Fundamentals of Astrodynamics and Applications

Second Edition

David A. Vallado
with contributions by
Wayne D. McClain

Space
Technology
Library

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Foreword

At Microcosm, when an astrodynamics problem arises it has become standard practice to "look it up in Vallado." In most cases, the answer is there, and in a form that can be readily used and applied to the spaceflight problem at hand. I believe it is for this reason that *Fundamentals of Astrodynamics and Applications* has so rapidly become the standard astrodynamics reference for those of us in the business of spaceflight.

Thus, it is truly a pleasure to welcome the revised and expanded second edition of "Vallado" to the Space Technology Library. While the first edition was an exceptionally useful and popular book throughout the community, there are a number of reasons why we believe the second edition will be even more so. The text has expanded to 958 pages. There are many reworked examples and derivations. Entirely new topics include ground illumination calculations, Moon rise and set, and a listing of relevant InterNet sites. There is an improved and expanded discussion of coordinate systems, orbit determination, and differential correction. Perhaps most important is that all of the software routines described in the book are now available for free in FORTRAN, PASCAL, and C on the book website, *www.smad.com/Vallado*. With extensive references to both specific equations and the section of the book where the topic is discussed, this software is the perfect practical companion to Vallado's book. This makes the second edition even more valuable as both a text and reference.

We hope that you will find this book to be as useful and productive as those of us at Microcosm have. To me, a good measure of its real utility is that nearly every engineer in our Space Systems Division has spent their own hard-earned money and bought a copy for themselves. The book is not only an excellent text, it is a superb reference that will be used over and over in practical astrodynamics.

To you, our readers, we are certain that you will enjoy the clarity and simplicity with which Dave both derives and explains modern, applied astrodynamics. To Dave Vallado, Welcome Aboard! We are delighted to have you join the Space Technology Library.

James R. Wertz
El Segundo, CA
February 2001
jwertz@smad.com

Preface to the Second Edition

My goal in writing this book is to augment and consolidate some of the information other astrodynamics books contain and, perhaps more importantly, to present and describe the computational fundamentals of astrodynamics. During the early 1960s, tremendous progress took place in astrodynamics, in part due to the U.S.'s drive to reach the Moon and to stay technologically superior during the Cold War. While the theories advanced by large steps, the computer code took only a "small step" because of available computing languages and hardware. But in the 1970s and 1980s, the computer industry took gigantic steps, so now is the time to capitalize on advancements in both areas.

You're probably wondering, does this book present anything new? Well, yes and no. The dynamics have been in effect since the beginning of time, and most of our mathematics hasn't changed much since its discovery over 300 years ago. Today, advances in computer technology allow students and practitioners to determine orbits with high precision on a PC, but they need a compendium of *well-documented* algorithms to use that power. Hence, this book.

The days of studying just one discipline are nearly over. Today's scientists should study computer science in *addition* to rigorous engineering principles—after all, that's how we do our business. As older managers retire and newer computer systems arrive, the need is more pressing than ever for a new look at *traditional* astrodynamics. The days of "well, it works" are long gone.

The keys for the future are standardizing and documenting, modularity, efficiency, and maintainability. This book attempts to use the new programming capabilities to exploit each of these areas. Standardizing and documenting can't be overemphasized. I've witnessed tremendous efforts in astrodynamics and, on every project, I always came away wishing that someone had written a few more steps, or had provided a test case to check how the program worked. Too often, books are written mainly for academics; writers forget that engineers need simple algorithms and examples to use in everyday operations. Operational "documentation" is often severely limited or nonexistent. Although I use standard notation as much as possible and work hard to match existing practices, it's impossible to be consistent with all sources.

I've included many derivations because I've seen too many engineers spend excessive time trying to recreate something that is "easy to verify." Recall the infamous scribbling of Pierre de Fermat (1601-1665) in the margin of a page where he omitted the proof because surely everyone could understand it and there wasn't enough room to say more (*Scientific American*, Oct. 1978, Oct. 1993, Oct. 1994, and Nov. 1997, 68-73). Fermat's last theorem remained a mystery for 350 years and though proven by Andrew Wiles in 1994, the lack of Fermat's written derivation generated controversy for several hundred years! Redoing fundamental problems is useful to learn the basics, but we can save steps (and therefore time and money) by presenting the material straightforwardly, with consistent notation.

Modularity is closely aligned with standardizing and documenting, but it's distinct. I've become accustomed to the old axiom "this program already does everything you need," only to find that your application requires just 5% of the code, and you can't pull out the routines you need because all routines are interconnected. In this book I present many independent algorithms. Admittedly, some routines will share algorithms, but with a little thought, this becomes only a minor issue.

Efficiency is important, yet elusive. During the development, we can consider some simple guidelines that will make routines run much faster. I've given you lots of examples and hints throughout the book.

Finally, at times it will seem as though I'm leading you by the hand through a problem, and at others, you'll need to scratch some intermediate equations in the margin (make sure you *do* record your notes!). My goal in writing this book has been (within some reasonable bounds for the overall length) to derive and detail each algorithm to about the same level to provide a solid foundation. Because individual abilities vary widely, you may find routines you've worked with to be too simple and unknown algorithms to be confusing. I've tried to minimize the confusion, and I'll appreciate your comments for improvement.

You may need some prerequisites for this book. It tries to bridge topics in astrodynamics from undergraduate courses with follow-on, graduate-level topics. It's intended for upper-level undergraduate classes in astrodynamics, as well as for some graduate-level classes. From my experience in laboratory research, operational environments, and instruction, I've tried to present the material with a practical flavor. The book covers theory, applications, and interpretations through numerous examples. The idea is to blend real-world operations with a theoretical understanding of the mechanics at work.

I've coded almost all the algorithms to verify each technique. Actual computer source code is available via the Internet. It details test cases and code in PASCAL, FORTRAN, and C. These languages should permit relatively easy conversion to other languages.

This book is intended to introduce astrodynamics to the undergraduate, and to lay a solid foundation for graduate-level studies. As such, it covers a wide scope of topics, some of which are primarily for academic purposes, while others are for general interest. Topics in astronomy are introduced where relevant to astrodynamics and the study of Earth-orbiting satellites, as well as for general information. Having a degree or studying astrodynamics, you'll invariably get questions about "what star is that?" This text probably will not prepare you for that task. However, you can consult any number of introductory astronomy books to provide an answer to that question!

The first 7 chapters relate primarily to two-body motion. In particular, the first five chapters have been re-ordered from the first edition to better match undergraduate curriculums. The text now begins with the two-body equations of motion, and Kepler's solutions. This permits initial analysis of orbits, and sets the stage for the coordinate and time system discussions in Chap. 3. The old Chap. 4 was broken into two chapters and combined with information form initial orbit determination and estimation. This combination provides a more consolidated discussion of observations and celestial phenomena.

The last 4 chapters discuss more advanced topics, and concentrate on propagation (numerical and analytical), and differential correction. These topics form the heart of orbit determination systems, whether they handle all 10,000 satellites in Earth orbit, or a single satellite. My placement of the material in this order isn't intended to restrict first or second courses from using these chapters. In fact, I'd expect a first course to incorporate topics such as predicting a satellite's location over a ground site (Chap. 11), or an introductory numerical propagation exercise (Chap. 8), or an initial orbit determination method (Gibbs or Herrick-Gibbs Chap. 7). I tired to keep all the similar material in the same chapter to avoid looking back several chapters to see the various approaches to accomplish a task.

I show many detailed derivations, culminating in an algorithm, suitable for programming a computer. I provide comments about computer programming because of the importance computers have become today in all disciplines. For the seasoned veteran, the algorithm may be trivial, and the computer coding comments may be unwarranted. For the student, these extra items may be the means to understanding, if not simply getting the program to work! The derivation ensures the assumptions, methodology, and rationale are clear for each technique.

For academic courses, several prerequisites are necessary. I have included a very brief review of vectors and matrices, and some numerical techniques in Appendix C. These are not intended to fully explain the concepts, but rather to introduce my notation and nomenclature.

For the casual reader, instructor, etc. who wishes further information in certain areas, I've listed many references in the reference section. I include the number of the chapter they are cited in, and the references indicate page numbers to help you find the citation quickly. I recommend a few texts that are especially good in certain areas. This list is *not* exhaustive!

Advanced Astrodynamics:	Battin (1987), Szebehely (1967), Stiefle and Scheifele (1971)
Analytical propagation:	Brouwer and Clemence (1961)
Attitude determination:	Wertz (1978)
Estimation:	Gelb (1989), Bierman (1977)
General Astrodynamics:	Escobal (1985), Danby (1992)
Gravity, geodesy:	Lambeck (1988), Kaula (1966)
Mathematics:	Kreyzig (1983), Numerical Recipes (1992), Sadler (1974)
Time, coordinates, astronomy:	Seidelmann (1992), Green (1988)

For academic courses, several pre-requisites are necessary. First, an understanding of analytical geometry, introductory calculus, differential equations, and basic computer programming are very useful. I mention several comments that apply to any computer programming language, and basic skills will be very useful. Data structures can also be helpful, depending on the additional computer programs that are assigned during a class. The estimation sections will benefit from introductory knowledge of statistics and probability theory. Although I explain several concepts, a single text can't explain everything!

Finally, although I've made every attempt to correct mistakes in the book, I'm sure there will be some errors. I encourage you to contact myself, or Wayne McClain by e-mail for any questions you discover. *All* comments are important!

valladodl@worldnet.att.net or wmcclain@draper.com

Questions regarding the publication of the book should be sent to Microcosm Press at bookproject@smad.com

Acknowledgments

I must credit the numerous people who made this project a success. First and foremost, I thank my parents who have encouraged me throughout my life. My early childhood memories are of the 1960s, when the Cold War raged, the Cuban missile crisis arose, and Vietnam erupted. To their credit, my parents somehow diverted my attention toward one of the most important events to occur in recent history: the effort to land a man on the Moon. I'll never forget that feeling I had, glued to the TV (VCRs and DVDs weren't available yet) as Neil Armstrong walked the surface of the Moon. That event, coupled with my father's example as an engineer on the Manhattan Project during World War II, compelled me to become an engineer. Space was the obvious choice as the final frontier to explore.

Any large work needs solid technical reviews, but I'm especially thankful for those of Wayne McClain. As the primary technical reviewer, he helped me refine this book into a polished presentation. His insight and extensive experience were irreplaceable. Also, his patience with my ever-changing ideas and endless questions was truly remarkable! He provided extensive technical contributions to the book resulting from his experience in the field. The Charles Stark Draper Laboratory also deserves credit for granting Wayne McClain the time necessary to provide the thorough review of this work.

I'd also like to credit the people who allowed me to adapt parts of their work. I've listed each person alphabetically to avoid any favoritism—every comment was useful!

- Salvatore Alfano provided many papers for this book on continuous-thrust modeling (with Jim Thorne), close approach (with David Negron), rise/set calculations, and the method of ratios (also with David Negron). These works add a lot of practical insight and applications to my presentations.
- Leon Blitzer graciously permitted use of his *Handbook For Orbital Perturbations* (1970) for the perturbations chapters. Although I changed the look of his handbook material, much of the content remains, and I tried to keep the original intent he achieved almost thirty years ago.
- Carole Boelitz permitted me to include sections from her thesis in the perturbations sections. She also provided valuable reviews and derivations.
- Chapter 10 was initially based on a handout Jack Ferguson developed at the U.S. Air Force Academy in 1978. Jack Ferguson, others, and I reworked this handout in 1992 and I've expanded it again for this application.
- Daniel Fonte also allowed me to use sections of his thesis and his recent technical papers to give great credibility to the perturbations section. This area has been neglected for much too long.
- Tom Logsdon and Warner Miller provided initial thoughts and insight for the GPS section for Chap. 11. Mike Gabor helped refine this chapter into its final form.
- Jim Thorne allowed me to use sections from his continuous thrust and series reversion for solution of the Lambert problem. Some examples are from his work.

- Vasiliy Yurasov provided details on the Russian Space Surveillance System, and their atmospheric models. He also submitted an update to the GOST model, which is contained in Appendix B. This represents a unique collection of data which was formerly difficult to obtain.

I received tremendous support from many people throughout the astrodynamics community. In particular,

- Gerry Baumgartner and Michael Gabor provided a very detailed examination of the complete text. They gave extensive derivations and provided a level of confidence in many of the difficult sections.

Other people worked entirely through the text, some more than once:

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Other individuals concentrated on certain chapters, relying on their expertise, which worked especially well considering the broad range of topics covered in the book:

- Carole Boelitz, Paul Cefola, Jack Ferguson, Jim Hall, Richard Hujsak, John Junkins, David Negron, Jr., Beny Neta, David Richardson, Ken Seidelmann, Wayne Smith, Sid Sridharan, and Jim Thorne.

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Any book written mostly by an engineer needs fine tuning to become a complete work. For the first edition, Daryl Boden assisted as a technical editor and provided technical insight for the text. Perry Luckett, a former professor of mine, really came to the rescue. He clarified ideas through more direct language, standardized or sharpened figures and tables, and kept my English in line!

I enjoyed continued support from the leadership at Air Force Research Laboratory, including Richard Davis, Eugene Dionne, Michael Heil, Henry Pugh, and Edward Duff. Their support for, and perseverance with, this long project was invaluable.

Several people formatted the first edition in FrameMaker Ver. 4.02. I began the typing and oversaw the operation throughout. Elsie Lugo deserves great credit for helping me create the first draft. Linda Pranke continued as production editor through the many intermediate drafts until the book was close to publication. She not only endured my cryptic notations but attended to every detail and caught inconsistencies that might have confused readers. She really made this book happen—despite my endless changes!

The first edition also owes many thanks to Margaret Hollander and Wiley Larson for getting the book published and on the streets.

Many people helped review the book by asking questions over the last few years. Their efforts were an important factor in finding errors and suggesting corrections. I am very grateful for their efforts and persistence to find me!

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I'd like to mention John Seago and Mike Gabor in particular as they helped refine the second edition with numerous comments and suggestions. Their insight, in-depth knowledge, and attention to detail were invaluable in making the second edition a reality.

I completed the second edition in FrameMaker Ver 5.5 which greatly improved the speed of processing the text. Finally, and certainly not least, Jim Wertz helped get this edition published. His counsel has been important throughout this effort, and I am very grateful for his assistance!

David A. Vallado
Denver, CO
February 2001
davallado@west.raytheon.com
valladodl@worldnet.att.net

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