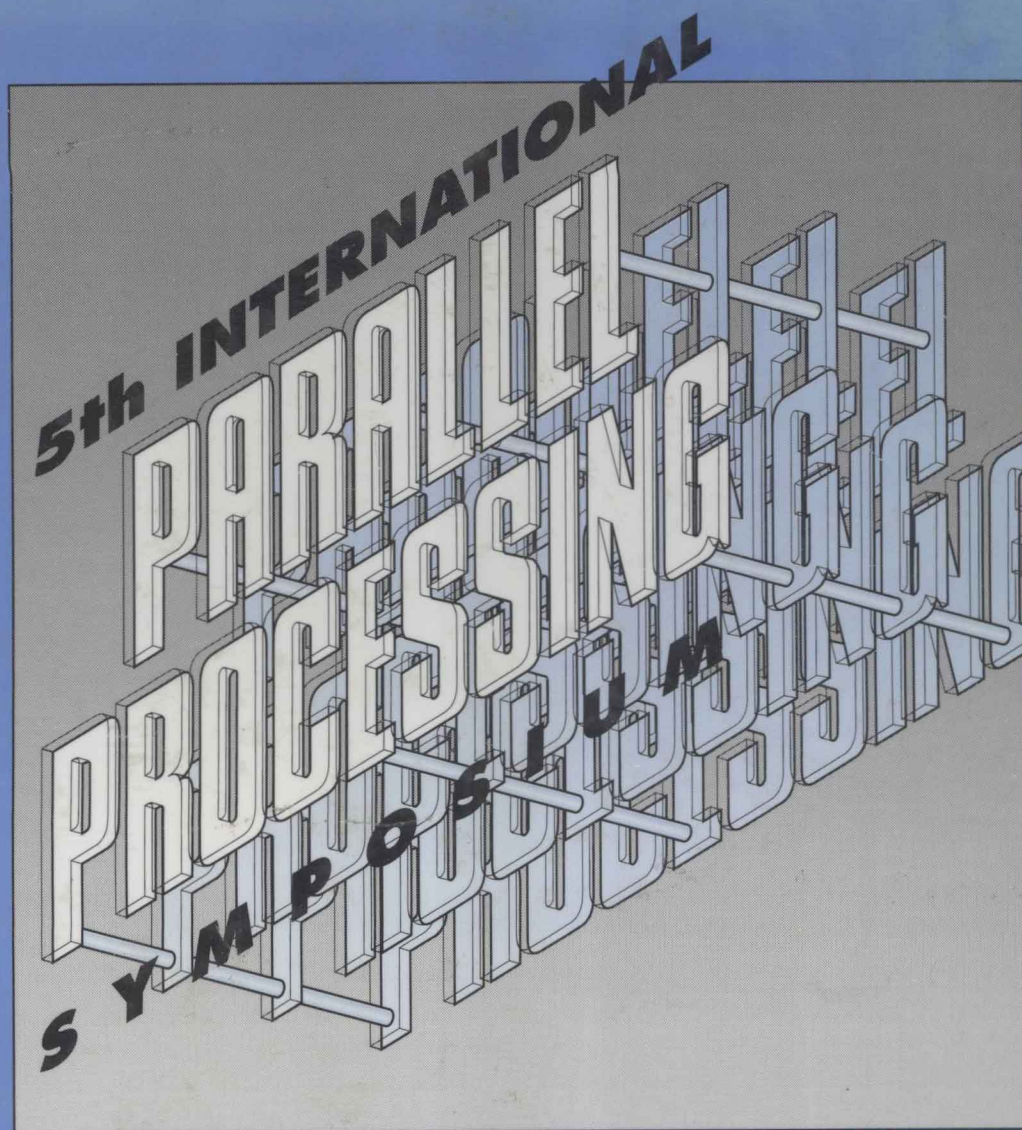


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APRIL 30 - MAY 2, 1991 • ANAHEIM, CALIFORNIA



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Proceedings

The Fifth International Parallel Processing Symposium

V. K. Prasanna Kumar, Editor
University of Southern California

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Foreword

The papers in this volume were presented at the 5th International Parallel Processing Symposium, held April 30 through May 2, 1991, in Anaheim, California. The symposium was sponsored by the IEEE Computer Society.

The program committee met on December 6, 1990, and selected these 112 papers from 243 manuscripts submitted in response to the Call for Papers. The selection was based on originality, technical quality, and relevance to the theme of the Symposium. The submissions were refereed, and many of them represent reports of continuing research. It is anticipated that most of these papers will appear in more polished and complete form in scientific journals.

The program committee wishes to thank all who submitted for consideration and those who evaluated the submissions.

*V. K. Prasanna Kumar
Program Chair*

*Prith Banerjee
Jacob Barhen
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Abe Waksman*



Message from the Chair

Welcome to the Fifth International Parallel Processing Symposium.

This year's event is the culmination of many hours of work from a dedicated group of individuals consisting of V. K. Prasanna Kumar, Program Committee Chairman-USC; Sally Jelinek, Publicity-Electronic Design Associated; Bill Pitts, Treasurer-Toshiba America; Susamma Barua, Exhibits and Local Arrangements-CSUF and George Westrom-Steering Committee Chair, Odetics, Inc. representing the Orange County IEEE Computer Society. As a result of growing interest from both the international community and from the United States, the name of the Symposium has been changed to reflect this broader participation. Because of the growth in attendance, the location was moved to the larger facilities of the Disneyland Convention Center. With the growing interest in research activity related to Parallel processing, the number of papers submitted has almost doubled to 243 papers. At the Symposium, 112 of these manuscripts will be presented in the technical and poster sessions. The Symposium theme continues to feature a blend of theoretical and applied parallel processing applications. This year the Symposium is co-located with the Federal Computer Conference. The Parallel Processing commercial exhibits will be combined with the FCC exhibits which will have almost 300 booths. In addition, three formal tutorials will be presented by Professor H. J. Siegal of Purdue University, Professors L. Bic and A. Nicolau of UCI, and Dr. Harold Szu of the Naval Surface Weapons Research Center.

Special recognition is given to Dr. V. K. Prasanna Kumar for his excellent work as Program Chairman. Through Dr. Prasanna Kumar's leadership this year's Program Committee and Keynote Speakers consist of the leading scientists in the parallel processing field. Special thanks are given to Sally Jelinek for not only coordinating all the publicity and the advanced program, but also for the many additional administrative tasks she undertook. I would like to extend special thanks to Professor H. T. Kung of Carnegie Mellon University for his recommendations for this year's Symposium. Equally, special thanks go to our keynote speakers, Professor Kai Hwang of USC, Professor David Kuck of the University of Illinois, and Dr. Harold Stone of IBM, who bring to the Symposium a wealth of creative experiences in parallel processing which they will share with us.

Finally, I would like to extend our thanks to the many universities and industrial firms who support the Parallel Processing Symposium, particularly the USC-Dept of EE-Systems, Encore Computer Corporation, Alliant Computer Systems Corporation, Grumman Data Systems, Hughes Aircraft, Intel Corporation, JPL, Motorola Corporation, Rockwell International, Toshiba America Information Systems, UCI, CalTech, and CSUF.

*Dr. Larry H. Canter
General Chairman*

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ALGORITHMS

Efficient Implementations of a Class of $\pm 2^b$ Parallel Computations on a SIMD Hypercube

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Abstract

We identify an important class of parallel computations, called $\pm 2^b$ - *descend*, with an efficient implementation on a hypercube. Given the input $A[0 : N - 1]$, a computation in this class consists of $\log N$ iterations. Iteration b , $b = \log N - 1, \dots, 0$, computes the new value of each $A[i]$ as a function of $A[i]$, $A[i + 2^b]$ and $A[i - 2^b]$. We obtain a general algorithm for implementing any computation in this class in $O(\log N)$ time on a SIMD hypercube. Our general descend algorithm results in an efficient $O(\log N)$ implementation of Batcher's odd-even merge algorithm on a hypercube. The best previously known implementation of odd-even merge on a SIMD hypercube requires $O(\log^2 N)$ time.

1 Introduction

A Boolean n -cube, also called n -dimensional hypercube, is a parallel computer with $N = 2^n$ processors numbered $0, 1, \dots, N - 1$. A pair of processors i, j are directly connected iff the binary representations of i and j differ in exactly one bit. Let $i = i_{n-1}i_{n-2} \dots i_0$ for $i \in [0 : N - 1]$, so

$$i \oplus 2^b = i_{n-1} \dots i_{b+1} \bar{i}_b i_{b-1} \dots i_0.$$

Processor i is connected to $\log N$ other processors: $i \oplus 2^b$, $0 \leq b \leq n - 1$.

The hypercube interconnection has n dimensions. Dimension b is the connections between $N/2$ pairs of processors $i, i \oplus 2^b$. The $N/2$ processors with $i_b = 0$ and the $N/2$ processors with $i_b = 1$ each form a *subcube*, having the structure of an $(n-1)$ -cube.

The processors can communicate only via the processor-to-processor interconnection. Two communication models of hypercube have been consid-

ered in the literature. In a SIMD model, all communications in a single step are restricted to a single dimension. In a MIMD model, this restriction does not exist.

There is a wide class of parallel algorithms which require processor i to communicate with processors $i \pm 2^b \bmod N$, $0 \leq i \leq N - 1$, where b is an integer determined by each stage of the algorithm.

Consider the complexity of performing a 2^b -permutation on a hypercube. (A 2^b -permutation of a sequence $a[0 : N - 1]$ sends the element initially at position i to position $i + 2^b \bmod N$. The inverse is called -2^b -permutation.) If we assume that element $a[i]$ is mapped onto processor i , then it is easy to see that at least $\log N - b$ routing steps are needed to perform a 2^b -permutation. (There are known $O(\log N - b)$ algorithms for performing a 2^b -permutation [8] on a hypercube.)

Considering the more general case where the elements may be mapped onto processors according to any one-to-one mapping π , the following bounds are known for performing a 2^b permutation on a hypercube:

- For a SIMD model, the lower bound of $\log N - b$ still holds [12].
- For a MIMD model, a 2^b permutation can be performed in $O(1)$ steps using an appropriate one-to-one mapping [5], [12].

The lower bound for a SIMD model suggests that if a parallel algorithm requires $f(N)$ communication steps of type $\pm 2^b$, it may require $O(f(N) \cdot \log N)$ steps on a hypercube.

A well-known example of a parallel algorithm requiring $\pm 2^b$ communications is Batcher's odd-even merge. To merge two sorted sequences of length $N/2$, this algorithm runs through $\log N$ iterations,