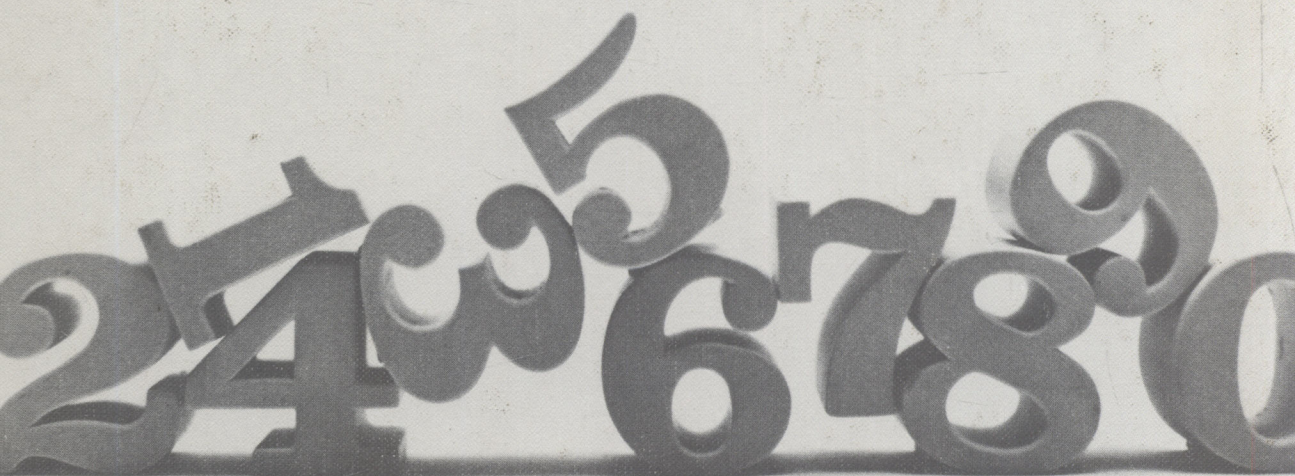


Introductory Mathematics

CHARLES P. McKEAGUE



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CUESTA COLLEGE

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Preface to the Instructor

This text is intended to be used in a lecture situation. Each section is written so that it can be discussed in a 45–50 minute class session. The text covers all the material usually taught in a basic mathematics or arithmetic class. In addition, there is a chapter on negative numbers and a chapter on solving linear equations.

The text has been written so it can be used for classes in arithmetic made up of students that will not go on in mathematics, and for those classes that will lead to beginning algebra. The properties and definitions used in algebra are used here. The vocabulary necessary to communicate effectively in mathematics is defined and explained clearly. The properties and definitions are highlighted so that reference back to them is easy. Although the emphasis of the text is on understanding the principles of mathematics and their application to problem solving, memorization is indicated wherever it is necessary.

Organization of the Text

The book begins with a preface to the student explaining what study habits are necessary to ensure success in arithmetic.

The rest of the book is divided into ten chapters. Each chapter is organized as follows:

1. A short introduction explains in a very general way what the student can expect to find in the chapter. The introduction also specifies the material covered previously that is needed to develop the concepts in the chapter.
2. The body of the chapter is divided into sections. Each section contains three main parts:
 - a. *Explanations*: The explanations are made as simple and intuitive as possible without sacrificing mathematical correctness. Many of the properties developed in the first two chapters are referred to in later chapters. The idea is to make as few definitions and rules as possible, and then refer back to them when new situations are encountered.

- b. *Examples*: The examples are chosen to clarify the explanations and preview the problem sets. They cover all situations encountered in the problem sets and can be referred to easily when trouble arises.
- c. *Problem Sets*: The problem sets contain three main categories of problems: drill problems, calculator problems, and review problems.

DRILL PROBLEMS: These are the standard, noncalculator problems that parallel the examples. The problems increase in difficulty as the problem set progresses, and, in most cases, each even-numbered problem is similar to the odd-numbered problem that precedes it.

CALCULATOR PROBLEMS: Beginning in Chapter 2, most problem sets contain several problems to be worked on a calculator. There is at least one calculator problem for each major type of problem in the section. If the course is to be taught without the use of calculators, these problems can easily be omitted without loss of continuity.

REVIEW PROBLEMS: At the end of each problem set there are a number of review problems. Whenever possible, these review problems cover concepts from preceding sections that are needed for the development of the following section.

- 3. The chapter summary and review lists the new properties and definitions found in the chapter. The review also contains a list of common mistakes, clearly marked as such, so that the student can learn to recognize and avoid them.
- 4. The chapter diagnostic test is designed to give the students an idea of how well they have mastered the material in the chapter. The problems are representative of all problems in the chapter. These diagnostic tests can also be used as pretests if desired.

I think you will find that this text will assist you in the classroom. It is written in a style that is both readable and mathematically correct.

Acknowledgments

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I want to thank the many people who reviewed this manuscript for their suggestions and contributions to this book. I was lucky to have such a fine group of instructors contribute their thoughts: Charles Beals, Hartnell College; Philip R. Denham, Riverside City College; Delvis A. Fernandez, Chabot College; Fred Lane, Los Angeles Valley College; Donald Mazukelli, Los Angeles Valley College; William Pearce, Kilgore College; Paul Pontius, Pan American University; Dorothy Schwellenbach, Hartnell College; Jean Vincenzi, Saddleback Junior College.

Preface to the Student

Many of the people who enroll in my introductory math classes are apprehensive at first. They feel that, since they have had a difficult time with mathematics in the past, they are in for a difficult time again. Some of them even feel that it's really no use at all—that they have never really *understood* mathematics and probably never will.

Are you like that?

Most people who have a difficult time expect something from mathematics that is not there, and they end up disappointed and frustrated with their experience in math because of it. I think most people believe that, since mathematics is a logical subject, they should be able to understand it without any trouble. Mathematics is probably the most logical subject there is. But that doesn't mean it is always easy to understand. Many times, a new topic in math will not be understandable the first time you read through it. Some topics must be read and thought about many times before they become understandable. If you find yourself becoming confused at times, it doesn't necessarily mean something is wrong. Confusion is sometimes part of the process of understanding. The frustration and disappointment takes place when we expect to understand a new topic and then find out that we don't understand anything about it. My advice is, "Don't expect to understand anything you read the first time you read it. Mathematics just doesn't work that way."

If you are interested in having a positive experience with arithmetic, then read through the following list carefully. The list explains the things you can do to make sure that you are successful in arithmetic. I think you will find that it is possible to be successful in arithmetic.

How to Be Successful

1. *Attend all class sessions on time.* There is no way to know exactly what goes on in class unless you are there. Missing class and then expecting to find out what went on from someone else is not the same as being there yourself. All you get

from someone else is their impression of what went on. It may not be the same experience you would have had if you had been there.

2. *Read the book.* It is usually best to read the section that will be lectured on before you go to class. Reading, even when you don't understand everything you read, is better than not reading at all.
3. *Work problems every day.* The real key to success in mathematics is working problems. The more problems you work, the better you get at working problems. The problems you are struggling with are the ones you should spend the most time on.
4. *Do it on your own.* Don't be misled into thinking that someone else's work is your own. Watching someone else work through a problem is not the same as working the same problem yourself. It's okay to get help when you need it. Just make sure that when it is all over you have done all the work yourself.
5. *Don't expect to understand it the first time around.* Remember, topics in mathematics are not always understandable the first time they are encountered. That's just the way it is in math. Expecting to understand everything you come across immediately will only lead to disappointment and frustration.
6. *Spend as much time as it takes for you to get to the level you want to attain.* There is no set formula for the exact amount of time you need to spend on arithmetic to master it. You will find out quickly—probably on the first test—if you are spending enough time studying. Even if it turns out that you have to spend 2 or 3 hours on each section to master it, then that's how much time you should take. Spending less time than that will not get you there.

Introductory Mathematics

Contents

Preface to the Instructor ix
Preface to the Student xiii

1 The Basics 1

1.1 Place Value and Names for Numbers 1
1.2 Basic Facts and Properties of Addition 6
1.3 Introduction to Subtraction 13
1.4 Basic Facts and Properties of Multiplication 17
1.5 Introduction to Division 23
1.6 Introduction to Exponents 28
Chapter 1 Summary and Review 31
Chapter 1 Diagnostic Test 33

2 Whole Numbers 35

2.1 Addition with Whole Numbers 35
2.2 Rounding Numbers 43
2.3 Subtraction with Whole Numbers 46
2.4 Powers of 10 51
2.5 The Distributive Property and Multiplication with Whole Numbers 56
2.6 Division with Whole Numbers 61
2.7 Order of Operations and Average 68
2.8 Prime Numbers and Factors 73
Chapter 2 Summary and Review 78
Chapter 2 Diagnostic Test 79

3 Fractions 83

3.1 Meaning and Properties of Fractions 83
3.2 Reducing to Lowest Terms 92

3.3	Multiplication with Fractions	97
3.4	Division with Fractions	103
3.5	Addition and Subtraction with Fractions	108
3.6	Complex Fractions	116
	Chapter 3 Summary and Review	119
	Chapter 3 Diagnostic Test	122
4	Mixed Numbers	125
4.1	Mixed-Number Notation	125
4.2	Multiplication and Division with Mixed Numbers	130
4.3	Addition with Mixed Numbers	134
4.4	Subtraction with Mixed Numbers	138
4.5	Combinations of Operations	141
	Chapter 4 Summary and Review	144
	Chapter 4 Diagnostic Test	146
5	Decimals	149
5.1	Decimal Notation and Place Value	149
5.2	Addition and Subtraction with Decimals	154
5.3	Multiplication with Decimals	158
5.4	Division with Decimals	162
5.5	Fractions and Decimals	167
5.6	Square Roots	173
	Chapter 5 Summary and Review	177
	Chapter 5 Diagnostic Test	180
6	Ratio and Proportion	183
6.1	Ratios as Fractions	183
6.2	Rates and Unit Pricing	188
6.3	Solving Equations by Division	191
6.4	Proportions	195
6.5	Applications of Proportions	200
	Chapter 6 Summary and Review	204
	Chapter 6 Diagnostic Test	206
7	Percent	209
7.1	Percents and Decimals	209
7.2	Percents and Fractions	213
7.3	Basic Percent Problems	218
7.4	General Applications of Percents, Including Income Tax	223
7.5	Sales Tax and Commission	228
7.6	Percent Increase or Decrease, and Discount	232
7.7	Interest	237
	Chapter 7 Summary and Review	242
	Chapter 7 Diagnostic Test	244

8	Measurement and the Metric System	247
8.1	Length	247
8.2	Area	253
8.3	Volume	263
8.4	Weight	270
8.5	Converting from One System to the Other	273
	Chapter 8 Summary and Review	279
	Chapter 8 Diagnostic Test	281
9	Negative Numbers	285
9.1	Positive and Negative Numbers	285
9.2	Addition with Negative Numbers	290
9.3	Subtraction with Negative Numbers	296
9.4	Multiplication with Negative Numbers	300
9.5	Division with Negative Numbers	305
	Chapter 9 Summary and Review	310
	Chapter 9 Diagnostic Test	312
10	Solving Equations	315
10.1	The Distributive Property and Algebraic Expressions	315
10.2	The Addition Property of Equality	321
10.3	The Multiplication Property of Equality	325
10.4	Linear Equations in One Variable	329
10.5	Word Problems	332
	Chapter 10 Summary and Review	337
	Chapter 10 Diagnostic Test	339
	Appendix 1: One Hundred Addition Facts	AP-1
	Appendix 2: One Hundred Multiplication Facts	AP-3
	Appendix 3: Using a Calculator	AP-5
	Answers to Odd-Numbered Problems	AN-1
	Index	IN-1

The Basics 1

The material in this chapter is needed to study basic mathematics. Some of it may seem very simple, and some of it may seem a little confusing. You may find yourself saying, “What has this got to do with arithmetic?” But you have to learn this material before you can go on to the rest of this book. There are things in this chapter that you will have to memorize. The facts and concepts that need to be memorized are always clearly indicated. Don’t expect these concepts to come to you just by reading them. You actually have to spend time memorizing them even if you understand them.

1.1 PLACE VALUE AND NAMES FOR NUMBERS

Our number system is based on the number 10 and is therefore called a “base 10” number system. We write all numbers in our number system using the *digits* 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The positions of the digits in a number determine the values of the digits. For example, the 5 in the number 251 has a different value from the 5 in the number 542.

The *place values* in our number system are as follows: The first digit on the right is in the *ones column*. The next digit to the left of the ones column is in the *tens column*. The next digit to the left is in the *hundreds column*. For a number like 542, the digit 5 is in the hundreds column, the 4 is in the tens column, and the 2 is in the ones column.

If we keep moving to the left, the columns increase in value. (Actually, each column is 10 times as large as the column on its right. We will say more about this when we cover multiplication.) The following diagram shows the name and

value of each of the first seven columns in our number system:

<i>Millions Column</i>	<i>Hundred Thousands Column</i>	<i>Ten Thousands Column</i>	<i>Thousands Column</i>	<i>Hundreds Column</i>	<i>Tens Column</i>	<i>Ones Column</i>
1,000,000	100,000	10,000	1,000	100	10	1

EXAMPLE 1 Give the place value of each digit in the number 305,964.

SOLUTION Starting with the digit at the right, we have:

4 in the ones column, 6 in the tens column, 9 in the hundreds column, 5 in the thousands column, 0 in the ten thousands column, and 3 in the hundred thousands column. ■

To find the place values of digits in larger numbers we can refer to Table 1.

TABLE 1

<i>Hundred Billions 100,000,000,000</i>	<i>Ten Billions 10,000,000,000</i>	<i>Billions 1,000,000,000</i>	<i>Hundred Millions 100,000,000</i>	<i>Ten Millions 10,000,000</i>	<i>Millions 1,000,000</i>	<i>Hundred Thousands 100,000</i>	<i>Ten Thousands 10,000</i>	<i>Thousands 1,000</i>	<i>Hundreds 100</i>	<i>Tens 10</i>	<i>Ones 1</i>
---------------------------------------------	----------------------------------------	-----------------------------------	-----------------------------------------	------------------------------------	-------------------------------	--------------------------------------	---------------------------------	----------------------------	-------------------------	--------------------	-------------------

EXAMPLE 2 Give the place value of each digit in the number 73,890,672,540.

SOLUTION The following diagram shows the place value of each digit.

<i>Ten Billions</i>	<i>Billions</i>	<i>Hundred Millions</i>	<i>Ten Millions</i>	<i>Millions</i>	<i>Hundred Thousands</i>	<i>Ten Thousands</i>	<i>Thousands</i>	<i>Hundreds</i>	<i>Tens</i>	<i>Ones</i>
7	3,	8	9	0,	6	7	2,	5	4	0

■

We can use the idea of place value to write numbers in *expanded form*. For example, the number 542 can be written in expanded form as

$$542 = 500 + 40 + 2$$

because the 5 is in the hundreds column, the 4 is in the tens column, and the 2 is in the ones column.

Here are more examples of numbers written in expanded form.

EXAMPLE 3 Write 5,478 in expanded form.

SOLUTION $5,478 = 5,000 + 400 + 70 + 8$ ■

EXAMPLE 4 Write 354,798 in expanded form.

SOLUTION $354,798 = 300,000 + 50,000 + 4,000 + 700 + 90 + 8$ ■

EXAMPLE 5 Write 56,094 in expanded form.

SOLUTION Notice that there is a 0 in the hundreds column. This means we have 0 hundreds. In expanded form we have:

$$56,094 = 50,000 + 6,000 + 90 + 4$$

Note that we don't have to include the 0. ■

EXAMPLE 6 Write 5,070,603 in expanded form.

SOLUTION The columns with 0 in them will not appear in the expanded form.

$$5,070,603 = 5,000,000 + 70,000 + 600 + 3$$
 ■

The idea of place value and expanded form can be used to help write the names for numbers. Naming numbers and writing them in words takes some practice. Let's begin by looking at the names of some two-digit numbers. Table 2 lists a few. Notice that the two-digit numbers that do not end in 0 have two parts. These parts are separated by a hyphen.

TABLE 2

Number	In English	Number	In English
25	Twenty-five	30	Thirty
47	Forty-seven	62	Sixty-two
93	Ninety-three	77	Seventy-seven
		50	Fifty

In mathematics, a collection of numbers is called a *set*. In this chapter and the next chapter we will be working with the set of *whole numbers*, which is defined as follows:

$$\text{Whole numbers} = \{0, 1, 2, 3, \dots\}$$

The dots mean “and so on,” and the braces $\{ \}$ are used to group the numbers in the set together.

Another way to visualize the whole numbers is with a *number line*. To draw a number line we simply draw a straight line and mark off equally spaced points along the line, as shown in Figure 1. We label the point at the left with 0, and the rest of the points, in order, with the numbers 1, 2, 3, 4, 5, and so on.

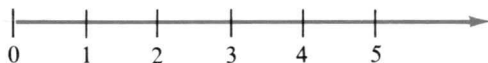


Figure 1

The arrow on the right indicates that the number line can continue in that direction forever. When we refer to numbers in these first two chapters, we will always be referring to the whole numbers, and all these numbers can be shown on a number line.

PROBLEM SET 1.1

Give the place value of each digit in the following numbers. (See Examples 1 and 2, and Table 1.)

- | | | | |
|----------|-----------|-------------|-------------|
| 1. 78 | 2. 93 | 3. 45 | 4. 79 |
| 5. 348 | 6. 789 | 7. 608 | 8. 450 |
| 9. 2,378 | 10. 6,481 | 11. 273,569 | 12. 768,253 |

Give the place value of the 5 in each of the following numbers.

- | | | |
|-----------------|-----------------|---------------------|
| 13. 458,992 | 14. 75,003,782 | 15. 507,994,787 |
| 16. 320,906,050 | 17. 267,894,335 | 18. 234,345,678,789 |
| 19. 4,569,000 | 20. 50,000 | |

Write each of the following numbers in expanded form. (See Examples 3–6.)

- | | | | |
|---------------|---------------|---------------|----------------|
| 21. 658 | 22. 479 | 23. 68 | 24. 71 |
| 25. 4,587 | 26. 3,762 | 27. 32,674 | 28. 54,883 |
| 29. 3,462,577 | 30. 5,673,524 | 31. 407 | 32. 508 |
| 33. 30,068 | 34. 50,905 | 35. 3,004,008 | 36. 20,088,060 |