Lecture Notes in Engineering

Edited by C. A. Brebbia and S. A. Orszag

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C. A. Brebbia, H. Tottenham, G. B. Warburton, J. M. Wilson, R. R. Wilson

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FOREWORD

The increasing size and complexity of new structural forces in engineering have made it necessary for designers to be aware of their dynamic behaviour. Dynamics is a subject which has traditionally been poorly taught in most engineering courses. This book was conceived as a way of providing engineers with a deeper knowledge of dynamic analysis and of indicating to them how some of the new vibrations problems can be solved. The authors start from basic principles to end up with the latest random vibration applications. The book originated in a week course given annually by the authors at the Computational Mechanics Centre, Ashurst Lodge, Southampton, England. Special care was taken to ensure continuity in the text and notations.

Southampton 1984

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CHAPTER 1

INTRODUCTION TO VIBRATION

by

G.B. Warburton

1. Introductory Remarks

In recent years the number of structures, for which the dynamic forces, likely to be encountered in service, have required investigation at the design stage, has increased. Several factors have contributed to this increase: growth in size of structures of various types; consequential increased importance of wind forces; efforts to reduce the effects of earthquakes on structures and to prevent total collapse; design of off-shore structures. Two important questions are: why is it essential to include dynamic effects in structural analysis and why is this a more difficult task than conventional (static) structural analysis?

Suppose that the stresses in a structure are known for: (a) a static force P at a particular location; (b) a force at the same location that varies in magnitude with time and has a maximum value of P. Then the dynamic magnification factor is the maximum stress at a point for (b) / the stress at the same point for (a). This factor depends upon how the force varies with time, the distribution of stiffness and mass in the structure and the damping present. In certain circumstances it will be very large; in others very small. Obviously, if there is any possibility of the dynamic magnification factor being significantly greater than unity, a dynamic analysis of the structure is necessary. This book is primarily concerned with methods of determining dynamic magnification factors for various types of loads and structures. However, no simple rules exist for these factors. Thus there are greater conceptual difficulties for dynamic problems than for comparable static problems, as the intuition and experience, which help an engineer to form a reasonable view of the safety of a structure under static forces, do not lead to an estimate of the relevant dynamic magnification factors. Also the time dependence of stresses, displacements etc. and the necessity to include mass and damping effects make dynamic analysis more complex than its static counterpart. There are also practical difficulties; some dynamic loads, e.g. wind forces, and most damping forces can only be estimated.

In addition to the possibility of elastic failure of a structure if dynamic effects are neglected, long-time repetition of dynamic stresses, whose magnitudes would be considered to be safe from static considerations, may lead to cumulative fatigue failures.

In this chapter the concepts that are relevant to vibration analysis of structures will be discussed briefly. Emphasis is on the response of structures to dynamic forces and how different types of force time variation influence the choice of method. Many of the concepts are introduced by considering the simplest vibrating structure; then, as this simple structure has limited practical applications, general structures are discussed. For these the normal mode method of determining response is given particular attention, because it illustrates the physical behaviour of structures better than other methods. Lastly dynamic interaction problems are discussed; here interaction exists between the vibrations of a structure and those of the underlying soil or the surrounding fluid. Many current practical problems, and also much current research effort, involve interaction effects.

Naturally in a single chapter the major topics of structural vibration can only be mentioned. Most of these topics will be studied in depth in subsequent chapters. It is hoped that their introduction here will illustrate their interrelationship and show how they contribute to the determination of stresses in complex structures caused by various types of dynamic excitation.

2. Single Degree of Freedom Systems: Equation of Motion and Types of Problem

Although the dynamic response of a practical structure will be complex, it is necessary to begin our study by considering the fundamentals of vibration of simple systems. A rough guide to the complexity of a dynamical system is the number of *degrees of freedom* possessed by the system. This number is equal to the number of independent coordinates required to specify completely the displacement of the system. For instance, a rigid body constrained to move in the X Y plane requires three coordinates to specify its position completely - namely the linear displacements in the X- and Y-directions and the angular rotation about the Z-axis (perpendicular to the plane X Y); thus this body has three degrees of freedom. The displacement of an elastic body, e.g. a beam, has to be specified at each point by using a continuous equation so that an elastic body has an infinite number of degrees of freedom. In a dynamical problem the number of modes of vibration in which a structure can respond is equal to the number of degrees of freedom, thus the simplest structure has only one degree of freedom.

Figure 1 shows the conventional representation of a system with one degree of freedom; it consists of a mass m constrained to move in the X-direction by frictionless guides and restrained by the spring of stiffness k. It is assumed that the mass of the spring is negligible compared to m. Thus the displacement of the system is specified completely by x, the displacement of the mass, and the system has one degree of freedom. For the purpose of analysing their dynamic response it is possible to treat some simple structures as systems with one degree of freedom.

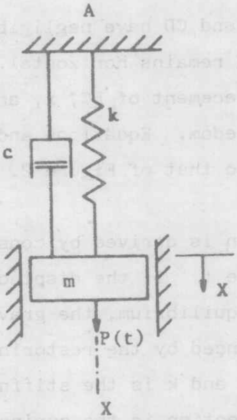

Figure 1 Single degree of freedom system

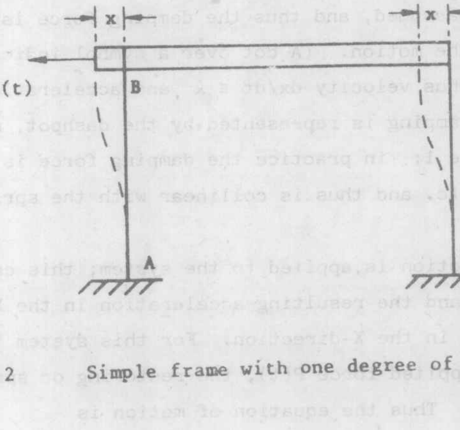

Figure 2 Simple frame with one degree of freedom

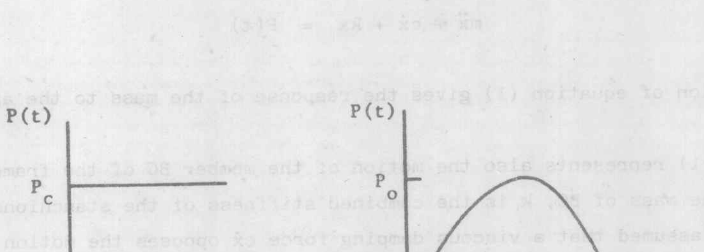

Figure 3 Examples of transient force excitation

In the simple frame of Figure 2 it is assumed that the horizontal member BC is rigid, that the vertical members AB and CD have negligible mass compared to that of BC, and that in any swaying motion BC remains horizontal. Then the motion of the system is given by the horizontal displacement of BC, x, and the frame can be treated as a system with one degree of freedom. Equations and results derived for the system of Figure 1 will be applicable to that of Figure 2.

The general equation of motion is derived by considering the forces acting on the mass m of Figure 1 at any time t. If the displacement of the mass, x, is measured from the position of static equilibrium, the gravity force mg need not be included in the equation as it is balanced by the restoring force in the spring kx_s where x_s is the static deflection of m and k is the stiffness of the spring or the force required to produce unit deflection in the spring; it is assumed that the spring is linear, i.e. k is a constant.

In any real system there will be some damping; this may take various forms, but here *viscous damping* will be assumed, and thus the damping force is proportional to the velocity $\dot{x}$ and opposes the motion. (A dot over a symbol indicates differentiation with respect to time; thus velocity $dx/dt \equiv \dot{x}$ and acceleration $d^2x/dt^2 \equiv \ddot{x}$.) Conventionally viscous damping is represented by the dashpot, shown in parallel with the spring in Figure 1; in practice the damping force is caused by internal friction in the spring etc. and thus is collinear with the spring force.

Newton's second law of motion is applied to the system; this can be expressed as the product of the mass and the resulting acceleration in the X-direction is equal to the net applied force in the X-direction. For this system the latter has three components, namely the applied force P(t), the restoring or spring force (-kx) and the damping force ($-c\dot{x}$). Thus the equation of motion is

$$m\ddot{x} = P(t) - kx - c\dot{x}$$

or

$$m\ddot{x} + c\dot{x} + kx = P(t) \qquad (1)$$

The solution of equation (1) gives the response of the mass to the applied force P(t).

Equation (1) represents also the motion of the member BC of the frame of Figure 2, if m is the mass of BC, k is the combined stiffness of the stanchions AB and DC, and it is assumed that a viscous damping force $c\dot{x}$ opposes the motion of BC. The equations of motion for various single degree of freedom systems are derived in Chapter 2.

Vibrations may be excited by impressed motion at the support. Considering Figure 2, to represent a simple structure, its response to vibrations transmitted through the ground by earthquakes, traffic, pile-drivers, hammers, explosions etc. is important in practice. Suppose that the support A in Figure 1 is given a vertical displacement $x_o(t)$ or the foundation AD in Figure 2 is given a horizontal displacement $x_o(t)$. In both cases the restoring force on the mass due to deformation of the spring or stanchions is $k(x-x_o)$. The damping force is proportional to the relative velocity across the dashpot (Figure 1) and is $c(\dot{x} - \dot{x}_o)$. If the force P(t), shown in Figure 1 and 2, is no longer acting, the equation of motion is:

$$m\ddot{x} = -k(x-x_o) - c(\dot{x}-\dot{x}_o)$$

i.e. $$m\ddot{x} + c\dot{x} + kx = kx_o + c\dot{x}_o \qquad (2)$$

Writing equation (2) in terms of the displacement of the mass relative to the support (i.e. the deformation of the spring or stanchions), $x_r = x - x_o$,

$$m\ddot{x}_r + c\dot{x}_r + kx_r = -m\ddot{x}_o \qquad (3)$$

The solution of equation (3) yields the relative displacement, which is proportional to the stress in the elastic member. This solution can be obtained when the base acceleration $\ddot{x}_o$ is specified. In practical problems relating to excitation due to imposed motion of the base, the acceleration is usually known, rather than the displacement and velocity, although the latter can be found by integration.

Equations (1), (2) and (3) are mathematically similar. Thus discussion of the different types of excitation, i.e. how the applied force or base motion varies with time, applies to all three equations. Solutions obtained from one equation can be used to infer solutions for either of the others. Only a change of nomenclature is required to interchange solutions between equations (1) and (3).

Considering the force P(t), shown in Figures 1 and 2, there are three main types of excitation: (i) Harmonic forces, such as $P(t) = P_o \sin \omega t$ or $P(t) = C\omega^2 \sin \omega t$, (the latter is typical of a component of the force produced by out of balance in a rotating machine). A force which is periodic but not harmonic can be expressed as a sum of harmonic terms, using Fourier series, and for a linear system the total response can be obtained by superposing the individual response from each harmonic component of the force. Thus forces which are periodic but not harmonic will not be considered further. (ii) Transient or aperiodic forces: usually these are forces which are applied suddenly or for a short interval of time; simple examples, illustrating the two types, are shown in Figure 3(a) and (b). (iii) Random forces:

the force P(t) cannot be specified as a known function of time, but can be described only in statistical terms; forces due to gusts of wind form an example of this type of excitation.

For (i) the steady-state response of the mass to the harmonic force is required. For (ii) the transient response is required, usually the maximum displacement of the mass or the maximum extension of the spring (the stress in the elastic member of the system is proportional to this extension), occurring during the period of application of the force or in the motion immediately following this period, will be of greatest interest. For (iii) the response can only be determined statistically.

Mathematically, the solution of equation (1) consists of two parts: the complementary function, which is obtained by solving the equation with the right hand side equal to zero, i.e. P(t) = 0, and the particular integral which depends on the form of P(t). Physically, the complementary function represents free damped vibrations, i.e. the vibrations that occur if the mass is given an initial displacement or velocity and released. The solution for free vibrations can be written

$$x = \exp(-\gamma\omega_n t)(A \sin \omega_d t + B \cos \omega_d t) \tag{4}$$

where $\omega_d = \omega_n(1 - \gamma^2)^{\frac{1}{2}}$ (5)

$$\omega_n^2 = k/m, \quad \gamma = c/c_c \text{ and } c_c = 2(km)^{\frac{1}{2}} = 2k/\omega_n = 2m\omega_n \tag{6}$$

In equation (4) the constants A and B are chosen to satisfy the initial conditions, i.e. the values of x and $\dot{x}$ at time $t = 0$. Equation (4) represents a damped oscillation: $x \to 0$ as $t \to \infty$. It has been assumed that the damping ratio $\gamma < 1$. In practice, $\gamma << 1$; thus from equation (5) $\omega_d \approx \omega_n$. Now ω_n is the (circular or radian) natural frequency of the system and is of great importance in vibration analysis. If an initial displacement is given to the mass in Figure 1 or 2, the frequency of the ensuing vibrations is strictly ω_d, but provided that $\gamma << 1$ it can be assumed that the natural frequency ω_n has been measured. The assumption that $\gamma << 1$ can be checked by determining γ from the rate of decay of successive oscillations. (See Chapter 2 for further details).

3. <u>Response</u>

The response of systems with one degree of freedom (Figures 1 and 2) to the various types of excitation force will be summarised.

Considering a *harmonic applied force*, i.e. $P(t) = P_o \cos \omega t$, where P_o is a constant and ω is the (radian) frequency of the force, equation (1) becomes

$$m\ddot{x} + c\dot{x} + kx = P_o \cos \omega t \quad (7)$$

The complete solution consists of free damped vibrations [equation (4)] and a particular integral. However, the former dies out and thus the steady state solution is given by the particular integral, which can be shown to be

$$x = \frac{P_o \cos(\omega t - \beta)}{[(k - m\omega^2)^2 + c^2\omega^2)]^{1/2}} \quad (8)$$

with $\tan \beta = \dfrac{c\omega}{k - m\omega^2}$.

Using definitions (6) and putting $r = \omega/\omega_n$, i.e. r is the ratio of the excitation frequency to the natural frequency, the steady state amplitude X from equation (8) is

$$\frac{kX}{P_o} = \frac{1}{[(1 - r^2)^2 + (2\gamma r)^2]^{1/2}} \quad (9)$$

Now P_o/k is the static deflection of the mass due to a static force P_o, so kX/P_o is the dynamic magnification factor. Equation (9) introduces the phenomenon of resonance. The dynamic magnification factor is a function of the frequency ratio r and the damping ratio γ. For γ small it has a sharp peak when $r = 1$ and this peak value, obtained by putting $r = 1$ in equation (9), is $1/2\gamma$. Thus for practical systems with low damping the dynamic magnification factor is very large when the excitation and natural frequencies coincide. However, well away from resonance the dynamic magnification factor is not large. (See Chapter 2 for further details).

Looking ahead to more complex structures, the viscous damping mechanism, shown in Figure 1 and used in the above equations, causes the response at higher frequencies (strictly higher resonances) to be underestimated. To overcome this difficulty viscous damping is replaced by *hysteretic damping*, i.e. the damping term $c\dot{x}$ in equation (1) is replaced by $h\dot{x}/\omega$, where h is the hysteretic damping constant and ω is the excitation frequency. With the viscous damper the energy dissipated per cycle increases linearly with the frequency, although the amplitude of vibration is kept constant. For a hysteretic damper the energy dissipated per cycle is independent of the frequency. For hysteretic damping equation (7) is replaced by

$$m\ddot{x} + h\dot{x}/\omega + kx = P_o \cos \omega t \quad (10)$$

If $h/k = \mu$, the steady state amplitude is

$$\frac{kX}{P_o} = \frac{1}{[(1 - r^2)^2 + \mu^2]^{1/2}} \qquad (11)$$

The maximum value of the dynamic magnification factor is $1/\mu$ and occurs when $r = 1$.

For a *general transient force* $P(t)$ the solution of equation (1) is given by the Duhamel integral, which is derived in Chapter 2, or by the convolution integral using Laplace transforms, and is

$$x = \frac{1}{m\omega_d} \int_o^t P(\tau) \exp[-\gamma\omega_n (t - \tau)] \sin \omega_d(t - \tau) d\tau \qquad (12)$$

In equation (12) it is assumed that at $t = 0$ the displacement and velocity of the mass are zero. If these conditions are not satisfied, free vibrations, equation (4) must be added with A and B determined from the non-zero conditions. [Equation (12) could be used to determine the complete response to a harmonic applied force, but other methods of solution are simpler.] Considering the step function force of Figure 3a, $P(t) = P_o$, $t > 0$, equation (12) is integrated and the response

$$\frac{kx}{P_o} = 1 - \exp(-\gamma\omega_n t) \left[\cos \omega_d t + \frac{\gamma}{(1 - \gamma^2)^{1/2}} \sin \omega_d t \right] \qquad (13)$$

This gives damped oscillations about the new mean position, given by $kx/P_o = 1$. The maximum response occurs when $\omega_d t = \pi$ and is given by

$$\left[\frac{kx}{P_o}\right]_{max} = 1 + \exp\left[\frac{-\pi\gamma}{(1 - \gamma^2)^{1/2}}\right] \qquad (14)$$

The variation of the dynamic magnification factor from equation (14) with the damping factor γ is shown in Table 1. For small damping the factor is relatively insensitive to γ. For comparison the maximum dynamic magnification factor associated with a harmonic force, namely $1/2\gamma$ for $r = 1$, is also given in the table.

Next consider the response of the system of Figure 1 to a sinusoidal force of finite duration, i.e.

$$P(t) = P_o \sin \frac{\pi t}{t_o}, \qquad 0 \leq t \leq Nt_o \qquad (15)$$

$$P(t) = 0, \qquad t > Nt_o$$

where N is an integer. [The force P(t) is shown in Figure 3(b) for N = 1].

The response for $t \lesssim Nt_o$ is obtained by substituting equation (15) in (12). For $t > Nt_o$ free damped vibrations occur and are given by equation (4) with A and B chosen to give appropriate continuity conditions at $t = Nt_o$. Figure 4 shows the dynamic magnification factor, i.e. the maximum value of kx/P_o with respect to time, plotted against t_o/T for zero damping ($\gamma = 0$) and n = 1, 2 and 4; $T(=2\pi/\omega_n)$ is the period of the system. Outside the range of values of t_o/T for which $(kx/P_o)_{max}$ has been plotted for N = 2 and 4, its behaviour is more complicated but values are significantly less than the peak values shown. When $t_o/T = 0.5$, $(kx/P_o)_{max} = N\pi/2$. Thus for an excitation force of two complete waves (N = 4) the dynamic magnification factor can be as large as 6.3, For the plotted ranges the maximum displacement occurs in the residual or free vibration era (i.e. $t > Nt_o$) if $t_o/T < 0.5$ and occurs in the forced vibration era (i.e. $t < Nt_o$) if $t_o/T > 0.5$.

For a *random variable* the spectral density shows the distribution of the harmonic content of the variable over the frequency range from zero to infinity. If the spectral density is specified, the mean value of the square of the variable can be obtained. For stationary ergodic random processes with Gaussian or normal probability distributions (these standard assumptions for random vibration theory are described in Chapter 14), if $S_p(\omega)$ and $S_x(\omega)$ are the spectral densities of the input force and response respectively for the system of Fig. 1, the mean square values are given by

$$< P^2(t) > \;=\; \frac{1}{2\pi}\int_o^\infty S_p(\omega)\,d\omega \qquad (16)$$

and

$$< x^2(t) > \;=\; \frac{1}{2\pi}\int_o^\infty S_x(\omega)\,d\omega \qquad (17)$$

It can be shown (Chapter 14) that the spectral densities are related by

$$S_x(\omega) \;=\; \frac{S_p(\omega)}{k^2[(1 - r^2)^2 + (2\gamma r)^2]} \qquad (18)$$

[If hysteretic damping replaces viscous damping in Fig. 1, $2\gamma r$ in equation (18) is replaced by μ]. If the spectral density of the force is known, the mean square value of the response is obtained from equations (17) and (18). The simplest force spectrum is: $S_p(\omega) = S_o$, a constant; i.e. the spectrum is uniform over the complete frequency range and is called white noise. The corresponding mean response is, from the calculus of residues,

$$< x^2(t) > \;=\; \frac{S_o\,\omega_n}{8\gamma\,k^2} \qquad (19)$$

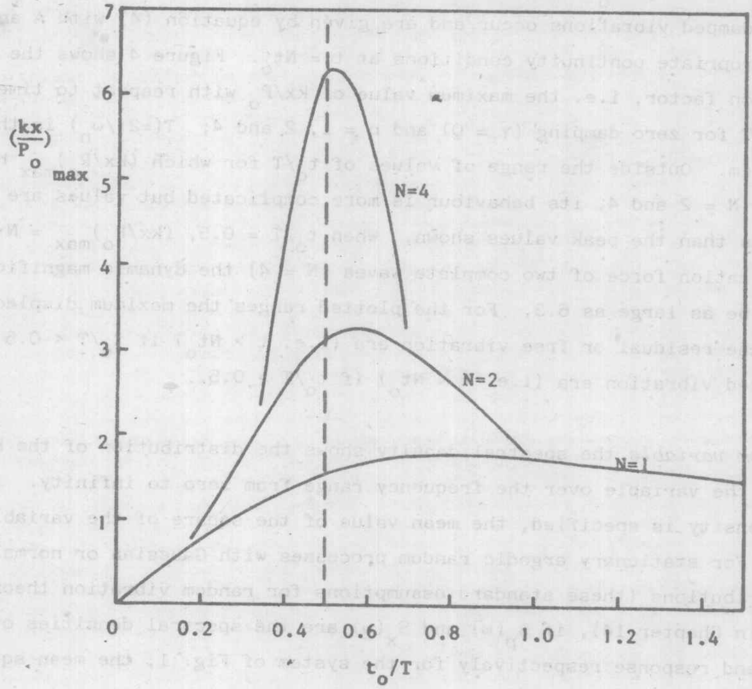

Figure 4 Dynamic magnification factor for a single degree of freedom system, subjected to a transient force:

$$P(t) = P_o \sin \pi t/t_o, \quad 0 \leq t \leq Nt_o$$
$$= 0, \quad t > Nt_o$$

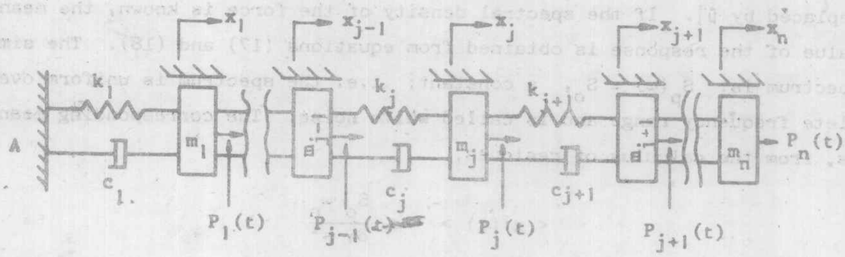

Figure 5 Multi degree of freedom system