Longman Mathematical Texts

Electricity

Second edition

The late **C. A. Coulson**
and **T. J. M. Boyd**

Professor of Applied Mathematics in the University of Wales at University College of North Wales, Bangor

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Preface

In preparing a new edition of **Electricity** my aims have been twofold. In the first place it seemed to me vital to preserve the essence of the original book, with its clarity and economy of style, which has served successfully to introduce a generation of students to the subject. Thus the first eight chapters are based firmly on Professor Coulson's book—though with some re-ordering and some change on emphasis—and in these the subject is developed along conventional lines. Alternative treatments, such as those in which Maxwell's equations are introduced at an early stage, are not without appeal but I have eschewed these since, on balance, I believe that the traditional development is still the best. Perhaps its most serious shortcoming lies in the fact that many students find a prolonged discussion of the physics leading up to Maxwell's equations rather lacklustre. In lecturing on the subject I try to counter this by appealing to contemporary applications of the basic theory and I have adopted the same approach in this book. Thus in Chapter 5, and again in Chapter 8, some simple illustrations from plasma physics are introduced to highlight ideas in magnetostatics and in electromagnetic induction.

In the second place in the interests of providing a balanced account of classical electrodynamics I felt that it was necessary to extend the coverage of the subject beyond that of the first edition to include a full discussion of electromagnetic waves and radiation. These, together with a short introduction to relativistic electrodynamics, make up the remainder of the book.

In preparing this edition I have adopted another feature of courses I have given on classical electrodynamics, namely the use of computer graphics as an aid to understanding. I am certain that many lecturers would agree that one of the difficulties in presenting a theoretical account of the subject to the undergraduates of today compared, say, with those for whom Professor Coulson wrote, stems from their lack of direct experience in carrying out many of the simple basic experiments in electromagnetism which were once an essential part of any school physics curriculum. It is now commonplace to find undergraduates with little feel for the magnitudes typical of electric and magnetic quantities and with only the haziest ideas of the electric or magnetic fields from simple configurations of charges or currents. I have felt for a long time that one of the best antidotes to some of these problems is to make as full use as possible of computer graphics. For the past few years I have taught courses on the subject with the aid of a graphics system and some specimen programs have been included in the book by way of illustration of what can be done. Others have been used to plot charged particle trajectories (Chapters 5 and 12), field lines of TE and TM modes in a rectangular waveguide (Chapter 10) and radiation patterns (Chapter 11). Since sophisticated graphics facilities are now widely available in most computing laboratories—and indeed in many university physics and engineering departments—there are few, if any, technical obstacles to adopting this teaching aid. Used imaginatively, I have found that the interest of students has been stimulated in innumerable ways.

The last four chapters of the book extend the subject beyond the coverage in the first edition. Chapter 9 includes a full discussion of electromagnetic waves and examples from plasma physics are used to illustrate the basic concepts of wave motion. This is followed in Chapter 10 by a treatment of wave propagation in bounded media which includes an illustration on dielectric waveguides from modern optical communications theory. Chapter 11 presents a discussion of electromagnetic radiation including both radiation from antennas and from accelerated point charges. The book ends with a chapter on relativistic electrodynamics without which no introductory account of the subject from a theoretical standpoint can be considered complete. There are of course very many topics such as the relation of the equations of macroscopic electrodynamics to the microscopic description, together with much of what is embraced by classical electron theory that have been omitted. These require more sophisticated discussion and are best left in my view to an advanced course in the subject.

Professor Coulson, in his original preface, wrote that no one really understands the subject until he or she has worked a good many examples in it. That is no less true today, and most of the examples in the original edition have been preserved and others added. Moreover I have added a number of worked examples in several chapters.

In giving thought to a revised edition of his book, Professor Coulson had decided to change from the Gaussian-cgs system of units of the earlier edition to SI units. I have followed his wishes in this matter though I believe it essential for students of the subject to be familiar with both systems and to this end a short appendix on units is included.

I am conscious of my debt to a number of people who have helped me very considerably by commenting on the various drafts of the book and by working examples. These include a number of research students at Bangor notably Gareth Humphreys-Jones, Ivan Moshkun and David Cooke. I have had invaluable help on computer graphics from Dr. G. A. Gardner and Philip Range and particularly from Terry Hewitt who has tested all the graphics programs used in the book, whether listed or not. I am also indebted to Mrs. M. Walker for her expert preparation of the typescript and to Professor Alan Jeffrey and Longman for their patience over the many delays which the work has suffered due to other commitments.

T. J. M. BOYD
Bangor, 1979.

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Preliminary survey

§1.1 Electrostatics

The fact that a piece of amber, when rubbed, will attract small particles of matter was known 2 500 years ago by Thales of Miletus. From this simple experimental fact has developed the whole science of electrostatics, which deals with the properties of electricity at rest. Indeed the very word electricity is derived from the Greek word for amber, *η'λεκτρον*. Since the beginnings of physics with the Milesian school of philosophers in the sixth century B.C., a great deal of experimental knowledge of electricity has accumulated, especially in the last 200 years. This knowledge has seldom been obtained in the most systematic order, so our approach in this book will not be to report the various experimental findings as they were first made, but rather to start with a general survey of the whole subject. Readers interested in a full history of the classical theory of electricity and magnetism are referred to [1].† The subject itself is properly described as **classical electrodynamics** and concerns all classical – as opposed to quantal – aspects of the subject, from electrostatics to magnetic effects and relativity.

Electricity – in this general sense – is the study of positive and negative charges and their interactions. Like charges repel each other, unlike charges attract. They do so with a force that is enormous compared with gravitation and is responsible for holding our material universe together. The smallest negative charge is that of the **electron** – the first of the elementary particles – whose existence had been anticipated by Larmor in 1894 and which was discovered and measured for the first time by J. J. Thomson in 1897. The name itself had been introduced by Stoney in 1881 in the sense of an elementary unit of electricity, either positive or negative, which he used to interpret Faraday's classical experiments on electrolysis carried out in 1833.

† See the Bibliography on p. 383

The smallest positive charge has the same magnitude as that of the electron and is found on two additional elementary particles, the **proton** and the **positron**. The proton (Greek for "first") was discovered and named by Rutherford in 1919 and is of course the nucleus of the hydrogen atom. The existence of atomic nuclei had been previously discovered by Rutherford in 1911. The positron, or positive electron, had its origins in Dirac's theory of the electron which showed that an electron can exist in a negative energy state. Dirac interpreted these negative energy states as positive electrons. Their existence was established by experiment in 1933 by Anderson and they were named positrons. Classical electrodynamics is not concerned with positrons since they are produced in the free state only under special conditions. On colliding with electrons in the presence of a nuclear field both particles suffer annihilation with the production of very short wavelength electromagnetic radiation.

All charges are integral multiples of these fundamental units, the magnitude of which is 1.6×10^{-19} coulomb. The smallness of the charge on the electron in relation to ordinary measurements is reflected in the fact that in a 60-watt lamp at 200 volts approximately 2×10^{18} electronic units of charge flow along the filament per second. In addition to the charge on the electron and proton the other property of these elementary particles which concerns us in classical electrodynamics is their mass. The mass of the electron is 9.109×10^{-31} kg; the proton is approximately 1 836 times heavier, at 1.673×10^{-27} kg. Of course both particles possess other properties such as spin and statistics. The spin of the proton for example is revealed by a study of the spectrum of the hydrogen atom. Associated with spin is a magnetic moment which will concern us, at least superficially, in Chapter 6, when we refer to the interpretation of ferromagnetism in terms of electron spin.

In classical – and in quantum – electrodynamics the electron is a point charge, i.e. it has no physical extension. Neither the electron nor the proton can be a point in the strict mathematical sense of the word. Indeed we shall meet a quantity in Chapter 11 which is called the classical radius of the electron and has dimensions of the order of 10^{-15} metre. In classical electrodynamics we are not concerned with phenomena on this scale length and so will take the elementary charges to be point particles. We must recognize however that this does involve a basic inconsistency in the theory (divergence of the self-energy of the electron, cf. §3.4) which lies outside the scope of this book but which is discussed in theories of classical charged particles [2].

Matter is composed of atoms built from protons and neutrons (yet another elementary particle of mass comparable to the proton but devoid of electric charge) which form the nuclei, and of electrons which form their outer structure. Atoms are typically of dimension 10^{-10} metre and their physical behaviour is described by quantum mechanics. In a cubic metre of an ordinary solid there are about 10^{29} atoms; in diamond, for example, there are 4×10^{29}. We know from elementary physics that as we heat matter to higher temperatures it usually passes first to a liquid phase and from this to a gaseous state. What happens when we continue to heat the gas is that the bonds between the nucleus and some of the outer electrons are broken and we obtain matter in a fourth state known as a **plasma**. In the most general sense a plasma is that state of matter in which enough free charged particles (i.e. those not bound to an atom) are present for its dynamical behaviour to be dominated by electromagnetic forces. As we continue to heat the matter, now in plasma form, more and more of the remaining electrons are stripped from the nucleus until eventually the atoms are fully ionized. Our sun and stars in general are hot enough for the matter to be almost completely ionized; interstellar gas is also ionized due to the action of stellar radiation. Indeed virtually all matter in the universe is in the plasma state. Moreover since its behaviour is in very many cases classical we shall make reference to it throughout this book to illustrate points in the theory but without discussing details of plasma physics [3].

A study of the forces which positive and negative charges exert on one another forms the content of Chapters 2 and 3. This may be said to represent the science of electrostatics proper.

§1.2 Electric currents

The study of electrostatics, or charges at rest, leads naturally to a study of electric currents, or charges in motion. The current may be caused by movement either of the positive or negative charges, or of both. Thus in a plasma positive ions carrying a positive charge move in one direction, and electrons carrying negative charge move in the opposite direction, so that the total current has two components. On the other hand in metals such as copper, the charge is carried entirely by electrons. It makes no difference to our formulation of the laws of current flow, developed in Chapter 4, which type of carrier bears the charge, for in all cases the current is measured by the rate at which the charge flows, i.e. the net amount crossing unit area in unit time. The direction of the current is determined by the sign of the product of the charge carrying the current and the mean drift velocity.

The definition of current in terms of a mean or average value of particle drift is an instance of a vital distinction between **microscopic** and **macroscopic** quantities. On a microscopic scale the charges are moving with a distribution of velocities. On a macroscopic scale however we consider only the average motion of charges within a small volume to obtain a mean drift velocity, which, when multiplied by the average number of charges and their respective charge, measures the electric current. For purposes of discussing the flow of current, materials may be placed in one of two categories – **insulators** or dielectric materials and **conductors**. An insulator (e.g. glass) is a substance in which it is extraordinarily difficult to cause electric current to flow. The explanation is simple, since in these materials all the electrons are firmly bound to the positive charges. As we cannot easily separate them it follows that no net flow of charge can take place. A conductor on the other hand is a substance in which a certain number of electrons (or negative ions in some instances) are easily separated from their associated positive charges and one or both can move under the influence of a force of the right kind. Thus in metals such as copper there are electrons (those in the outermost structure of the copper atoms) known as **free** or **conduction** electrons, which are able to flow freely through the material, giving rise to a current while the positive charges remain fixed. In a plasma the positive charges – positive ions – also move and contribute to the current, though their contribution is usually negligible in comparison to that of the electrons. This is due to the inertia of the ions relative to that of electrons so that they acquire a much smaller drift velocity.

In **electrolytes**, on the other hand, each molecule of electrolyte separates spontaneously into positive and negative ions and the total current is the sum of these two currents.

There is a further very important class of conductors – on which a whole technology is based – known as **semiconductors**.