Hu Huang

Dynamics of Surface Waves in Coastal Waters

Wave-Current-Bottom Interactions

海岸水域表面波动力学

波–流–海底相互作用

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Contents

1
Preliminaries

The history of the study on surface water waves, from the early work in 1687 by Newton to the pioneer resonant interaction theory in 1967 by Phillips, is briefly reviewed first, involving two important coastal current models: one-equation model—the mild-slope equation and its variants approximating the vertical structure of surface waves and averaging over variable depth; a shallow water approximation—the Boussinesq-type equations which reduce a three-dimensional problem into a two-dimensional one. Then three kinds of the formulations on surface water wave problems, i.e. the classical, the Lagrangian and the Hamiltonian, are described in outline.

1.1 Water Wave Theories in Historical Perspective

70.8% of the earth's surface is covered by oceans, the great theoretical and practical importance of water waves cannot be overestimated. Surface water waves, subjected to gravity force, surface tension, and other forces, are the most easily observed and studied; however, there is still a lot we don't know about these waves, particularly in coastal waters where uneven bottom topography plays a distinctive and vital role in wave propagation.

Historically [9], the subject of water waves traces back to the work by Newton in *Principia* (1687), against hydrostatics by Archimedes in 3 B. C. Much later, while accompanied by nonlinear water waves considered by Gerstner, the linear wave theory reached a real level of advances by the works of Laplace, Lagrange, Poisson, and Cauchy. Following this is the period of substantial contributions by Russell on the nonlinear solitary experiments, Green, Kelland, Airy on the nonlinear shallow water equations, and Earnshaw. Then, publishing his great paper in 1847 [42], Stokes ushered in a new era of his own weakly nonlinear water waves [9,10]. Later, the KdV equation of an important development appeared explicitly in 1895 by Korteweg and de Vries, but implicitly in 1872 by Boussinesq [33], that is, the Boussinesq equations.

Modern water wave theory began with weak, nonlinear interactions among gravity waves on the surface of deep water [36], which were confirmed and extended by Hasselmann [15], subsequently culminating in the Zakharov formulation or the wave turbulence theory [24,41,46–48] incorporating the effects of cubic or quartet interactions without limitations on spectral width on deep-intermediate water. In shallow coastal water, the nonlinear wave field is dominated by near-resonant quadratic interactions involving triplets of waves. It is the main wave-current-bottom interactions that have made rich and progressive coastal wave modeling since the late 1960s, albeit less mature relative to the well-established deep-water wave models [18,23]. At present, there is a wide variety of viewpoints to describe coastal water waves, such as the linear and nonlinear, the deterministic and stochastic, the time and frequency domains, the phase-resolving (for rapidly varying waves) and phase-averaged (for slowly varying waves), and parabolic approximation.

An overview of the current main and typical coastal wave models is as follows.

1.1.1 The Mild-Slope Equations

Linear theory all along plays a guiding and basic role in constructing theories. Take the mild-slope equations for example. The mild-slope equations simplify the refraction and diffraction of the linear surface waves in water of intermediate, variable depths by approximating the vertical structure of the motion in which a specific, preselected, depth function that corresponds to propagating waves in water of constant depth is adopted, and averaging over the depth by a vertical integration concerned essentially with Galerkin's method and variational principles. The original mild-slope equation was derived independently by Eckart [12], Berkhoff [5], and Smith and Sprinks [40]. Many of its extended counterparts have since been added, but most of them deal with pure wave motion apart from a few extensions on wave-current interactions by, for example, Kirby [21]. Huang [17] recently showed that the classical mild-slope equation of Berkhoff [5], the mild-slope equation for wave-current interactions by Kirby [21], the modified mild-slope equation by Chamberlain and Porter [7], and the hierarchy of partial differential equations by Miles and Chamberlain [34], can arise from an elaborate system of approximations to wave-current interactions over uneven bottoms.

Because any one-equation model cannot capture all features of the problem, the coupled-mode system, an infinite set of coupled equations, has been investigated by presenting a multi-mode approximation, such as the evanescent mode, the bottom mode, and the propagating wave mode [3,8,30,38].

Some recent entries into extensive literature in the linear mild-slope equations are provided in [11,16,20,25,37].

There also exist a number of deterministic and stochastic nonlinear mild-slope equations involving resonance in both wave-wave interactions and wave-bottom interactions played dominantly by Bragg scattering [1,2,13,19,43].

1.1.2 The Boussinesq-Type Equations

Boussinesq (1872) once advanced a theory for shallow water waves over a horizontal bottom, much later it was developed to the classical Boussinesq equations for an uneven bottom by Mei and LeMéhauté [31], Madsen and Mei [28], and Peregrine [35]. The current Boussinesq-type equations, featuring prominently in reducing the three-dimensional problem to a two-dimensional one, have attracted considerable attention over the past 20 years, thus giving rise to a number of enhanced and higher-order Boussinesq-type equations with the objective of improving linear and nonlinear properties [22,27,29], and allowing for wave propagation in almost all finite water depths.

Theoretically, Boussinesq-type equations are rich in almost every aspect of wave transformation over variable depth and in ambient (depth-uniform) currents, such as short-crested waves [14]. It is probably the richness that has practically made the present higher-order Boussinesq-type equations dauntingly complex in form. What should be the next step in the right direction by comparison with directly using the Navier-Stokes equations?

1.2 The Governing Equations

When water waves begin to propagate across the surface of water initially at rest, the motion is in effect irrotational. Consider that incompressible inviscid fluid is in irrotational motion over a rigid, impermeable bottom of varying quiescent depth $h(x,y)$, x and y denoting horizontal Cartesian coordinates. The vertical coordinate, z, is measured positively upwards with the free surface elevation at $z=\zeta(x,y,t)$. The governing equations for wave motion are then given as

$$\nabla^2\Phi+\frac{\partial^2\Phi}{\partial z^2}=0,\quad -h\leqslant z\leqslant\zeta, \tag{1.2.1}$$

$$\frac{\partial\Phi}{\partial t}+\frac{1}{2}\left[(\nabla\Phi)^2+\left(\frac{\partial\Phi}{\partial z}\right)^2\right]+\frac{p_\mathrm{a}-\mathscr{T}}{\rho}+g\zeta=0,\quad z=\zeta, \tag{1.2.2}$$

$$\frac{\partial\zeta}{\partial t}+\nabla\Phi\cdot\nabla\zeta=\frac{\partial\Phi}{\partial z},\quad z=\zeta, \tag{1.2.3}$$

$$\frac{\partial\Phi}{\partial z}+\nabla\Phi\cdot\nabla h=0,\quad z=-h, \tag{1.2.4}$$

where Φ is the velocity potential, g the gravitational acceleration, ρ the fluid density, p_a the atmospheric pressure, $\nabla=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y}\right)$, $\mathscr{T}$ represents the surface tension effect

$$\mathscr{T}=\gamma\frac{\zeta_{xx}(1+\zeta_y^2)+\zeta_{yy}(1+\zeta_x^2)-2\zeta_{xy}\zeta_x\zeta_y}{(1+\zeta_x^2+\zeta_y^2)^{\frac{3}{2}}}=\gamma\nabla\cdot\left[\frac{\nabla\zeta}{(1+|\nabla\zeta|^2)^{\frac{1}{2}}}\right], \tag{1.2.5}$$

in which γ is the surface tension coefficient.

If $p_\mathrm{a}=\gamma=0$, a combined condition for Φ arises from eliminating ζ from the two free surface conditions (1.2.2) and (1.2.3)

$$\frac{\partial^2\Phi}{\partial^2}+g\frac{\partial\Phi}{\partial z}+\left[\frac{\partial}{\partial t}+\frac{1}{2}\nabla\Phi\cdot\nabla+\frac{1}{2}\frac{\partial\Phi}{\partial z}\frac{\partial}{\partial z}\right]\left[|\nabla\Phi|^2+\left(\frac{\partial\Phi}{\partial z}\right)^2\right]=0,\quad z=\zeta. \tag{1.2.6}$$

The pressure $p(x,y,z,t)$ is given by Bernoulli equation

$$\frac{\partial\Phi}{\partial t}+\frac{1}{2}\left[(\nabla\Phi)^2+\left(\frac{\partial\Phi}{\partial z}\right)^2\right]+\frac{p-p_\mathrm{a}}{\rho}+gz=0,\quad -h\leqslant z\leqslant\zeta. \tag{1.2.7}$$

In water of infinite depth, the kinematic boundary condition on the bottom (1.2.4) is replaced by

$$\left|\left(\nabla+\frac{\partial}{\partial z}\right)\Phi\right| \to 0, \quad z \to -\infty. \tag{1.2.8}$$

1.3 Lagrangian Formulation

Fundamentally, mechanics can be classified as two main branches: Lagrangian mechanics and Hamiltonian mechanics, based respectively on variational principles and the energy concept. Luke's variational principle [26] for irrotational motion is

$$\delta \iint L\mathrm{d}x\mathrm{d}y\mathrm{d}t = 0, \tag{1.3.1}$$

with the Lagrangian

$$L = -\rho \int_{-h}^{\zeta} \left\{ \frac{\partial \Phi}{\partial t} + \frac{1}{2}\left[(\nabla\Phi)^2 + \left(\frac{\partial \Phi}{\partial z}\right)^2\right] + gz \right\} \mathrm{d}z. \tag{1.3.2}$$

It should be noted that for rotational flow there already existed Bateman's variational principle [4] containing the results of Seliger and Whitham [39] and Luke [26], and the averaged Lagrangian of Whitham [44,45] rests on Luke's variational principle.

1.4 Hamiltonian Formulation

Zakharov [46], Broer [6], and Miles [32] independently discovered the Hamiltonian theory of surface waves in which the canonical variables are the velocity potential $\varphi(x,y,t)$ at, and the displacement ζ of, the free surface,

$$\frac{\delta \mathscr{H}}{\delta \varphi} = \rho \frac{\partial \zeta}{\partial t}, \quad \frac{\delta \mathscr{H}}{\delta \zeta} = -\rho \frac{\partial \varphi}{\partial t}, \tag{1.4.1}$$

with

$$\mathscr{H} = \iint \mathrm{d}x\,\mathrm{d}y H$$
$$= \frac{1}{2}\rho \iint \mathrm{d}x\,\mathrm{d}y \left\{ g\zeta^2 + \int_{-h}^{\zeta} \mathrm{d}z \left[(\nabla\Phi)^2 + \left(\frac{\partial\Phi}{\partial z}\right)^2 \right] \right\}. \quad (1.4.2)$$

For surface capillary-gravity waves, (1.4.2) can be extended as

$$\mathscr{H} = \frac{1}{2}\rho \iint \mathrm{d}x\,\mathrm{d}y \left\{ g\zeta^2 + \int_{-h}^{\zeta} \mathrm{d}z \left[(\nabla\Phi)^2 + \left(\frac{\partial\Phi}{\partial z}\right)^2 \right] + 2\gamma\left(\sqrt{1+|\nabla\zeta|^2} - 1\right) \right\}. \quad (1.4.3)$$

In (1.4.1), ζ and φ can be respectively replaced by their Fourier decomposition coefficients, ζ_k and φ_k, that is

$$\frac{\delta\mathscr{H}}{\delta\varphi_k} = \rho\frac{\partial\zeta_k}{\partial t}, \quad \frac{\delta\mathscr{H}}{\delta\zeta_k} = -\rho\frac{\partial\varphi_k}{\partial t}. \quad (1.4.4)$$

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2
Weakly Nonlinear Water Waves Propagating over Uneven Bottoms

A number of errors involving the expressions for the concepts, the algebraic operations, and the typographical in the derivation of the third-order evolution equations of Liu and Dingemans are pointed out and corrected. These modified equations are then extended to fourth-order with the stability analysis for uniform Stokes waves in which there exist certain fourth-order terms concerning the stability of finite depth waves, and to including the effects of ambient currents. These extensions result in a couple of equations, i.e. a general third-order evolution equation for the envelope of a modulated wave train with a current over an uneven bottom and an equation of the second-order (locked and free) long waves induced by the modulated wave train.

2.1 Modified Third-Order Evolution Equations of Liu and Dingemans

To a great extent, evolution equations in coastal waters depend on how to describe topographical features [11]. The more typical the bottom topography is, the more general the evolution equations are. Liu and Dingemans (hereafter referred to as L & D) [12] studied the second-order long waves generated by a modulating wave train over such an uneven bottom as

$$h = h_0(\mathbf{x_1}) + \delta^2 h_1(\mathbf{x}), \tag{2.1.1}$$

$$O(\delta) = O(\frac{\nabla h_0}{kh_0}) = O(\frac{\lambda}{\Lambda}) \ll 1, \quad O(kh_1) = O(\delta^2), \quad O(\frac{h_1}{\Lambda}) = O(\delta^3), \tag{2.1.2}$$

$$\mathbf{x} = (x, y), \quad \mathbf{x_1} = (x_1, y_1) = (\delta x, \delta y), \quad t_1 = \delta t, \tag{2.1.3}$$

where δ denotes the small modulation parameter, $O(\delta) = O(\varepsilon)$, ε is the small nonlinearity parameter, λ and Λ are respectively the wave length of the carrier wave and the horizontal length scale.

A number of errors are unfortunately found in the derivation of L & D and now corrected in their numbering equations (adding R notation) as follows [8,10]

$$\sum_{m=0}^{\infty} \frac{(\delta^2 h_1)^m}{m!} \frac{\partial^m}{\partial z^m} \left[\frac{\partial \Phi}{\partial z} + \delta \nabla_1 h_0 \cdot \nabla \Phi + \delta^2 \nabla h_1 \cdot \nabla \Phi \right] = 0, \quad (z = -h_0), \tag{R3.6}$$

$$\frac{\partial \phi^{(n,m)}}{\partial z} = F^{(n,m)}, \qquad (z = -h_0), \tag{R3.28}$$

$$H^{(2,0)} = \frac{\partial \phi^{(1,0)}}{\partial t_1} + \left(k_0^2 - k_\infty^2\right) \left| \phi^{(1,1)} \right|_{z=0}^2, \qquad k_\infty = \frac{\omega_0^2}{g}, \tag{R4.7}$$

$$\frac{\partial}{\partial t_1} \left(\frac{A^2}{\omega_0} \right) + \nabla_1 \cdot \left(\mathbf{C_g} \frac{A}{\omega_0} \right) = 0, \tag{R4.17}$$

$$G^{(3,0)} = \frac{\partial}{\partial t_1} \left[g \zeta^{(2,0)} \right] + 2 \nabla_1 \cdot \left[\omega_0 \mathbf{k_0} |\phi^{(1,1)}|^2 \right]_{z=0}, \tag{R4.28}$$

$$g \nabla_1 \cdot \int_{-h_0}^{0} \nabla_1 \phi^{(1,0)} \, \mathrm{d}z + \frac{\partial}{\partial t_1} \left[g \zeta^{(2,0)} \right] + 2 \nabla_1 \cdot \left[\omega_0 \mathbf{k_0} |\phi^{(1,1,)}|^2 \right]_{z=0} = 0, \tag{R4.29}$$

$$\frac{\partial \zeta^{(2,0)}}{\partial t_1} + \nabla_1 \cdot \left[h_0 \nabla_1 \phi^{(1,0)} + \frac{\mathbf{k_0}}{2\omega_0} g \, |A|^2 \right] = 0, \tag{R4.30}$$

$$\frac{\partial^2 \zeta^{(2,0)}}{\partial t_1^2} - \nabla_1 \cdot \left[g h_0 \nabla_1 \zeta^{(2,0)} \right] = \nabla_1 \cdot \left[h_0 \nabla_1 \left(\frac{\omega_0^2 |A|^2}{4 \sinh^2 q} \right) \right] - \frac{\partial}{\partial t_1} \nabla_1 \cdot \left(\frac{\mathbf{k_0} g |A|^2}{2\omega_0} \right), \tag{R4.31}$$