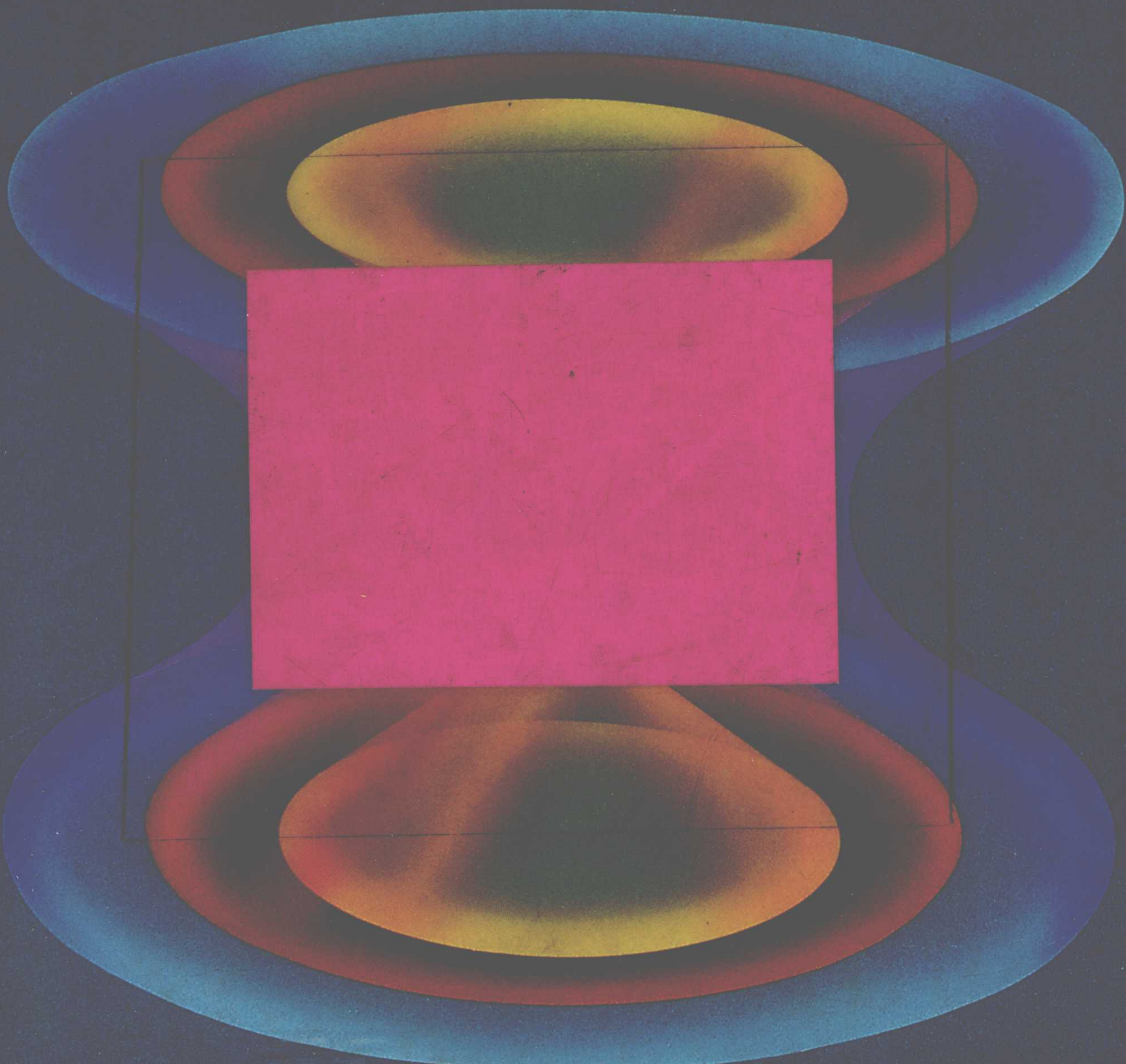
CALCULUS

ROBERT SEELEY



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HBJ

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PREFACE

Mathematics, and particularly calculus, provides an essential language and intellectual framework for science. With the explosive growth of science and technology comes a growing need for mastery of calculus, at least as a language, and better yet, as a way of thinking.

The wide variation in backgrounds and talents of calculus students presents a real challenge both to the instructor and to the textbook used. This book responds by exposing the main ideas of calculus as quickly as possible, before elaborating all the details. For example, a chapter on the applications of calculus to problems of graphing, optimization, and motion precedes any real discussion of limits. The idea is to exploit the students' geometric intuition, which is generally much better developed than their facility in calculation.

In the course of class testing over the years, I have found only good consequences of this approach. Students easily grasp the basic ideas of the applications of calculus and, with the understanding of this background, develop the maturity and motivation necessary for confronting the technical questions of limits. The same philosophy determines the treatment of infinite series and other topics. The textbook presents infinite series as a means of calculation *before* developing the abstract convergence tests.

Such minor reordering of traditional topics does not imply a lax treatment of the subject. Theoretical questions are treated with the prevailing standards of rigor; they are just treated somewhat later than usual. The difficult parts of the theory—such as the proofs of the Intermediate Value Theorem and Extreme Value Theorem and the justification of term-by-term operations on infinite series—are postponed to an appendix. The $\varepsilon - \delta$ definition of function limits is also in the appendix because very few students are ready for it early in the course. But the $\varepsilon - N$ definition of sequence limits is in the main body of the text in Chapter 11; by that point the definition seems natural.

The transcendental functions are presented in the order that is currently standard. The trigonometric functions are discussed early and provide a variety of examples for limits and derivatives. The exponential and logarithmic functions follow integrals, but could actually be covered at any time after Chapter 4. (Limits involving these functions are treated by l'Hopital's Rule in Chapter 4.)

Differential equations, discussed as slope fields, are introduced as soon as possible, beginning with antiderivatives in Chapter 2. They provide significant examples and problems throughout the text. The final chapter presents complex numbers and their use in solving constant coefficient equations, particularly the damped oscillator equation.

The textbook uses several pedagogical features to reinforce topic presentations:

- *An extensive art program* illustrates concepts and problems with two- or full-color figures.
- *Examples*, essential to illustrate and clarify general principles, are used extensively. They are clearly set off from the text discussions and are worked in great detail. Students should be encouraged to do their own work in similar detail to clarify thinking and avoid careless errors.
- Section *summaries* precede the problem sets for that section so that students can easily review as they work the problems.
- The *problems* are grouped in "A," "B," and "C" sets in order of increasing maturity. "A" problems are straightforward applications of the text material, "B" problems require more initiative or originality to work, and "C" problems are for students who show greater mathematical sophistication.
- *Review problems* end each chapter.
- *Answers* to problems marked with an asterisk (*) are given at the end of the book. (Answers to all the problems are given in the Instructor's Manual.)

ACKNOWLEDGMENTS

A book such as this is not created single-handedly. I owe a great debt to my students for their patience in using class notes and for their suggestions. My family, too, has contributed a good share of patience. My son Karl did much of the word processing, while Joe provided answers to many problems. I am grateful to my colleagues, particularly to Matt Gaffney for a multitude of suggestions and conversations about the book, to Steve Parrott for reading all of one version, and to Paul Salomaa for providing many answers; but most of all I am grateful to Dennis Wortman, who undertook the major task of writing the Instructor's Manual.

The book would not have been published without the support of several people. John Parker of Harcourt Brace Jovanovich kept an interest as the book developed. It has had essential support from a sequence of editors. Shelley Langman, Wesley Lawton, Ted Buchholz, and Richard Wallis presided over its development and provided support for the class testing, and Mike Johnson brought it into the world. The conscientious and cooperative book team included Robert Watrous, manuscript editor; Karen Denhams, production editor; Martha Gilman, designer; Vicki Kelly, art editor; Lesley Lenox, production manager; and Pamela Whiting, associate editor.

Comments from reviewers have, of course, been essential. They all have my grateful thanks: Charles C. Alexander, University of Mississippi; Glen D. Anderson, Michigan State University; Thomas F. Banchoff, Brown University; Douglas B. Crawford, College of San Mateo; Daniel S. Drucker, Wayne State University; Roger W. Hansell, University of Connecticut; James E. Hodge, Angelo State University; Kendell Hyde, Weber State College; Eleanor L. Kendrick, San Jose City College; Eleanor Killam, University of Massachusetts,

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Robert Seeley

SUPPLEMENTS

For Students

Student Solutions Manual by Dennis Wortman contains solutions for those problems that have answers in the text. (These are marked in the text with an asterisk.) The solutions are worked out with unusual care and completeness, and will be particularly instructive to those who want this extra help.

For Instructors

Instructor's Manual with Solutions by Dennis Wortman contains the worked-out solutions for *all* the problems in the text.

Computerized Testbank (Micro-Pac Genie) by Microsystems Software, Ltd., is the most complete test-generating and author system with graphics on the market. This system is available for the IBM PC, XT or compatible system, and the Macintosh.

Testbank is also available in a printed version.

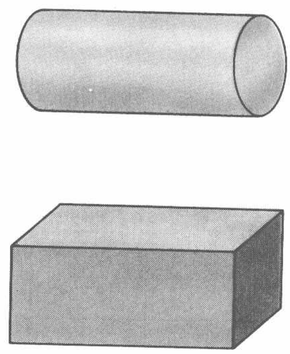
COMPUTER SUPPLEMENTS FOR INSTRUCTORS AND STUDENTS

CALCULUS 3.0 from True Basic is intended for users who wish to expand their basic understanding. Easy-to-use features include a simple, menu-driven interface and context-sensitive pop-up HELP and GLOSSARY screens. A dialog box is used to enter numbers and functions and to execute problems; a graph box displays plotted functions.

Calculus encourages free exploration of topics including the concept of limits, tangents, l'Hopital's Rule, as well as exploring and visualizing parametric equations and differential equations. It is available for Apple Macintosh, IBM PC, and IBM PS/2.

INTERACTIVE CALCULUS from Math Lab allows students to experience complex mathematical ideas in a graphic and intuitive setting. Concepts such as limits, differentiation, and integration are numerically and graphically performed in a single keystroke. It is available for Apple II, IBM PC, and IBM PS/2.

CALCAIDE, a computer software program by Elizabeth Chang, Hood College, is a microcomputer disk for IBM PC and IBM PS/2 systems with interactive color graphing and numerical computation programs for the mainstream calculus course.



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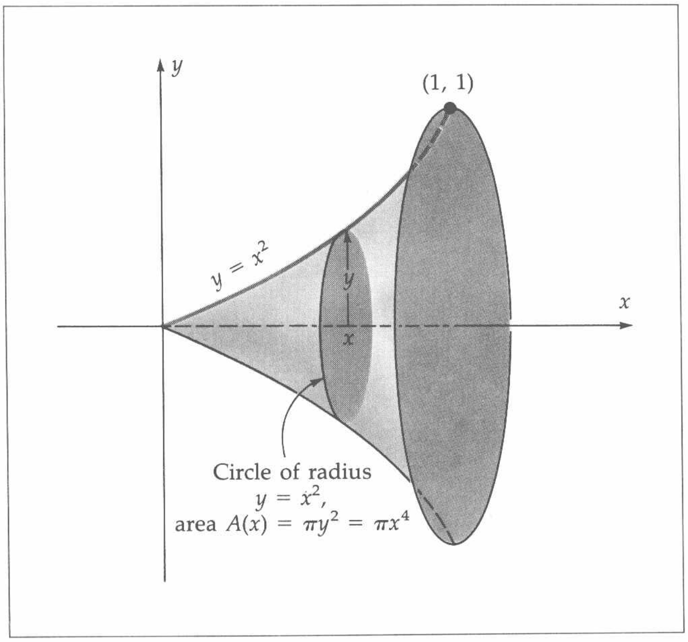
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Circle of radius
y = x²,
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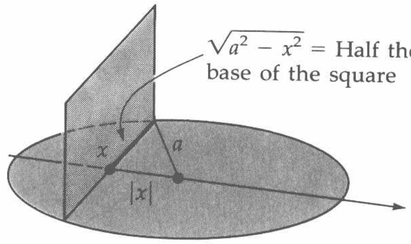

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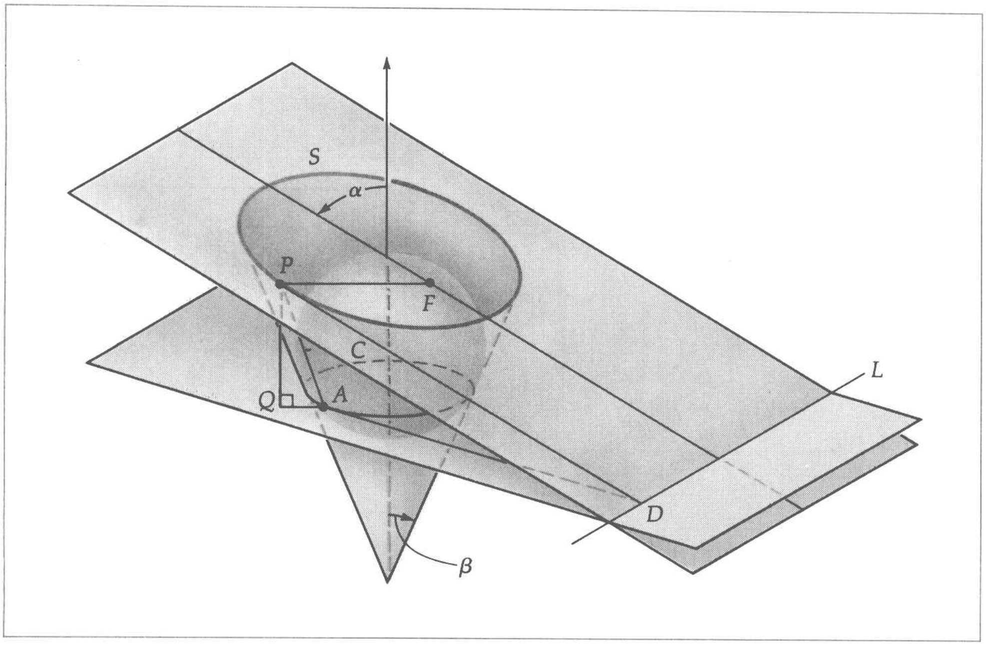
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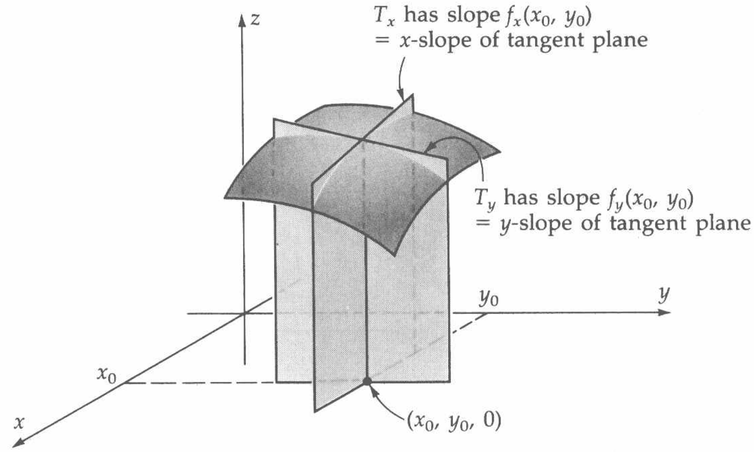

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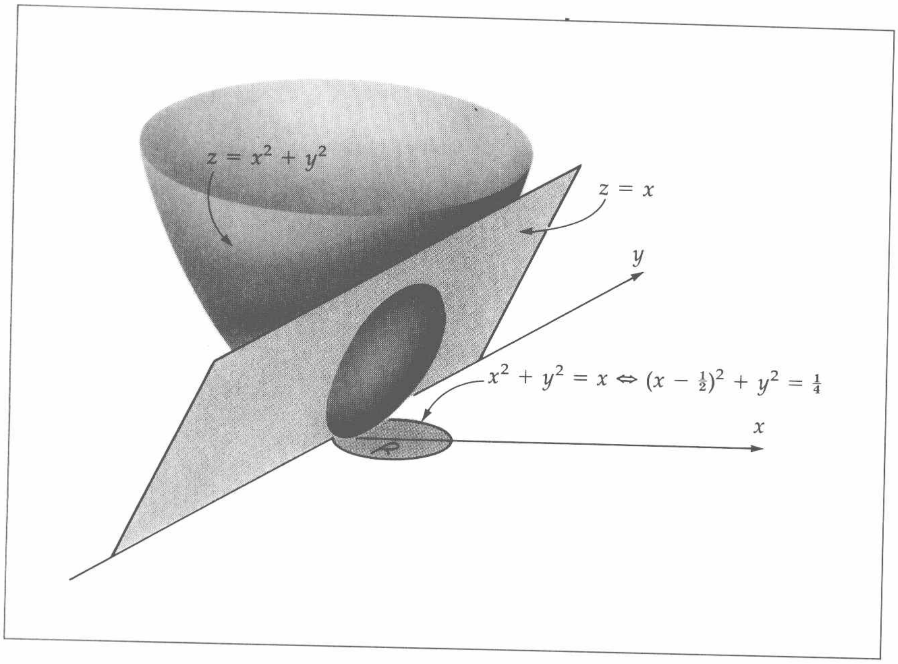

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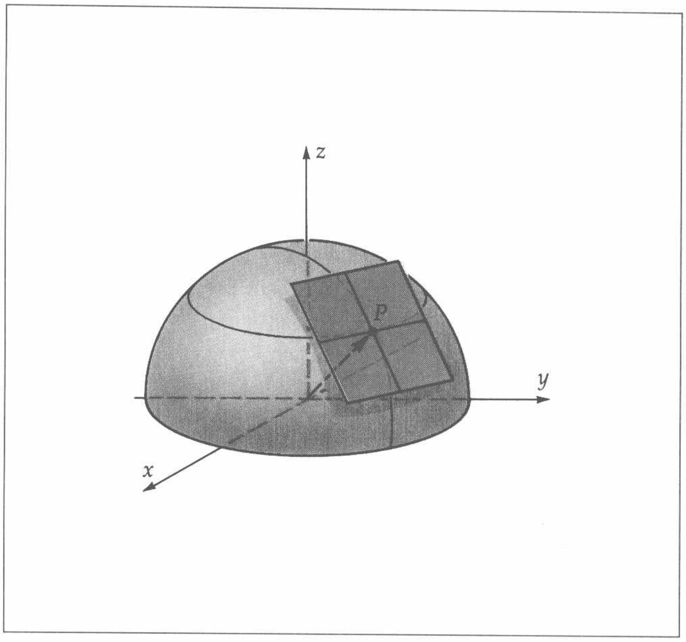
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BACKGROUND

1.1

COORDINATES

We begin by reviewing basic ideas and notation concerning coordinate systems. (Some of you know all of this, and all of you know some of it; that's the nature of a review.) Then we use coordinates to investigate two "optimization" problems, typical of those to be solved by calculus later, in Chapters 2–4.

The Number Line

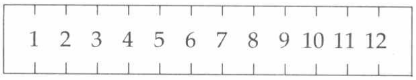


FIGURE 1
Humble ruler.

The humble ruler (Fig. 1) suggests the basic premise of analytic geometry: *The points of a straight line can be identified with numbers.* This appealing idea gave rise to some deep questions: What is a line? What is a number? Fortunately the questions have been answered[1], and we accept this premise as the basis for uniting algebra and geometry.

To identify numbers with points on a horizontal line, choose a unit of distance and an origin. Then label each point according to its distance from the origin, with negative numbers to the left and positive ones to the right (Fig. 2). You now have a **coordinate axis**, or a **number line**. The axis is the line itself; the number associated with a point on the line is a coordinate.

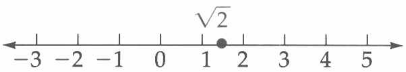


FIGURE 2
Number line.

On the coordinate axis, the distance from point x to the origin is the **absolute value of x**, denoted $|x|$ (Fig. 3):

$$|-2| = 2, \qquad |2| = 2, \qquad |-3| = 3, \qquad |0| = 0.$$

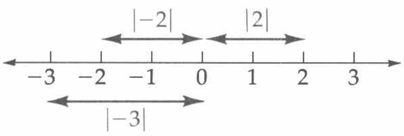


FIGURE 3
$|x|$ = distance from x to 0.

The absolute value $|x|$ gives the "size" of x but not its sign; that is, both 2 and -2 have the same absolute value 2. In algebraic terms, the absolute value is defined by

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$$

That $|x| = x$ when $x \geq 0$ is clear; but to understand the definition when $x < 0$, try for example $x = -2$:

$$|-2| = 2 = -(-2)$$

so indeed $|x| = -x$ when x is the negative number -2.

With a little thought and experimentation, you can understand two properties of the absolute value:

$$|xy| = |x| \cdot |y|$$

and

$$|x + y| \leq |x| + |y|.$$

[1] One of the best answers was given by Richard Dedekind in 1872; *see* James R. Newman *The World of Mathematics,* Vol. 1, pp 525–36.

FIGURE 4
$a < b$ means a is to the left of b.

Numbers are *ordered* according to their positions on the line, as shown in Figure 4:

$$a < b \text{ means } a \text{ is to the left of } b.$$

So $-100 < 2$; in words, -100 is less than 2. To compare size without regard to sign, use the absolute value; the inequality

$$|2| < |-100|$$

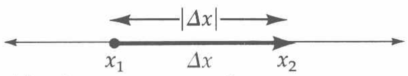
says that 2 is "smaller" than -100.

The *difference* of two numbers x_1 and x_2 is denoted Δx:

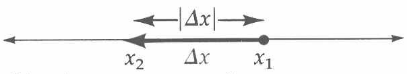
$$\Delta x = x_2 - x_1.$$

(The symbol Δ is the Greek capital letter delta, which corresponds to our letter D, for difference. Thus Δx is the *difference* of two values of x, not the product of a number Δ times a number x.) To form the difference, you need to know which number is first and which is second, that is, you need an *ordered* pair: x_1 first and x_2 second. Then Δx is *positive* if $x_2 > x_1$, and *negative* if $x_2 < x_1$. As in Figure 5 we indicate Δx by an arrow from x_1 to x_2; it points to the right if $\Delta x > 0$, and to the left if $\Delta x < 0$. The length of the arrow is the absolute value $|\Delta x|$, which gives the *distance* between x_1 and x_2:

$$\text{distance from } x_1 \text{ to } x_2 = |x_2 - x_1| = |\Delta x|.$$



(a) $\Delta x = x_2 - x_1 > 0$, so arrow points to the right.

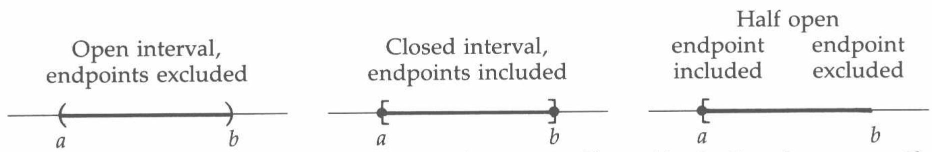


(b) $\Delta x = x_2 - x_1 < 0$, so arrow points to the left.

FIGURE 5
Arrows for Δx point *from* x_1 *to* x_2.

Segments on the line are called **intervals** of numbers. It can be crucial to know whether the endpoints are included or not. Square brackets indicate an included endpoint, parentheses an excluded one (Fig. 6). For example,

$$[a, b) = \text{all numbers } x \text{ between } a \text{ and } b, \text{ including } a \text{ and excluding } b.$$



(a) $(a, b) = \{x: a < x < b\}$. (b) $[a, b] = \{x: a \le x \le b\}$. (c) $[a, b) = \{x: a \le x < b\}$.

FIGURE 6
Intervals: Open, closed, half open.

If both endpoints are included, the interval is called **closed**; if both are excluded, it is **open**.

To describe intervals, and sets in general, we use the concise "set-builder notation." With this, the interval $[a, b)$ is described as

$$\{x: a \le x < b\}$$

Typical member of set. — Condition determining whether x is in the set.