Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

1051

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Proceedings

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Springer-Verlag
Berlin Heidelberg New York Tokyo

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Proceedings of a conference
held in Aarhus, Denmark, August 1–7, 1982

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Springer-Verlag
Berlin Heidelberg New York Tokyo 1984

Editors

Ib H. Madsen
Robert A. Oliver
Mathematics Institute, Aarhus University
8000 Aarhus C, Denmark

AMS Subject Classifications (1980): 55-XX, 57-XX, 18F25

ISBN 3-540-12902-2 Springer-Verlag Berlin Heidelberg New York Tokyo
ISBN 0-387-12902-2 Springer-Verlag New York Heidelberg Berlin Tokyo

Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.
2146/3140-543210

Lecture Notes in Mathematics

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PREFACE

The 4th Aarhus conference on algebraic topology in connection with the I.C.M. was held August 1.-7. 1982 at the Mathematics Institute, Aarhus University. The conference was supported by the Danish Natural Science Research Council, the Aarhus University Research Fund and the Danish Mathematical Society.

The conference was structured with plenary talks in the morning together with special sessions in the afternoon in three parallel running tracks. The special sessions were divided according to subject into four categories:

Algebraic K-theory and L-theory
Geometry of manifolds
Homotopy theory
Transformation groups.

Titles of all talks given at the conference are listed below.

These Proceedings contain papers which were presented at the conference, and some related papers. All papers have been refereed and we take this opportunity to thank the many referees. We would like to thank the City of Aarhus for inviting the participants of the conference and companions to a soupé in the Town Hall. Thanks also go to the Aarhus Congress bureau for arranging the accomodations for the participants. Especially we would like to thank Kirsten Boddum and Sonja Eld who handled the administrative and secretarial duties.

Aarhus, October 1983

Ib Madsen, Bob Oliver

LIST OF PARTICIPANTS

Frank Adams, Cambridge
Doug Anderson, Syracuse
David Anick, Berkeley
Amir Assadi, Virginia
Michèle Audin, Paris
Tony Bahri, Rutgers
Tony Bak, Bielefeld
Andrew Baker, Chicago
Jean Barge, Nantes
Eva Bayer-Fluckiger, Genève
Steffen Bentzen, Aarhus
Bernhelm Booss, Roskilde
C.F. Bödigheimer, Heidelberg
Nourreddine Boudriga, Paris
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Shaun Bullett, London
Dan Burghelea, Ohio State
Marcel Bökstedt, Bielefeld
Gunnar Carlsson, San Diego
Manuel Castellet, Barcelona
Ruth Charney, Yale
Monica Clapp, Mexico
Francis Clarke, Swansea
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Fred Cohen, Kentucky
Frank Conolly, Notre Dame
Jim Davis, Princeton
Mike Davis, Columbia
Tammo tom Dieck, Göttingen
Albrecht Dold, Heidelberg
Simon Donaldson, Oxford
Karl Heinz Dovermann, Purdue
Johan Dupont, Aarhus
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Sam Gitler, Mexico
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A NORM THEOREM FOR K_2 OF GLOBAL FIELDS

Anthony Bak

A.S. Merkurjev and A.A. Suslin have provided recently in [7] and [11] an important tool for studying problems envolving the functor K_2 applied to division rings. The tool is a reduced norm homomorphism $N_D : K_2(D) \longrightarrow K_2(K)$ where K is any commutative field and D is an arbitrary finite, central, K-division algebra. One promising area of application for the new tool is that of arithmetic. In [7], the authors take already a step in this direction by proving the following theorem [7, 17.4] : If K is a global field and if the index $\sqrt{[D:K]}$ of D/K is square free then the $\operatorname{coker}(N_D) \simeq (\mathbb{Z}/2\mathbb{Z})^{|\Sigma|}$ where Σ is the set of all real primes of K which do not split D. Furthermore, they conjecture that the restriction imposed on the index of D/K is unnecessary. The purpose of this note is to verify their conjecture.

The main theorem of this note will establish a result slightly sharper than that conjectured by Merkurjev and Suslin. The sharpened result is suggested by the Hasse-Schilling norm theorem (cf. [10, 33.15]). We recall this theorem next. If K is a global field, we shall let v denote any noncomplex prime of K. We let K_v denote the completion of K at v and we let $D_v = D \otimes_K K_v$. We let $\mu(K)$ (resp. $\mu(K_v)$) denote the group of all roots of unity of K (resp. K_v) and we set $m = |\mu(K)|$ and $m_v = |\mu(K_v)|$. We let $\left(\frac{\cdot}{v}\right) : K_v^{\bullet} \times K_v^{\bullet} \longrightarrow \mu(K_v)$ denote the m_v'th power norm residue symbol on K_v. Finally, we let Σ_D denote the set of all real primes v of K which do not split D and if Σ is any finite (possibly empty) set of noncomplex primes of K, we let $\Sigma(K) = \{c \mid c \in K^{\bullet},\ c \in (K_v^{\bullet})^{m_v} \text{ for all } v \in \Sigma\}$. The Hasse-Schilling norm theorem says the following: <u>If</u> K <u>is a global field and if</u> $\mathrm{Nrd}_D : D^{\bullet} \longrightarrow K^{\bullet}$ <u>denotes the usual reduced norm homomorphism on</u> $D^{\bullet}$ <u>then the sequence below is exact</u> $D^{\bullet} \xrightarrow{\mathrm{Nrd}_D} K^{\bullet} \longrightarrow K^{\bullet}/\Sigma_D(K)^{\bullet} \longrightarrow 1$. The following theorem is the analogon for K_2 of the theorem above.

THEOREM 1. <u>If</u> K <u>is a global field then the sequence below is exact</u>

$$K_2(D) \xrightarrow{N_D} K_2(K) \xrightarrow{\amalg\left(\frac{\cdot}{v}\right)} \coprod_{v \in \Sigma} (\pm 1) \rightarrow 1 .$$

It is worth mentioning in connection with the arithmetic applications of N_D that the theorem above is sufficient to resolve the ambiguity of (±1) appearing in certain cases of the solution [2], [3], [4] to the congruence subgroup and metaplectic problems for classical groups of K-rank > 1. The resolution takes the form conjectured in these papers.

We prepare now for the proof of Theorem 1. It will be assumed that the reader is familiar with the definition of the functor K_2 and with Matsumoto's presentation of K_2 of a field in terms of symbols. A good reference for these materials is Milnor [8].

If K is a global field and if Σ is a finite set of noncomplex primes of K, we define the group $K_2(\Sigma(K)) = K^\bullet \otimes \Sigma(K)^\bullet / \langle (1-a) \otimes a \mid a \in \Sigma(K)^\bullet, (1-a) \neq 0 \rangle$. By Matsumoto's theorem (cf. [8, §11]), it is clear that there is a canonical homomorphism $K_2(\Sigma(K)) \longrightarrow K_2(K)$ which is an isomorphism whenever Σ is the empty set. The next result will be required in the proof of Theorem 1.

THEOREM 2. <u>If</u> K <u>is a global field and if</u> Σ <u>is a finite set of noncomplex primes of</u> K <u>then the zero-sequence</u>

$$K_2(\Sigma(K)) \longrightarrow K_2(K) \xrightarrow{\amalg \left(\frac{\cdot}{v}\right)} \coprod_{v \in \Sigma} \mu(K_v) \longrightarrow 1$$

<u>is exact</u>, <u>except possibly at</u> $K_2(K)$; <u>here</u>, <u>its homology has order at most</u> 2. <u>Moreover</u>, <u>if</u> $8 \nmid m_v$ <u>for all</u> $v \in \Sigma$ <u>then the sequence is exact</u>.

I do not know if the condition that $8 \nmid m_v$ for all $v \in \Sigma$ is necessary in order that the sequence above be exact.

PROOF. By the Moore reciprocity law (cf. [6]), there is an exact sequence

$$\begin{array}{ccccccc} & & \coprod_v \zeta_v & \longmapsto & \prod_v \zeta_v^{\frac{m_v}{m}} & & \\ (*) \quad K_2(K) & \xrightarrow{\lambda} & \coprod_v \mu(K_v) & \xrightarrow{\partial} & \mu(K) & \longrightarrow & 1 \\ (a,b) & \longmapsto & \coprod_v \left(\frac{a,b}{v}\right) & & & & \end{array}$$

and by a generalization [4, 3.2] of the Moore reciprocity law, there is an exact sequence

$$\coprod_{v\notin\Sigma} \zeta_v \longmapsto \prod_{v\notin\Sigma} \zeta_v^{\frac{m_v}{m}}$$

$$(**) \qquad K_2(\Sigma(K)) \xrightarrow{\lambda_\Sigma} \coprod_{v\notin\Sigma} \mu(K_v) \xrightarrow{\partial_\Sigma} \mu(K) \longrightarrow 1 .$$

$$(a,b) \longmapsto \coprod_{v\notin\Sigma} \left(\frac{a,b}{v}\right)$$

Consider the following commutative diagram

$$\begin{array}{ccccccc} K_2(K) & \xrightarrow{\lambda} & \coprod_{v} \mu(K_v) & \xrightarrow{\partial} & \mu(K) & \longrightarrow & 1 \\ \uparrow & & \uparrow & & \uparrow & & \\ K_2(\Sigma(K)) & \xrightarrow{\lambda_\Sigma} & \coprod_{v\notin\Sigma} \mu(K_v) & \xrightarrow{\partial_\Sigma} & \mu(K) & \longrightarrow & 1 . \end{array}$$

The diagram induces a homomorphism $\ker(\lambda_\Sigma) \longrightarrow \ker(\lambda)$. We shall show that

$$\left|\ker(\lambda)/\mathrm{image}(\ker \lambda_\Sigma)\right| \le \begin{cases} 2 & \text{in general} \\ 1 & \text{if } 8 \nmid m_v \text{ for all } v\in\Sigma . \end{cases}$$

Once this has been done, the theorem will follow by chasing the commutative diagram above.

By a result of Tate [12, (33)], $\bigcap_n (K_2(K))^n$ is a subgroup of $\ker(\lambda)$ such that $\left|\ker(\lambda)/\bigcap_n (K_2(K))^n\right| \le 2$. Let k denote the least common multiple of all m_v such that $v\in\Sigma$. If $a,b \in K^\bullet$ then it follows from the definition of $K_2(\Sigma(K))$ that the symbol (a,b^k) of $K_2(K)$ lies in image$(K_2(\Sigma(K))$. Thus, $\bigcap_n (K_2(K))^n \subseteq (K_2(K))^k \subseteq \mathrm{image}(K_2(\Sigma(K))$. This establishes the first assertion of the theorem. By another result of Tate [12, (33)] (cf. also [5, Théorème 9 and Corollaire]), $\ker(\lambda) \subseteq (K_2(K))^n$ providing $8 \nmid n$. If $8 \nmid m_v$ for all $v\in\Sigma$ then clearly $8 \nmid k$. Thus, $\ker(\lambda) \subseteq (K_2(K))^k \subseteq$ image$(K_2(\Sigma(K))$. This establishes the second assertion of the theorem.

The next result will also be required in the proof of Theorem 1. Let $\mathrm{Nrd}_D : D^\bullet \longrightarrow K^\bullet$ denote the usual reduced norm homomorphism on $D^\bullet$. Let $K_2(\mathrm{Nrd}_D(D)) = K^\bullet \otimes \mathrm{Nrd}_D(D^\bullet)/\langle(1-a)\otimes a \mid a \in \mathrm{Nrd}_D(D^\bullet),\ 1-a \neq 0\rangle$. By

Matsumoto's theorem, it is clear that there is a canonical homomorphism $K_2(\mathrm{Nrd}_D(D^{\bullet})) \longrightarrow K_2(K)$ which is an isomorphism whenever $\mathrm{Nrd}_D(D^{\bullet}) = K^{\bullet}$. If K is a local or a global field then by a theorem [9, 2.2] of Rehmann and Stuhler, there is a homomorphism $\psi_D : K_2(\mathrm{Nrd}_D(D^{\bullet})) \longrightarrow K_2(D)$, $(a, \mathrm{Nrd}_D(\beta)) \longmapsto (a, \beta)$.

PROPOSITION. <u>If</u> K <u>is a local or a global field then the diagram below commutes</u>

$$\begin{array}{ccc} & K_2(D) & \\ {\scriptstyle\psi_D}\nearrow & & \searrow{\scriptstyle N_D} \\ K_2(\mathrm{Nrd}_D(D^{\bullet})) & \longrightarrow & K_2(K) . \end{array}$$

PROOF. If E is any field extension of K which splits D, let $D_E = D \otimes_K E$. For a unique natural number n, there is an E-isomorphism $D_E \longrightarrow \mathbb{M}_n(E)$ where $\mathbb{M}_n(E)$ denotes the full matrix ring of $n \times n$-matrices with coeeficients in E. The induced isomorphism $K_2(D_E) \longrightarrow K_2(\mathbb{M}_n(E))$ is independent of the choice of E-isomorphism above. Furthermore, there is a unique Morita isomorphism (cf. [10, 16.18 and §37]) $K_2(\mathbb{M}_n(E)) \longrightarrow K_2(E)$. The composite of the two isomorphisms above will be denoted by $K_2(D_E) \xrightarrow{\simeq} K_2(E)$. By [4, 2.5], the diagram below commutes

$$\begin{array}{ccccc} & & K_2(D) & \longrightarrow & K_2(D_E) \\ & {\scriptstyle\psi_D}\nearrow & & & \downarrow\simeq \\ K_2(\mathrm{Nrd}_D(D^{\bullet})) & \longrightarrow & K_2(K) & \longrightarrow & K_2(E) . \end{array}$$

By Suslin [11, 5.7], the diagram below commutes

$$\begin{array}{ccc} K_2(D) & \longrightarrow & K_2(D_E) \\ {\scriptstyle N_D}\downarrow & & \downarrow\simeq \\ K_2(K) & \longrightarrow & K_2(E) . \end{array}$$

Moreover, by Merkurjev-Suslin [7, §7] (cf. also [11, 3.6]), there is an E

such that the homomorphism $K_2(K) \longrightarrow K_2(E)$ is injective. The proposition follows now, by chasing the following commutative diagram

$$\begin{array}{ccccc} & & K_2(D) & \longrightarrow & K_2(D_E) \\ & \overset{\psi_D}{\nearrow} & \Big\downarrow N_D & & \Big\downarrow \simeq \\ K_2(Nrd_D(D^{\bullet})) & \longrightarrow & K_2(K) & \longrightarrow & K_2(E) \end{array} .$$

COROLLARY. <u>If</u> K <u>is a global field with no real primes or if</u> K <u>is a nonarchimedean local field then</u> $Nrd_D(D^{\bullet}) = K^{\bullet}$ <u>and the homomorphism</u> $\psi_D : K_2(K) \longrightarrow K_2(D)$ <u>splits the homomorphism</u> $N_D : K_2(D) \longrightarrow K_2(K)$.

PROOF. If K is a global field with no real primes then by the Hasse-Schilling norm theorem cited above, $Nrd_D(D^{\bullet}) = K^{\bullet}$ and if K is a nonarchimedean local field then by a simple norm theorem (cf. [10, 33.4]), $Nrd_D(D^{\bullet}) = K^{\bullet}$. The remaining assertion of the corollary follows now from the proposition.

PROOF OF THEOREM 1. It is worth mentioning at the outset that because of the corollary above, one can restrict, if he likes, his attention to the case that K is a number field with at least one real prime. Recall that $\Sigma_D = \{v \mid v$ is a real prime of K, v does not split $D\}$. By the Hasse-Schilling norm theorem, $\Sigma_D(K) = Nrd_D(D^{\bullet})$. Let $M = \text{image}(K_2(\Sigma_D(K)) \longrightarrow K_2(K))$ and $N = \text{image}(N_D : K_2(D) \longrightarrow K_2(K))$. By Theorem 2, it suffices to show that $M = N$. By the proposition above, $M \subset N$. If λ is defined as in (*) above then the proof of Theorem 2 shows that $\ker(\lambda) \subset M$. Thus, it is enough to show that $\lambda(M) \supset \lambda(N)$.

Consider the following commutative diagram

$$\begin{array}{ccc} K_2(D) & \longrightarrow & \prod_v K_2(D_v) \\ N_D \Big\downarrow & & \Big\downarrow \prod_v N_{D_v} \\ K_2(K) & \longrightarrow & \prod_v K_2(K_v) \\ & \overset{\lambda}{\searrow} & \Big\downarrow \prod \left(\frac{,}{v}\right) \\ & & \prod_v \mu(K_v) \end{array}$$