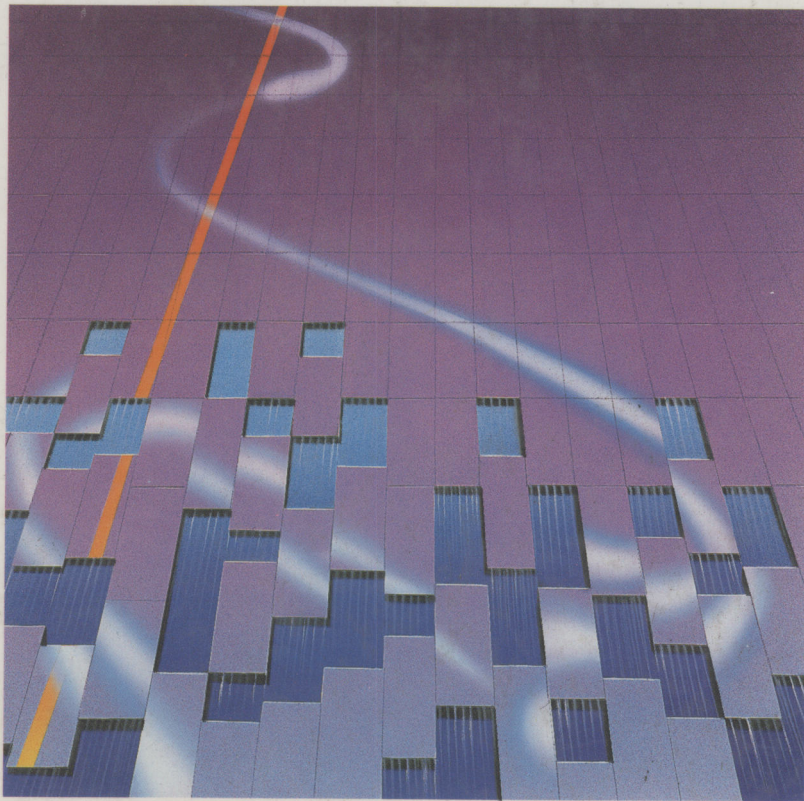


Digital Signal Processing

A Practical Approach



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Preface

Purpose of this book

This book was born out of our experience in teaching practically oriented courses in digital signal processing (DSP) to undergraduate students at the University of Plymouth and the Sheffield Hallam University, and to application engineers in industry for many years. It appeared to us that many of the available textbooks were either too elementary or too theoretical to be of practical use for undergraduates or application engineers in industry. As most readers will know from experience, the gap between learning the fundamentals in any subject and actually applying them is quite wide. We therefore decided to write this book which we believe undergraduates will understand and appreciate and which will equip them to undertake practical digital signal processing assignments and projects. We also believe that higher degree students and practising engineers and scientists will find this text most useful.

Our own research work over the last two decades in applied DSP has also inspired the contents, by identifying practical issues for discussion and presentation to bridge the gap between theoretical concepts and practical implementation, and by suggesting application examples, case studies, and problems.

The current great interest and developments in DSP both in industry and academia are likely to continue for the foreseeable future. The availability of numerous digital signal processors recognizes the commercial potential of DSP. Its major attraction lies in the ability to achieve guaranteed accuracy and perfect reproducibility, and in its inherent flexibility compared with analogue signal processing. In industry, many engineers lack the necessary knowledge and expertise in DSP to utilize the immense potential of the very powerful digital signal processors now available off the shelf. This book provides insight and practical guidance to enable engineers to design and develop practical DSP systems using these devices.

In academia, DSP is generally regarded as one of the more mathematical topics in the electrical engineering curriculum, and based on our experiences of teaching we have reduced the mathematical content to what we consider useful, essential, and interesting; we have also emphasized points of difficulty. Our experiences indicate that students learn best if they are aware of the practical relevance of a subject, and while more theoretical texts are essential for completeness and reference as the student matures in the subject, we believe in

producing graduates equipped also with practical knowledge and skills. This book was written with these considerations in mind.

The book is not a comprehensive text on DSP, but it covers most aspects of the subject found in undergraduate electrical, electronic or communication engineering degree courses. A number of DSP techniques which are of particular relevance to industry are also covered and in a few years, we believe, these will find their way into undergraduate curricula. These include techniques such as adaptive filtering and multirate processing.

The emphasis throughout the book is on the practical aspects of DSP. C language programs are provided to enable readers to explore the concepts presented in the book, to design and analyse their own DSP systems, and to gain a deeper understanding of DSP.

Commercial DSP software is available and plays a key role in the design and analysis of DSP systems. However, with most commercial software, it is difficult actually to find out or follow how a given operation is performed. The programs given in this book are useful in verifying results obtained manually. For example, by inserting break points the user can check the intermediate results of computation and follow the way the computation is made. Having acquired a sound knowledge of the principles, commercial software packages with good graphics support and user-friendly interfaces may then be used in most designs.

Main features of the book

- Provides an understanding of the fundamentals, implementation, and applications of DSP techniques from a practical point of view.
- Clear and easy to read, with mathematical contents reduced to that which is necessary for comprehension.
- DSP techniques and concepts are illustrated with practically oriented worked examples.
- Provides practical guidance to enable readers to design and develop actual DSP systems. Complete design examples and practical implementation details are given, including assembly language programs for the TMS320C10 and C25 processors.
- Provides C language implementation of many DSP algorithms and functions including programs for
 - digital FIR and IIR filter design,
 - finite wordlength effect analysis of user-designed IIR filters,
 - converting from cascade to parallel realization structures,
 - correlation computation,
 - discrete and fast Fourier transform algorithms,
 - inverse z -transformation,

- frequency response estimation, and
- multirate processing systems design.
- PC-based C programs are available on a computer disk to encourage readers to participate more actively in the learning process (see the section ‘How to obtain the program disk for this book’ in this preface for details).
- Contains many real-world application examples.
- Contains many end-of-chapter problems.
- Use of realistic examples to illustrate important concepts and to reinforce the knowledge gained.

The intended audience

The book is aimed at engineering, science and computer science students, and application engineers and scientists in industry who wish to gain a working knowledge of DSP. In particular, final year students studying for a degree in electronics, electrical or communication engineering will find the book valuable for both taught courses as well as their project work, as increasingly a greater proportion of student project work involves aspects of DSP. Postgraduates studying for a master’s degree or PhD in the above subjects will also find the book useful.

Undergraduate students will find the fundamental topics very attractive and, we believe, the book will be a valuable source of information both throughout their course as well as when they go into industry.

Large commercial or government organizations who undertake their own internal DSP short courses could base them on the book. We believe the book will serve as a good teaching text as well as a valuable self-learning text for undergraduate, graduate and application engineers.

Contents and organization

Chapter 1 contains an overview of DSP and its applications to make the reader aware of the meaning of DSP and its importance. The chapter presents, from a practical point of view using real-world examples, many fundamental topics which form the cornerstone of DSP, such as sampling and quantization of signals and their implications in real-time DSP. Discrete-time signals and systems are introduced in this chapter, and discussed further in Chapter 3.

Discrete transforms, particularly the discrete and fast Fourier transforms (FFT), provide important mathematical tools in DSP as well as relating the time and frequency domains. They are introduced and described in Chapter 2 with a discussion of some applications to put them in context. The derivation of the discrete Fourier transform (DFT) from the Fourier transform and

the exponential Fourier series provides a logical justification for the DFT which does not require coverage of the discrete Fourier series which would unnecessarily increase the length of the book (and the amount of work for the student!). The discussion has also been restricted to the description and implementation of the transforms. In particular, the topic of windowing has not been included in this chapter but is more appropriately discussed in detail in Chapter 10 on spectrum analysis.

In Chapter 3 the basics of discrete-time signals and systems are discussed. Important aspects of the z -transform, an invaluable tool for representing and analysing discrete-time signals and systems, are discussed. Many applications of the z -transform are highlighted, for example its use in the design, analysis and computation of the frequency response of discrete-time signals and systems. As in the rest of the book, the concepts as well as applications of the z -transform are illustrated with fully worked examples.

Correlation and convolution are fundamental and closely related topics in DSP and are covered in depth in Chapter 4. The authors consider an awareness of all the contents of this chapter to be essential for DSP, but after a preliminary scanning of the contents the reader may well be advised to build up his or her detailed knowledge by progressing through the chapter in stages. The contents might well be spread over several years of an undergraduate course.

Chapters 5, 6 and 7 include detailed practical discussions of digital filter design, one of the most important topics in DSP, being at the core of most DSP systems. Digital filter design is a vast topic and those new to it can find this somewhat overwhelming. Chapter 5 provides a general framework for filter design. A simple but general step-by-step guide for designing digital filters is given.

Techniques for designing FIR (finite impulse response) filters from specifications through to filter implementations are discussed in Chapter 6. Several fully worked examples are given throughout the chapter to consolidate the important concepts. A complete filter design example is included to show how all the stages of filter design fit together.

IIR (infinite impulse response) filter design is discussed in detail in Chapter 7, based on the simple step-by-step guide. The effect of quantization on filter performance in its various forms is discussed. In particular, ADC (analogue-to-digital) noise, coefficient quantization errors and their effects on frequency response and stability, arithmetic roundoff errors and overflow and procedures to minimize the effects of these errors are discussed.

Multirate processing techniques allow data to be processed at more than one sampling rate and have made possible such novel applications as single-bit ADCs and DACs (digital-to-analogue converters), and oversampled digital filtering, which are exploited in a number of modern digital systems, including for example the familiar compact disc player. In Chapter 8, the basic concepts of multirate processing are explained, illustrated with fully worked examples and by the design of actual multirate systems.

In Chapter 9, key aspects of adaptive filters are described, based on the LMS (least-mean-squares) and RLS (recursive least-square) algorithms which

are two of the most widely used algorithms in adaptive signal processing. The treatment is practical with only the essential theory included in the main text.

In Chapter 10, the important topic of spectrum estimation and analysis used to describe and study signals in the frequency domain is described with the emphasis being on the traditional nonparametric approach. Readers who are particularly interested in spectral analysis should study both Chapters 10 and 2 as Chapter 10 draws on explanations and worked examples given in Chapter 2. Those who master the contents of these chapters will be well placed to become competent in the analysis of signals in the frequency domain. However, it is worth noting that while the chapter introduces nonparametric methods in depth, the more modern parametric methods are expected to become more important in the future. These methods have not been covered in detail and the reader is referred to the given references.

In the last decade, tremendous progress has been made in DSP hardware, and this has led to the wide availability of low cost digital signal processors. For a successful application of DSP using these processors, it is necessary to appreciate the underlying concepts of DSP hardware and software. Chapter 11 discusses the key issues underlying general- and special-purpose processors for DSP, the impact of DSP algorithms on the hardware and software architectures of these processors, and the architectural requirements for efficient execution of DSP functions. We have used the Texas Instruments devices, TMS320C10, TMS320C25 and TMS320C30, as well as other well-known processors, to illustrate specific points, where possible.

In Chapter 12, we describe the hardware development environment used to implement some of the DSP algorithms described in previous chapters. Two low cost target boards based on the Texas Instruments TMS320C10 and TMS320C25 processors are described. A number of applications of DSP are described in the form of case studies. The presentation draws on many concepts discussed in earlier chapters.

How to use the book

A useful approach for undergraduate teaching will be to cover the materials in Chapter 1, to provide the understanding of fundamental topics such as the sampling theorem and discrete-time signals and systems, and to establish the benefits and applications of DSP. Then discrete transforms should be introduced, starting with the DFT and FFT (Chapter 2), and the z -transform (Chapter 3). Aspects of Chapters 10 and 4 may be used to illustrate the application of the DFT and FFT. After an introduction to correlation processing using a selection of materials from Chapter 4, a detailed treatment of digital filters should be undertaken.

In our experience students learn more when they are given realistic assignments to carry out. To this end we would encourage substantial assignments on, for example, filter design, the inverse z -transform, the DFT and FFT. Laboratory work should also be designed to demonstrate and reinforce

the techniques taught. It is important that students actually participate as well as attend lectures.

For postgraduate students the approach could be the same but the pace will be more brisk, and the more specialist topics of multirate processing and adaptive filters will also be included.

How to obtain the program disk for this book

The C-language programs and TMS320C10/25 assembly language programs described in this book are available on a disk for the IBM PC (or compatibles), as 3 1/2" high density (1.44 Mbyte). The C-language programs are available in both executable form and as source codes. A C compiler is required to run the source codes, but not to run the executable codes. The programs are written in standard ANSI C under Borland Turbo C version 2.0. The TMS320C25 codes in the disk require the Texas Instrument SWDS (software development system) package to run. The TMS320C10 codes run on the target board described in Chapter 12; they will require minor modifications to run on other systems.

The cost per disk is £27.50 or \$50 (including sales tax). This price also includes illustrative examples of how to use the C-language programs.

To obtain the program disk, please send your order together with a cheque drawn on a UK bank, bankers draft or proof of direct bank transfer to:

Digital Signal Processing: A Practical Approach
PEP Research & Consultancy Limited
Charles Cross Centre
Constantine Street
Plymouth
Devon PL4 8DE
UK

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A valid VAT invoice will be issued on receipt of payment.

Defective disks will be replaced if returned within a reasonable time, but refunds are unfortunately not possible.

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The practical nature of the book made it difficult to keep to deadlines. Each chapter took much longer to write than we had imagined or planned for. We thank the acquisition editor, Tim Pitts, for his patience and encouragement.

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Emmanuel Ifeachor
Barrie Jervis
April 1993

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