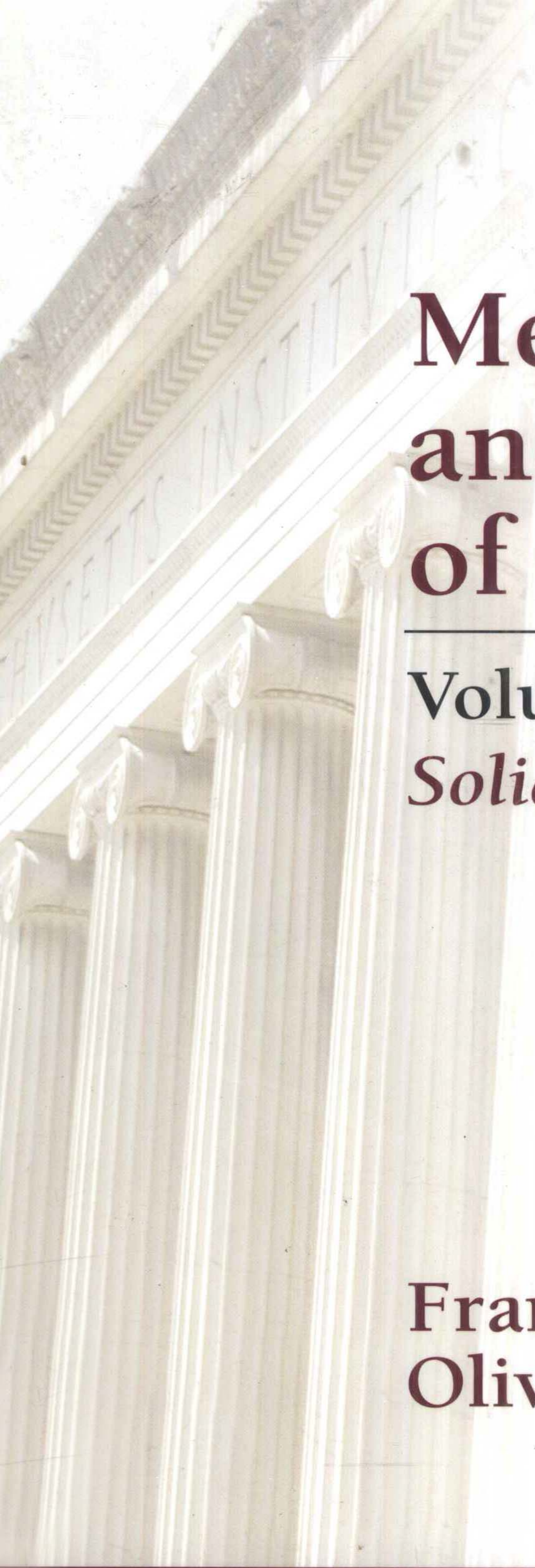

Mechanics and Durability of Solids

Volume I

Solid Mechanics

Franz-Josef Ulm
Olivier Coussy

MIT/Prentice Hall Series on Civil, Environmental, and Systems Engineering

Library of Congress Data Available

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Prentice Hall

Printed in the United States of America
10 9 8 7 6 5 4 3 2 1

ISBN 0-13-0479578

Pearson Education Ltd., *London*
Pearson Education Australia Pty. Ltd., *Sydney*
Pearson Education Singapore, Pte. Ltd.
Pearson Education North Asia Ltd., *Hong Kong*
Pearson Education Canada, Inc., *Toronto*
Pearson Educacion de Mexico, S.A. de C.V.
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Conversion Factors to SI Units

English	SI	SI symbol	To convert from English to SI multiply by	To convert from SI to English multiply by
		Area		
square inch	square centimeter	cm^2	6.452	0.1550
square foot	square meter	m^2	0.09290	10.76
acre	hectare	ha	0.4047	2.471
		Length		
inch	centimeter	cm	2.54	0.3937
foot	meter	m	0.3048	3.281
mile	kilometer	km	1.6093	0.6214
		Volume		
cubic inch	cubic centimeter	cm^3	16.387	0.06102
cubic foot	cubic meter	m^3	0.02832	35.32
gallon	cubic meter	m^3	0.003785	264.2
gallon	liter	L	3.785	0.2642
		Mass		
pound mass	kilogram	kg	0.4536	2.205
slug	kilogram	kg	14.59	0.06854
		Force		
pound	newton	N	4.448	0.2248
kip (1000 lb)	newton	N	4448	224.8
		Density		
slug/cubic foot	kilogram/cubic meter	kg/m^3	515.4	1.94×10^{-3}
		Work, Energy, Heat		
foot-pound	joule	J	1.356	0.7376
Btu	kilojoule	kJ	1.054	0.9479
Btu	kilowatt-hour	kWh	0.000293	3413
therm	kilowatt-hour	kWh	29.3	0.03413

Conversion Factors to SI Units (*continued*)

English	*SI*	*SI symbol*	*To convert from English to SI multiply by*	*To convert from SI to English multiply by*
		Power, Heat Rate		
horsepower	kilowatt	kW	0.7457	1.341
foot-pound/sec	watt	W	1.356	0.7376
Btu/hour	watt	W	0.2929	3.414
		Pressure		
pound/square inch	kilopascal	kPa	6.895	0.1450
pound/square foot	kilopascal	kPa	0.04788	20.89
feet of H_2O	kilopascal	kPa	2.983	0.3352
inches of Hg	kilopascal	kPa	3.374	0.2964
		Temperature		
Fahrenheit	Celcius	°C	$5/9(°F - 32)$	$9/5 \times °C + 32$
Fahrenheit	kelvin	K	$5/9(°F + 460)$	$9/5 \times K - 460$
		Velocity		
foot/second	meter/second	m/s	0.3048	3.281
mile/hour	meter/second	m/s	0.4470	2.237
mile/hour	kilometer/hour	km/h	1.609	0.6215
		Acceleration		
foot/second squared	meter/second squared	m/s^2	0.3048	3.281
		Torque		
pound-foot	newton-meter	$N \cdot m$	1.356	0.7376
pound-inch	newton-meter	$N \cdot m$	0.1130	8.85
		Viscosity, Kinematic Viscosity		
pound-sec/square foot	newton-sec/square meter	$N \cdot s/m^2$	47.88	0.02089
square foot/second	square meter/second	m^2/s	0.09290	10.76
		Flow Rate		
cubic foot/second	cubic meter/second	m^3/s	0.02832	35.32
cubic foot/second	liter/second	L/s	28.32	0.03532

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Preface

This textbook is the first of two volumes dealing with Mechanics and Durability of Solids. It provides an introduction to continuum mechanics and material modeling of engineering materials based on first energy principles. The second volume extends the approach to fracture and durability mechanics of solids. The overall theme of both volumes is a unified 'mechanistic' approach that uses energy concepts for modeling a large range of engineering material behavior, while generating the basis of a common language with other core disciplines in engineering sciences.

The first volume is composed of four parts: (I) Deformation and Strain; (II) Momentum Balance, Stress and Stress States; (III) Elasticity and Elasticity Bounds; (IV) Plasticity and Yield Design. Parts I and II introduce the two pillars of continuum mechanics and focus on geometrical and physical interpretation of strain and stresses, starting with the finite deformation theory, which is consistently linearized. Part III is dedicated to non-dissipative material behavior, with a focus on thermoelasticity and variational methods in elasticity and its application to heterogeneous material systems. Part IV starts with 1D plasticity, introducing ideal plasticity, hardening plasticity, and associated energy transformations. It is within the energy approach that the 1D Think models are extended to three dimensions, introducing the notion of associated and non-associated plasticity. Finally, the plastic collapse is introduced, leading to the development of the upper and lower bound theorem of limit analysis as bounds of the maximum admissible dissipation at plastic collapse of material systems and structures.

From the onset, our approach to writing this textbook was nourished by the multicultural flavor of our educational backgrounds: the pragmatism of the traditional German Engineering Mechanics education and the modern mathematical eloquence of "La Mécanique Rationelle." In such an endeavor, the need for a common language is critical. We developed this language over the years with our students on blackboards through 1D Think Models in France, Germany, Brazil, and finally at M.I.T. The outcome of this cultural adventure is this textbook; it is situated at the interface of Applied Mechanics and Engineering Mechanics.

The first ideas about writing this textbook go back to France, where we taught Continuum Mechanics together to undergraduate students in a joint program of L'École Normale Supérieure de Cachan and Université de Marne-La-Vallée. But it was M.I.T. that gave us the occasion to develop a spoken language into lecture notes for undergraduate and graduate students. Still, some of the Problem Sets in this textbook have a much longer history, rooted in the teaching of "La Mécanique Rationelle" by the most gifted educators, who instilled in us the beauty of Mechanics: Jean Mandel, Paul Germain, Jean Salençon, Yves Bamberger, Bernard Halphen; and with our colleagues and friends: Patrick de Buhan, Luc Dormieux, and many more. By recycling some of the Problem Sets from our drawers into this textbook, we trust that we remain true to our roots.

We wish to thank Professor Stein Sture of the University of Colorado at Boulder and Professor John Rudnicki of Nortwestern University for their assistance in reviewing the textbook.

We trust that this textbook will be a source of imagination.

FRANZ-JOSEF ULM
Cambridge, Massachusetts

OLIVIER COUSSY
Paris, France

Greek Alphabet and Transliteration

Name of Letter	Greek Alphabet		Transliteration
Alpha	A	α	a
Beta	B	β	b
Gamma	Γ	γ	g
Delta	Δ	δ ∂	d
Epsilon	E	ϵ	e
Zeta	Z	ζ	z
Eta	H	η	ē
Theta	Θ	θ ϑ	th
Iota	I	ι	i
Kappa	K	κ	k
Lambda	Λ	λ	l
Mu	M	μ	m
Nu	N	ν	n
Xi	Ξ	ξ	x
Omicron	O	o	o
Pi	Π	π	p
Rho	P	ρ	r; *initially*, rh
Sigma	Σ	σ s	s
Tau	T	τ	t
Upsilon	Υ	υ	u; *except after* a, e, ë, i, *often* y
Phi	Φ	ϕ φ	ph
Chi	X	χ	kh
Psi	Ψ	ψ	ps
Omega	Ω	ω	ō

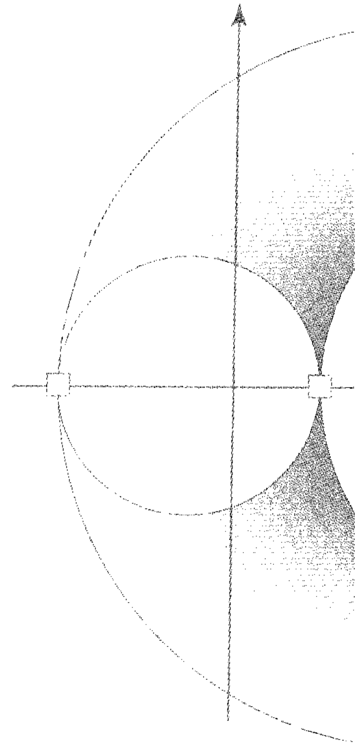
Contents

PART ONE

DEFORMATION AND STRAIN

CHAPTER 1

Description of Finite Deformation



This first chapter is devoted to the description of finite deformation of continuous material systems. By this we mean material systems, of which the behavior is described by means of continuum mechanics. This first chapter develops the essential mathematical ingredients for the description of the deformation without restriction on the order of magnitude of the deformation. These are the deformation gradient and the transport formulas of a material vector, of an elementary volume, and of an oriented surface. In addition, based on the analysis of the transport of the scalar product of two material vectors in deformation, the appropriate strain measures are derived: the Cauchy dilatation tensor and the Green–Lagrange strain tensor. They account for length and angle variations due to deformation and are invariant with respect to rigid body motion. Finally, the link between strain tensors and displacement is derived.

1.1 THE CONTINUUM MODEL

Continuum mechanics is concerned with the continuous description of the transformation of a material point within a system. A system is the part of the world in which we have a special interest; we denote this domain Ω. In engineering, the system is the structure under consideration (e.g., in civil engineering a tunnel, a bridge, or a foundation) and the interest that we have in it as engineers is the analysis of its deformation, stresses, and so on when subjected to loading. Every structural system is characterized by a length scale, defining the structural dimension (substratum height H, foundation width B, tunnel radius R, etc.), as sketched