NUMERICAL METHODS USING FORTRAN

HARRIS

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Preface

The person who employs a digital computer to solve the important problems of engineering and science must have an understanding of both a *programming language* and some basic concepts of *numerical methods*. In the process of learning, and in turn, communicating these two disciplines, it became obvious that the most effective way of illustrating the usefulness and importance of each was to provide a suitable marriage of the two. It is to this end that this book has been written. When programming and numerical methods are thus related and then applied to challenging problems on the digital computer, the study of the combination becomes an exciting experience.

The practicing engineer or scientist will find the content of the book to be self-teaching; the technical student will find the aid of the classroom advantageous. This book may be used alone or as a prelude to a more rigorous treatment of numerical analysis. The study of this book will give the reader a sufficient mastery of the use of the digital computer in numerical methods so that he will be able to solve moderately complex problems in engineering and science. For most purposes, it would seem necessary that the first five chapters of this text be read in their order of presentation. After Chapter 5, one or more of the chapters may be omitted; or the order may be modified to fit the needs of the particular study effort.

Of the several algebraic-type languages used to program digital computers, only *Fortran* is considered in this text. The essentials of the Fortran language appear in the body of the text, and many of the auxiliaries appear in Appendix 1. This produces an effective arrangement of study materials. Many readers will find it advantageous to make only occasional reference to this appendix. In the Fortran language, there are variations that are dependent upon a particular computer facility and local computational procedures. Furthermore, the Fortran language is in the process of evolution. As a result, the reader will need to supplement this book with some instructions, perhaps two pages, that apply to the particular computer facility that will be used to solve problems related to this book.

The Fortran checklist of Appendix 2 is a very valuable aid as a reference, and as a medium of convenient review. As one checks the Fortran statements in his program, he will find this appendix to be an effective aid.

Although some of the content of this book can be learned easily with a mathematical background of only trigonometry and algebra, a calculus background is needed to understand the entire content.

Most of the problems of this book are placed within the bounds of the respective chapter, and each represents an integral part of the thought process. Therefore, it is important for the reader to solve the problem at the time that it confronts him during the study of the material.

The author wishes to express his gratitude to a number of people for their help in the preparation of this book. In particular, he would like to thank Professors Bruce Barnes, F. N. Peebles, Lewis Conta, and Clifford L. Sayre who reviewed the manuscript in part or in whole; Mrs. Joyce Hansen for invaluable assistance in proofreading the manuscript; and personnel of the University of Utah Computer Center—Paul Tuan, Joseph Rice, and Edwin Dallin—for counsel and advice.

L. D. Harris

January, 1964

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CHAPTER I

Solving Numerical Problems on the Digital Computer

1.1 Elements of the Digital Computer

Throughout this text, our first concern is with the method of solving the moderately extensive and complicated numerical problem. It is assumed that the digital computer is a basic tool which will be used in arriving at the solution of the problem; therefore, brief consideration will be given to some of the salient mechanisms employed by the computer in solving the numerical problem. If the reader is interested in a detailed treatment of principles of operation of the digital computer, he might refer to one or more of the many texts and papers that treat the subject.[1,2,3] Most of us appreciate the fact that the digital computer has an enormous ability to make relatively simple, a calculation which would otherwise be extremely long and laborious. However, it may not be so obvious that this terrific calculating machine is also an extremely unintelligent brute. This imbecilic machine must be precisely directed or commanded by human beings. The precisely stated set of commands necessary to direct the computer in solving our problems is called a *program*. The person who prepares this set of instructions or commands is, of course, a programmer. Perhaps you, as the reader, are discouraged with the prospect of becoming a programmer, because it looks as if it would take more time to prepare the detailed instructions for the computer to follow than it would to solve the original numerical problem without using the computer—and such *can* be the case. On the other hand, a skilled programmer in a few hours of time can write a computer program that will solve a problem of such a magnitude that, if it were solved by ordinary methods, the individual operating the desk calculator might easily use hundreds or thousands of hours of his time before reaching a solution.

The digital computer performs additions, subtractions, divisions, and multiplications at a terrific speed and with great accuracy. By virtue of its ability to perform these four manipulations and by its ability to make certain logical decisions, the computer is able to calculate a wide assortment of functions such as $\sin x$. You must understand, however, that the computer must be instructed, or *programmed*, in great detail in order to perform these calculations.

Some thought suggests that in order for the digital computer to serve our needs, it must have the ability to receive information from the programmer. It must have the ability to READ[4] *programs*, and

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[1] McCormick, E. M., *Digital Computer Primer*, McGraw Hill Book Company, Inc., New York, 1959.

[2] Siegel, Paul, *Understanding Digital Computers*, John Wiley and Sons, Inc., New York, 1961.

[3] Bartee, Thomas C., *Digital Computer Fundamentals*, McGraw Hill Book Co., Inc., New York, 1960.

[4] In a sense, some words are understood by the computer. In this text, such words are written with capital letters. Shortly, this point will become clear to the reader.

it also must have the ability to READ *input* data that become involved in the internal computation. In a similar fashion, the digital computer must be able to communicate calculated results to us. Data and programs are *input* quantities READ into the computer, and calculated answers or results are *output* quantities that the computer must be able to PRINT, PUNCH, or WRITE out.

A little additional thought suggests that the digital computer must have facilities for storing *data* and *programs*. Conceivably, the computer might READ in one instruction, execute this one instruction, and then PRINT the answer out. But essentially no practical computational problem is executed by such a simple-minded procedure. In the practical computer, therefore, it becomes necessary to store full programs, significant amounts of *input* data, calculated *results from intermediate steps* in the total computation, and a significant number of computer answers that will ultimately be *output*. The device or system used for storing these pieces of vital information is called a *memory*.

Then, in addition, in order that the computer can perform its rather complex function in an organized fashion, there must be some mechanism for correlating or *controlling* the separate functions of input, output, memory, and arithmetic. These various functions, including that of *control*, are displayed in a simple graphical representation in Fig. 1.1. It is not practical for us to study the intricate details of how the memory unit stores information, how the control

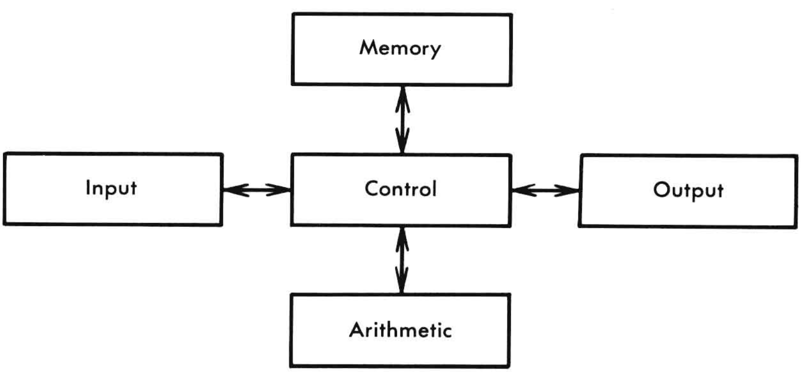


Fig. 1.1. **The functions of a digital computer.**

unit performs its necessary functions, how the input function is achieved, or how any other detail of the total computer function is executed. On the other hand, a few additional comments relative to each of the various functions displayed in Fig. 1.1 can be very useful to us as we learn how to *program* the digital computer to make the desired numerical analysis.

1.2 Input

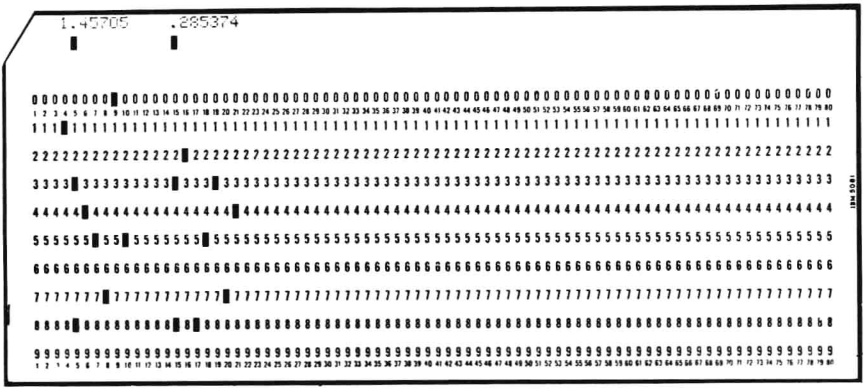
Magnetic tape, paper tape, and punched cards are the common media for carrying information (data and program) from the outside world through the input mechanism to the internal functions of the computer. Figure 1.2 displays two punched cards and a piece of punched paper tape. The input mechanism of the computer is able to receive information from cards and tape by reading the punched holes. A particular computer may have only the mechanism for reading cards; another computer may be able to read from both cards and paper tape. Card (a) Fig. 1.2 is typical of a data card containing two pieces of data. On the other hand, card (b) contains one instruction or one *statement* of a *program*.

1.3 Output

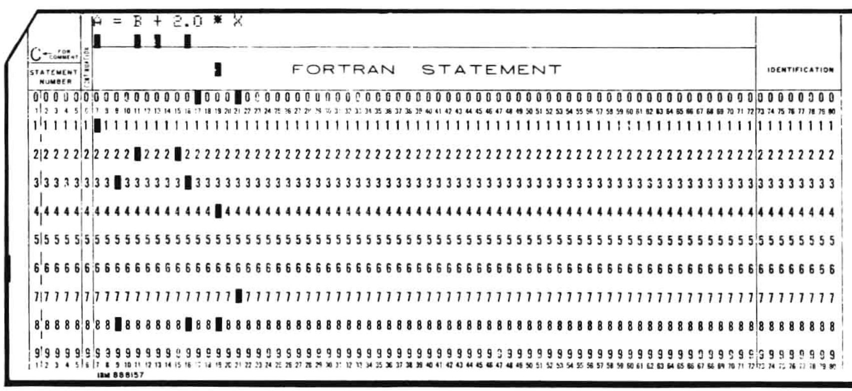
Output information can be punched in cards and in paper tape. It is also common to output information through a *line printer* onto sheets of paper.

1.4 Memory

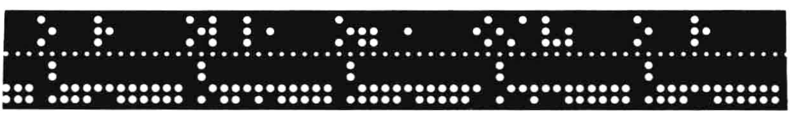
Programs, input data, output data, and other data (needed temporarily as the computation proceeds) are stored in the *memory*. The memory is such that it is capable of storing both alphabetic and numerical information; however, the alphabetic information is stored in an equivalent numerical form. Of course, the sign and the decimal associated with the number can be stored. Obviously information must be stored in memory in an organized fashion so that the computer may refer to a particular memory cell for the particular information stored therein. In the computer vernacular, we say that each storage cell has an *address*. Later, in our discussion of the



(a) **Data card.**



(b) **Statement of a program on a card.**



(c) **Punched paper tape.**

Fig. 1.2. **Two punched cards and a piece of punched paper tape. The computer reads the holes, black rectangles on cards and white circles on paper tape. Printed characters on cards aid humans to read punched information.**

digital computer, we will find a rather simple mechanism for addressing these cells in an organized fashion so that the computer can be programmed to determine what information is stored in any particular cell.

1.5 Control Unit

As the wording suggests, the *control unit* interprets the program in supervising, organizing, and sequencing the separate functions of input, output, arithmetic, and memory.

In this very brief look into how the digital computer operates, we have avoided becoming involved in the many intricate details. If the reader desires to know more of the details, he is referred to the previously cited references.[1,2,3]

CHAPTER II

The Program

2.1 What Is the Program?

The digital computer program is a set of precisely stated instructions which tells the computer how to execute the desired computations. It is the medium by which the programmer or analyst communicates the numerical problem to the internal workings of the computer. Although we have taken only a secondary interest in the materials of the first chapter, we will take an intense interest in learning the language through which we can communicate our instructions to the unsophisticated giant. The program is written in a special language which has an extremely limited and inflexible vocabulary.