Vicenç Torra Yasuo Narukawa Yuji Yoshida (Eds.)

## Modeling Decisions for Artificial Intelligence

4th International Conference, MDAI 2007 Kitakyushu, Japan, August 2007 Proceedings



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# Modeling Decisions for Artificial Intelligence

4th International Conference, MDAI 2007 Kitakyushu, Japan, August 16-18, 2007 Proceedings







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Volume Editors

Vicenc Torra **IIIA-CSIC** 

Campus UAB s/n, 08193 Bellaterra, Catalonia, Spain

E-mail: vtorra@ijia csic es

Yasuo Narukawa Toho Gakuen 3-1-10 Naka, Kunitachi, Tokyo, 186-0004, Japan E-mail: narukawa@d4.dion.ne.ip

Yuji Yoshida University of Kitakyushu Faculty of Economics and Business Administration 4-2-1 Kitagata, Kokuraminami, Kitakyushu 802-8577, Japan E-mail: yoshida@kitakyu-u.ac.jp

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#### **Preface**

This volume contains papers presented at the 4th International Conference on Modeling Decisions for Artificial Intelligence (MDAI 2007), held in Kitakyushu, Japan, August 16-18. This conference followed MDAI 2004 (Barcelona, Catalonia, Spain), MDAI 2005 (Tsukuba, Japan), and MDAI 2007 (Tarragona, Catalonia, Spain) with proceedings also published in the LNAI series (Vols. 3131, 3558, and 3885).

The aim of this conference was to provide a forum for researchers to discuss the theory and tools for modeling decisions as well as applications that encompass decision-making processes and information-fusion techniques.

The organizers received 193 papers from 21 different countries, from Asia, Europe, America, Africa, and Australia, 42 of which are published in this volume. Each submission received at least two reviews from the Program Committee and a few external reviewers. We would like to express our gratitude to them for their work. The plenary talks presented at the conference are also included in this volume.

The conference was supported by the University of Kitakyushu, the UNESCO Chair in Data Privacy, the Japan Society for Fuzzy Theory and Intelligent Informatics (SOFT), the Catalan Association for Artificial Intelligence (ACIA), the European Society for Fuzzy Logic and Technology (EUSFLAT), and the City of Kitakyushu.

May 2007

Vicenç Torra Yasuo Narukawa Yuji Yoshida

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#### An Overview of Fuzzy Relational Calculus and Its Applications

Etienne E. Kerre

Fuzziness & Uncertainty Modelling,
Department of Applied Mathematics & Computer Science,
Ghent University, Krijgslaan 281 - S9,
B-9000, Belgium
etienne.kerre@ugent.be
http://www.fuzzy.ugent.be

The calculus of relations has been very important during the past 40 years from theoretical as well as from practical point of view. The development of fuzzy set theory, particularly in the framework of relational calculus has undoubtly increased the interest in this domain of science.

In this paper I will give a brief overview of the most recent colourings of the classical theory of relations and convince the reader about the high applicability of fuzzy relational calculus in providing examples from information retrieval, relational databases, approximate reasoning, preference modelling, medical diagnosis. Most of the work presented here has been developed in my research team at Ghent University during the past 30 years.

#### 1 New Concepts in Classical Relational Calculus

The concept of a relation is fundamental since only in a few steps one can introduce this concept in the framework of set theory. Indeed as soon as the meaning of the so-called classifier  $\{z|P\}$ , i.e., the class of all objects z that satisfy a given property P, has been introduced (intuitively in the sense of Cantor or axiomatically in the sense of Gödel-Bernays-Von Neumann or Zermelo-Fraenkel) one may define a singleton, a doubleton, an ordered pair, the cartesian product of two sets and finally a relation from X to Y as a subset of the cartesian product  $X \times Y$ . This concept may be extended to a relation between n universes  $X_1, X_2, \ldots, X_n$  as a subset of  $X_1 \times X_2 \times \ldots \times X_n$ . We all are familiar with a special kind of relations, namely the functional relations or shortly functions. It is hard to image mathematics without the concept of a relation, in particular without the concept of a function. Some auxiliary notions with respect to a relation R from X to Y are:

- The domain dom(R) consisting of all elements of X that are coupled by R to at least one element of Y.
- The range rng(R) consisting of all elements of Y that are linked to at least one element of X.

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- The inverse  $R^{-1}$  consisting of all ordered pairs (y, x) such that (x, y) belongs to R.
- The R-afterset of x (denoted xR) consisting of all elements of Y that are linked to x.
- The R-foreset of y (denoted Ry) consisting of all elements of X that are linked to y.

Due to the last two notions introduced by Bandler and Kohout [1] in the 80's, a lot of new concepts concerning images and compositions could be introduced. These concepts substantially enlarge the toolkit of relational calculus.

Since relations are sets all set-theoretic operations such as union, intersection, complementation, difference, symmetric difference as well as the natural join operation can be applied to relations. For example suppose that  $R_1$  and  $R_2$  are relations from X to Y, then the union  $R_1 \cup R_2$  consists of all ordered pairs (x, y) that belong to  $R_1$  or to  $R_2$ . It is interesting to note that all these operations can be directly applied to aftersets and foresets, i.e., the  $(R_1 \cup R_2)$ -afterset of  $x \in X$  equals the union of  $xR_1$  and  $xR_2$ , i.e., the family of aftersets  $(xR)_{x\in X}$  contains all relevant information concerning the relation R and similarly for the family of foresets  $(Ry)_{y\in Y}$ .

Important notions in mathematics like continuity and measurability are based on the concept of direct and inverse image of a set under a (functional) relation.

Let R be a relation from X to Y, A a subset of X and B a subset of Y, then

- the direct image of A under R is given by:

$$R(A) = \{y | (\exists x \in A)((x, y) \in R)\}$$

or equivalently using after- and foresets:

$$R(A) = \{y | A \cap Ry \neq \emptyset\}$$
$$= \bigcup_{x \in A} xR$$

- the inverse image of B under R is given by:

$$\begin{split} R^{-1}(B) &= \{x | (\exists y \in B) ((x,y) \in R) \} \\ &= \{x | B \cap xR \neq \emptyset \} \\ &= \underset{y \in B}{\cup} Ry \end{split}$$

Inspired by the work of Bandler-Kohout [1] on the new compositions, De Baets-Kerre [2-5] have introduced some new images that could be defined using after- and foresets:

- the subdirect image of A under R:

$$\begin{array}{lll} R^{\triangleleft}(A) = \{y|A \cap Ry \neq \emptyset & and & A \subseteq Ry\} \\ = \{y|A \neq \emptyset & and & Ry \neq \emptyset & and & A \subseteq Ry\} \end{array}$$

- the superdirect image of A under R:

$$\begin{array}{lll} R^{\triangleright}(A) = \{y|A \cap Ry \neq \emptyset & and & Ry \subseteq A\} \\ = \{y|A \neq \emptyset & and & Ry \neq \emptyset & and & Ry \subseteq A\} \end{array}$$

- the squaredirect image of A under R:

$$R^{\square}(A) = \{y | A \cap Ry \neq \emptyset \quad and \quad A = Ry\}$$
  
= \{y | A \neq \Omega \quad and \quad Ry \neq \Omega \quad and \quad A = Ry\}

Finally we mention the most important operation on relations, namely composition or product and its useful extensions introduced by Bandler-Kohout [1] and slightly modified by De Baets-Kerre [2-5].

Let  $R_1$  be a relation from X to Y and  $R_2$  a relation from Y to Z, then:

- the round product of  $R_1$  and  $R_2$  (read  $R_1$  before  $R_2$ ,  $R_1$  followed by  $R_2$ ) is defined as the relation from X to Z given by:

$$R_1 \circ R_2 = \{(x, z) | (\exists y)((x, y) \in R_1 \text{ and } (y, z) \in R_2) \}$$

or equivalently using after- and foresets:

$$R_1 \circ R_2 = \{(x, z) | xR_1 \cap R_2 z \neq \emptyset\}$$

- the subproduct of  $R_1$  and  $R_2$ :

$$\begin{array}{lll} R_1 \triangleleft R_2 = \{(x,z) | xR_1 \cap R_2z \neq \emptyset & and & xR_1 \subseteq R_2z \} \\ = \{(x,z) | xR_1 \neq \emptyset & and & R_2z \neq \emptyset & and & xR_1 \subseteq R_2z \} \end{array}$$

- the superproduct of  $R_1$  and  $R_2$ :

$$R_1 \triangleright R_2 = \{(x, z) | xR_1 \cap R_2 z \neq \emptyset \quad and \quad R_2 z \subseteq xR_1\}$$
  
= \{(x, z) | xR\_1 \neq \empty \text{ and } \quad R\_2 z \neq \empty \text{ and } \quad R\_2 z \subseteq xR\_1\}

- the squareproduct of  $R_1$  and  $R_2$ :

$$R_1 \square R_2 = \{(x, z) | xR_1 \cap R_2 z \neq \emptyset \quad and \quad xR_1 = R_2 z \}$$
  
= \{(x, z) | xR\_1 \neq \empty \text{ and } \quad R\_2 z \neq \empty \text{ and } \quad xR\_1 = R\_2 z \}

#### 2 A Brief Outline of Fuzzy Relational Calculus

Since the old Greeks scientists have recognized that binary or black-or-white logic is not sufficient to model our knowledge which is mostly pervaded with imprecision. We have to wait until 1965 when Lotfi Zadeh introduced the concept of a fuzzy set in his seminal paper entitled "Fuzzy Sets",in order to model imprecise terms as "sets" with unsharp bounderies where the transition from belonging to not belonging is rather gradual than abrupt. In the same spirit Zadeh introduced the concept of a fuzzy relation from a universe X to a universe Y as

a fuzzy set R in the cartesian product  $X \times Y$  where R(x, y) denotes the strength of relationship between  $x \in X$  and  $y \in Y$ .

More formally a fuzzy relation R from X to Y is a mapping from  $X \times Y$  into the unit interval [0,1], attaching to every ordered pair (x,y) in  $X \times Y$  a degree of relationship R(x,y) belonging to [0,1].

The basic concepts introduced in section 1 can be generalized or fuzzified as follows. Let R be a fuzzy relation from X to Y, then:

- the domain of R is a fuzzy set in X given by:

$$dom(R)(x) = \sup\{R(x, y) | y \in Y\}, \forall x \in X$$

- the range of R is a fuzzy set in Y given by:

$$rng(R)(y) = \sup\{R(x,y)|x \in X\}, \forall y \in Y$$

– the inverse  $R^{-1}$  of R is the fuzzy relation from Y to X given by:

$$R^{-1}(y,x) = R(x,y), \forall (y,x) \in Y \times X$$

- the R-afterset of  $x \in X$  is the fuzzy set in Y given by:

$$xR(y) = R(x, y), \forall y \in Y$$

- the R-foreset of  $y \in Y$  is the fuzzy set in X given by:

$$Ry(x) = R(x, y), \forall x \in X$$

All the set-theoretic operations have been extended in an infinite number of ways to fuzzy sets and a fortiori to fuzzy relations using the concepts of triangular norms and conorms introduced by Schweizer-Sklar in the framework of probabilistic metric spaces. A triangular norm T is a  $[0,1]^2 - [0,1]$  mapping satisfying commutativity, associativity, monotonicity and the boundary condition  $T(x,1) = x, \forall x \in [0,1]$ . A triangular conorm S is defined in a similar way but with the boundary condition  $S(x,0) = x, \forall x \in [0,1]$ . The T-intersection (S-union) of two fuzzy relations  $R_1$  and  $R_2$  from X to Y is defined as a fuzzy relation from X to Y given as:

$$R_1 \cap_T R_2(x,y) = T(R_1(x,y), R_2(x,y))$$

$$R_1 \cup_S R_2(x,y) = S(R_1(x,y), R_2(x,y))$$

for all  $(x, y) \in X \times Y$ .

All operations on fuzzy relations may be defined using after- and foresets, i.e.:

$$x(R_1 \cap_T R_2) = xR_1 \cap_T xR_2, \forall x \in X$$

$$(R_1 \cup_S R_2)y = R_1 y \cup_S R_2 y, \forall y \in Y$$

In order to fuzzify the concepts of images and compositions we need an extension of the classical intersection or conjunction operation and the binary