# COMPUTERS IN TEACHING MATHEMATICS

Peter Kelman · Art Bardige · Jonathan Choate
George Hanify · John Richards · Nancy Roberts
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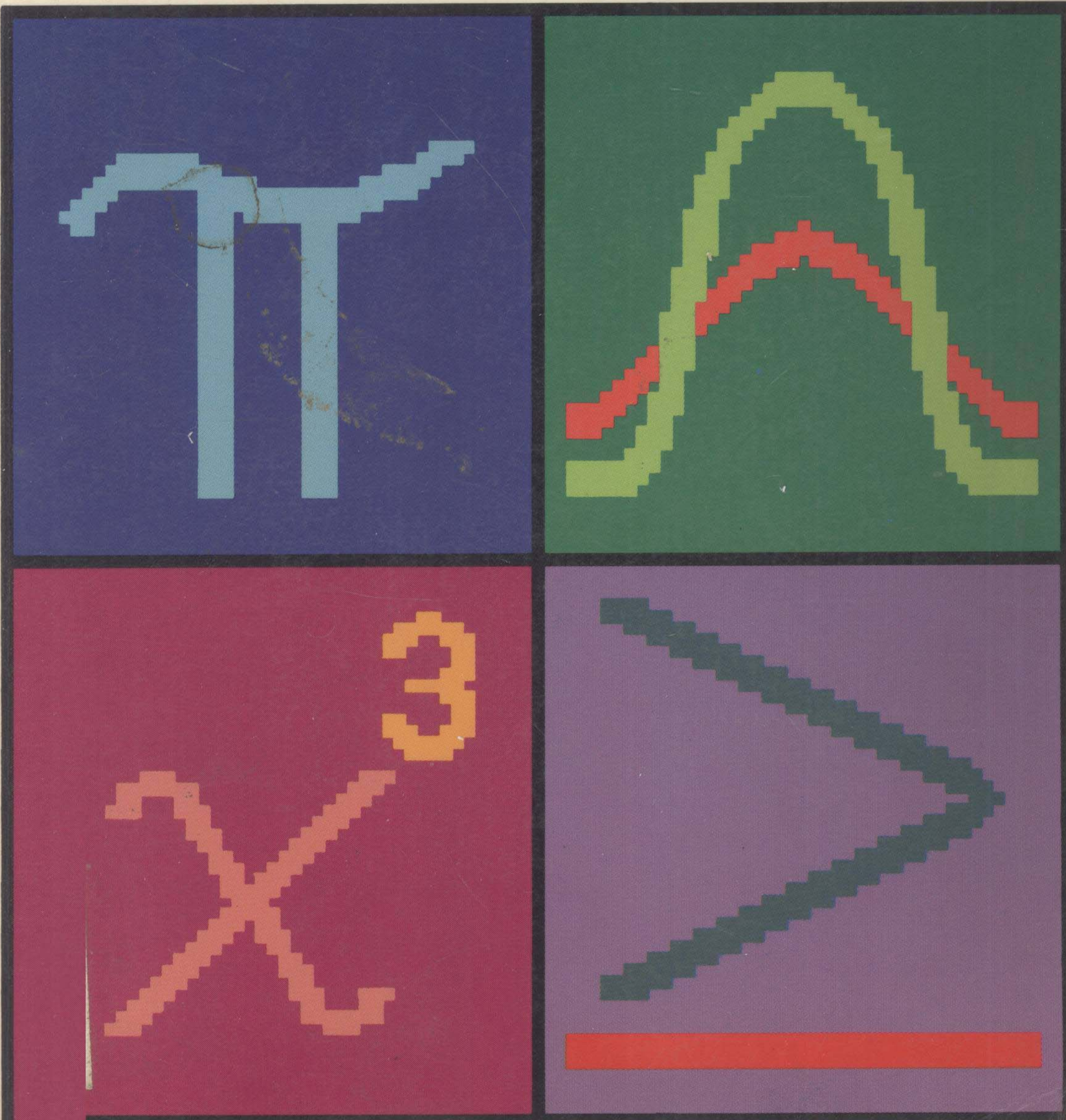

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# Foreword

The computer is a rich and complex tool that is increasingly within the financial means of schools. Like any educational tool, it comes with inherent advantages and disadvantages, is more appropriate for some uses than others, is more suited to some teaching styles than others, and is neither the answer to all our educational ills nor the end of all that is great and good in our educational system. Like any tool, it can be used well or poorly, be overemphasized or ignored, and it depends on the human qualities of the wielder for its effectiveness.

The purpose of the Addison-Wesley Series on *Computers in Education* is to persuade you, as educators, that the future of computers in education is in your hands. Your interest and involvement in educational computer applications will determine whether computers will be the textbook, the TV, or the chalkboard of education for the next generation.

For years, textbooks have dominated school curricula with little input from classroom teachers or local communities. Recently, television has become the most influential and ubiquitous educator in society yet has not been widely or particularly successfully used by teachers in school. On the other hand, for over one hundred years the chalkboard has been the most individualized, interactive, and creatively used technology in schools.

Already, textbook-like computerized curricula are being churned out with little teacher or local community input. Already, computers are available for home use at prices comparable to a good color television set and with programs at the educational level of the soaps. If teachers are to gain control over computers in education and make them be their chalkboards, the time to act is now.

Each book in the *Computers in Education* series is intended to provide teachers, school administrators, and parents with information and ideas that will help them begin to meet the educational challenge computers present. Taken as a whole, the series has been designed to help the reader:

- Appreciate the potential and the limits of computers in education.
- Develop a functional understanding of the computer.
- Overcome apprehension about and fear of the computer.
- Participate in efforts to introduce and integrate computers into a school.
- Use the computer creatively and effectively in teaching and administration.
- Apply a discerning and critical attitude toward the myriad computer-related products offered in increasing volume to the education market.
- Consider seriously the ethical, political, and philosophical ramifications of computer use in education.

*Practical Guide to Computers in Education* is the basic primer for the series. *Computers in Teaching Mathematics* is one of a number of books in the second tier of the series, each dealing with computer applications in particular educational contexts. Others include *Computers and Reading Instruction*, *Computers, Education, and Special Needs*, and *School Administrator's Guide to Computers in Education.* Still other titles are planned for this part of the series, including ones on computers and writing, business education, science, social studies, and the elementary school classroom. Each book in this second tier picks up where the *Practical Guide* leaves off. Each is more focused and provides far more practical detail to educators seriously considering computer use in their schools and curricula.

*Computers in Teaching Mathematics* is an exciting and challenging volume in this series. It recognizes the central role mathematics educators have played in bringing computers into the schools, and it presents a challenge to them to take a leadership role in the next phase of the educational computing revolution. The book is a unique blend of practical suggestions and forward-looking ideas. It provides a wealth of basic information mathematics teachers need in order to integrate computers into their schools and curricula, and it also explores in detail the potential of computers to transform the mathematics curriculum.

As series editor, I am excited to see this book in print. I believe mathematics educators will find it useful and challenging, and I look forward to seeing the next chapter in the educational computing revolution written by those who take up this challenge.

Peter Kelman

# *Preface*

We wrote this book on the premise that mathematics teachers are, and should be, at the forefront of the educational computing revolution. With the exception of two chapters (2 and 8) this book provides mathematics teachers with concrete ideas and suggestions for infusing computers into the mathematics curriculum. Chapter 1, "Transforming the Mathematics Curriculum" establishes the two major foci of the book: transforming process and transforming content. It introduces three themes that permeate the book: problem solving, computer graphics, and programming. Problem solving is the subject of Chapter 3, computer graphics is the focus of Chapter 4, and programming is discussed in Chapter 7.

Computer-based problem solving and computer graphics are presented as aspects of transforming the processes of mathematics. Programming, applied mathematics, and computer science are presented as aspects of transforming the content of the mathematics curriculum.

Each chapter differs in purpose and style, as well as subject. Chapter 2, "Traditional Computer Assisted Instruction," is the only chapter dealing with traditional uses of the computer in mathematics. Through fictional vignettes, it raises issues of educational philosophy and practice with regard to computer use in mathematics education.

Chapter 3, "Problem Solving: Transforming A Process," provides four detailed examples of how commercially available programs can alter traditional problem-solving activities in mathematics classrooms.

Chapter 4, "Using Computer Graphics in Mathematics," is filled with actual activities and exercises that use the graphics computer to teach mathematical concepts. Teachers will find a wealth of ideas with enough detail to implement many of them immediately.

Chapter 5, "Applied Mathematics: Transforming Content," presents two detailed case studies, one a fictional composite, the other true. Both show how the content of the mathematics curriculum could be radically altered by computer use.

Chapter 6, "Computer Science: Mathematics in the Computer," is a mini-course in computer science for teacher and students. Written with humor and in excruciating detail, it takes readers into the heart of the machine, the land of bits and bytes, so that they may understand the mathematics in the computer.

Chapter 7, "Programming and Computer Languages," is a polemic. It urges readers to consider programming not as a language learning activity, but as a construction activity, one in which creativity and problem solving play major roles.

Chapter 8, "The Mathematics Teacher as Computer Sponsor," is a short handbook for mathematics teachers who wish to be, or simply find themselves being, the educational computing leader of their school. It is filled with advice and concrete information to help teachers with that challenging task.

A large resources section assists the readers in that task and with implementing computer activities in their own mathematics courses.

Looking back to when we started work on this book, we find that our ideas on how we should approach the subject of computers in mathematics changed dramatically. We had thought about beginning gently, spending half the book on traditional uses of the computer and traditional mathematics goals and methods. But, as we shared knowledge and experiences with each other, we decided that computers held such great potential for bringing about change in the way we teach and learn mathematics we should dedicate the book to that purpose. So if at times we seem to get carried away, try to understand the excitement of working with so many educators whose experiences with computers are so promising.

As with any book that covers new ground, we were influenced by the work and ideas of many others. We benefitted especially from the suggestions of Bruce Vogeli, Judah Schwartz, Twila Slesnick, and Janet McDonald, but we take full responsibility for the final product and its message. As will be evident when you read the book, we have been particularly inspired by Logo and MicroDYNAMO, programming languages that could stand the mathematics curriculum on its head.

Writing a book with eight authors requires a great deal of logistical support. We could not have completed this book without the herculean efforts of various members of the Intentional Educations staff, most particularly Carol Trowbridge, Carol Nuccio, and Barbara Nielsen. Others at Intentional whose help was important to us include: Jack Reed, Janice Lewbin, Jane Caminos, and Susan Strand.

Finally, the support and patience of our families brought us through when deadlines loomed and meals were missed.

*March, 1983*

Peter Kelman
Art Bardige
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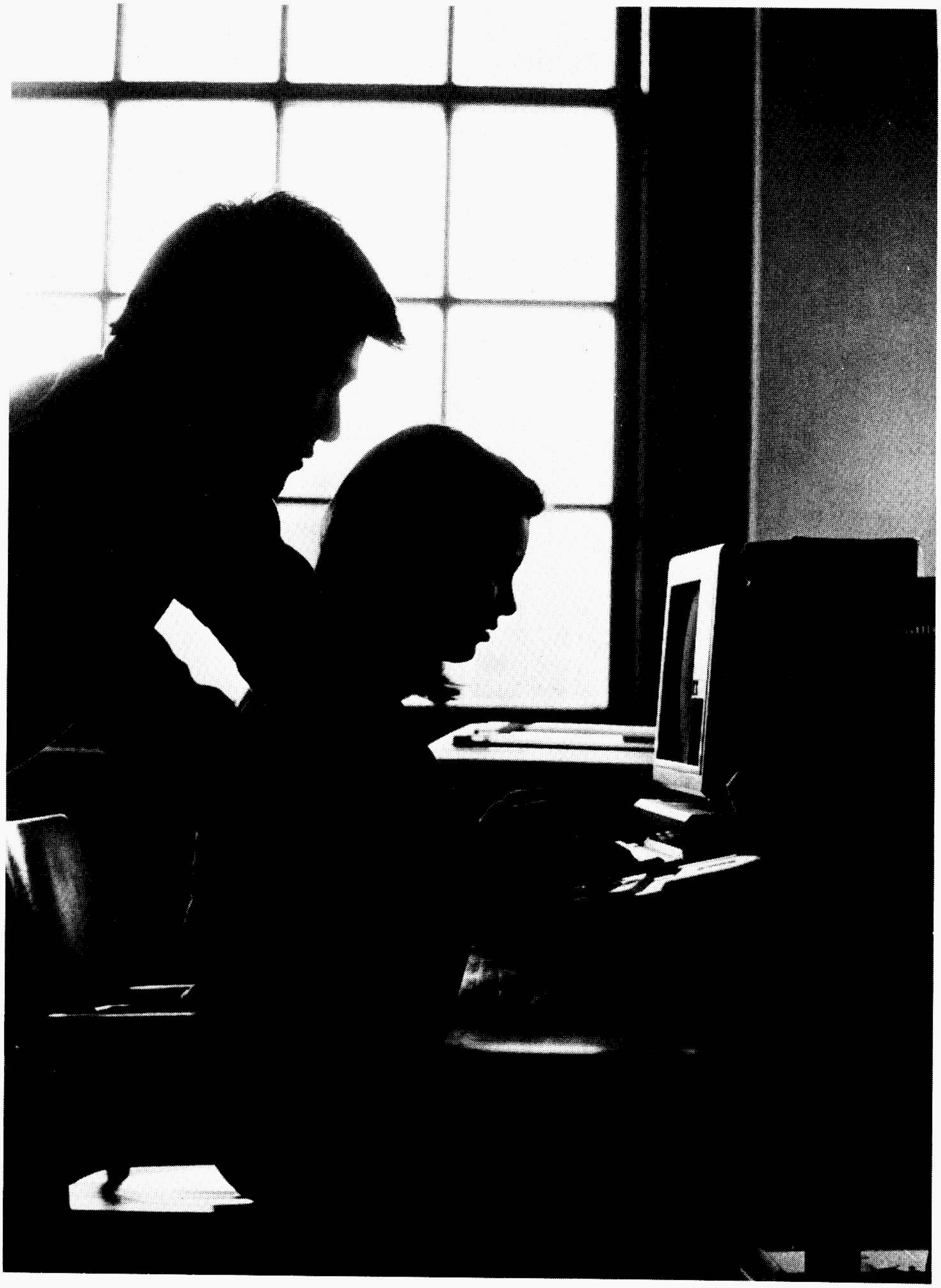

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# 1 Transforming the Mathematics Curriculum

Mathematics teachers were first to bring computers into the pre-college curriculum. In the early 1960s, a handful of forward-looking mathematics teachers, scattered across the country, arranged for their best students to have access to terminals of large **timesharing** computer systems. The students learned to program, usually in FORTRAN, the first **high-level computer language.** There was little else to do with computers in education at that time.

We've come a long way in the last twenty-five years. As nearly everyone is aware, computers themselves have changed dramatically, declining incredibly in size and cost, and increasing comparably in power and flexibility. As a result, computers have become part of virtually every aspect of our lives, including education in schools and at home.

Once again, mathematics teachers are at the forefront of computer implementation in schools. In fact, they have never ceased to be. Mathematics educators were among the first to use computers as an aid to instruction in the traditional curriculum. They pioneered so-called **computer assisted instruction** (CAI). The first large-scale computerized curriculum projects were in mathematics. The first educational computer games were mathematics games. And now, the entire mathematics curriculum appears to be the first of the traditional school curriculum areas to be undergoing substantial transformation because of computers.

All of this may not seem very surprising, since computers were designed to do mathematics and are fundamentally mathematics machines. Nevertheless, the inventive imagination mathematics educators have brought to educational computing is impressive, and the resulting innovations far from inevitable. Educational computing in mathematics might have remained dominated for some time by CAI and BASIC programming courses, if it hadn't been for the commitment of mathematics educators, in schools and universities, to unlocking the full educational potential of computers.

This is not meant to imply that there is anything wrong with CAI or BASIC programming courses; it is simply that these applications make only limited use of the vast capabilities of today's computers. Nevertheless, until very recently, educational computing consisted exclusively of these two types of application. As with all technological breakthroughs, people first viewed computers in terms of what they were used to. Just as the first automobiles were called horseless carriages and were designed to look and feel like buggies, the first computerized curricula were called drill and practice and tutorials and were designed to look and feel like workbooks and textbooks.

But, a new day is dawning for educational computing; traditional CAI is slowly giving way to a new generation of educational **software** and a new image of how the computer can be used educationally. The new software, dubbed "learningware" by some, do not emulate traditional curricular methods and materials: textbooks, workbooks, chalkboards, filmstrips, overhead projectors. Rather, they exploit the vast memory, logical structures, and impressive graphics capabilities of computers to produce an interactive, flexible, and powerful medium for teaching and learning. The corresponding new image of the computer in education is as a tool for learning, rather than as the latest audiovisual device. And for mathematics education, this new image includes the computer as a tool for *doing* mathematics.

However, in the last thirty years, new technologies and techniques have promised similarly dramatic changes in the way students are taught and learn. Radio, typewriters, film, mimeograph stencils, spirit masters, television, overhead transparencies, film strips, slide-tape presentations, film loops, Polaroid cameras, photocopiers, electronic calculators, thermofax machines, videotape, and others all have raised high hopes for educational improvement. And, all have failed to make much of a difference in children's learning or even in the ways we teach. So, teachers have a right to be skeptical about claims that computers will usher in a new age in education.

Nevertheless, the potential exists for computers to transform how and what is taught and learned in schools, and mathematics educators are the most likely group to play a major role in unlocking that potential for their students and themselves.

## TRANSFORMING THE PROCESSES OF MATHEMATICS EDUCATION

### The Computer as Problem-Solving Tool

Computers were originally designed to be used to solve scientific and business problems. As a result, they are powerful problem-solving devices with which students can probe problems, store and retrieve data, test out solutions, simulate problem situations, and calculate results. These problem-solving capacities of the computer enable students to work with much more interesting and complicated problems than they were able to before. They can ask "What if?" questions and attack problems in any number of ways. Most important, with computers, students have a new-found freedom to explore, to test strategies, and to play, all of which are at the heart of problem solving.

In the past, the difficulties of teaching problem solving have prompted educators to invent various recipes for it, some resembling old snake oil cures. The "scientific method," the best known of these recipes, has had surprising vitality despite the insistence of creative scientists that it is pure fiction and that, as a method, it hinders the problem solver more than it helps. These recipes each offer the one "true" path to all problem solving. But there is no one way to solve problems; there are many paths, many techniques, and many tools for exploring problems, identifying patterns, and finding solutions. People who learn problem solving well are like actors who play many different roles. They have wide and varied repertoires of tools with which to define and solve problems. The computer culture has appropriated, incorporated, and enhanced many of the most powerful of these tools. These exist primarily in sophisticated forms as business software that can be used by high school students. But, some problem-solving tools are available now which are designed explicitly for educational settings, and more of this type will be coming soon.

Numerical-analysis tools developed for business and scientific applications, such as *VisiCalc* and *TK!Solver*, can be adapted for use by students

| | A | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | |
| 2 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 3 | | | | | | | | | | | |
| 4 | 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 6 | 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| 7 | 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| 8 | 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 9 | 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| 10 | 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |
| 11 | 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |
| 12 | 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| 13 | 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |
| 14 | | | | | | | | | | | |
| 15 | | | | | | | | | | | |

Fig. 1.1

in mathematics classes. With them, students can keep track of and separate variables, store and display data, or even generate arithmetic tables, such as the multiplication table in Fig. 1.1. Generating such tables can give students insight into their construction, and may enable them to recognize useful arithmetic patterns. One of the most powerful features of *VisiCalc* is the ease with which students can alter programs so that, for example, a whole number multiplication table can be easily and instantly converted into a decimal multiplication table, as in Fig. 1.2. Numerical analysis programs can also help students solve problems by making it easy for

| | A | C | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | |
| 2 | 0 | .1 | .2 | .3 | .4 | .5 | .6 | .7 | .8 | .9 | 1 |
| 3 | | | | | | | | | | | |
| 4 | .1 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 | .1 |
| 5 | .2 | .02 | .04 | .06 | .08 | .1 | .12 | .14 | .16 | .18 | .2 |
| 6 | .3 | .03 | .06 | .09 | .12 | .15 | .18 | .21 | .24 | .27 | .3 |
| 7 | .4 | .04 | .08 | .12 | .16 | .2 | .24 | .28 | .32 | .36 | .4 |
| 8 | .5 | .05 | .1 | .15 | .2 | .25 | .3 | .35 | .4 | .45 | .5 |
| 9 | .6 | .06 | .12 | .18 | .24 | .3 | .36 | .42 | .48 | .54 | .6 |
| 10 | .7 | .07 | .14 | .21 | .28 | .35 | .42 | .49 | .56 | .63 | .7 |
| 11 | .8 | .08 | .16 | .24 | .32 | .4 | .48 | .56 | .64 | .72 | .8 |
| 12 | .9 | .09 | .18 | .27 | .36 | .45 | .54 | .63 | .72 | .81 | .9 |
| 13 | 1 | .1 | .2 | .3 | .4 | .5 | .6 | .7 | .8 | .9 | 1 |
| 14 | | | | | | | | | | | |

Fig. 1.2