Fracture Mechanics Criteria and Applications

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KLUWER ACADEMIC PUBLISHERS
DORDRECHT / BOSTON / LONDON

Library of Congress Cataloging-in-Publication Data

Gdoutos, E. E., 1948-
Fracture mechanics criteria and application / E.E. Gdoutos.
p. cm. -- (Engineering application of fracture mechanics ; 10)
ISBN 0-7923-0605-8 (U.S. : alk. paper)
1. Fracture mechanics. I. Title. II. Series.
TA409.G37 1990
620.1'126--dc20 90-4214

ISBN 0-7923-0605-8

Published by Kluwer Academic Publishers,
P.O. Box 17, 3300 AA Dordrecht, The Netherlands

Kluwer Academic Publishers incorporates the publishing programmes of D. Reidel, Martinus Nijhoff, Dr W. Junk, and MTP Press.

Sold and distributed in the U.S.A. and Canada
by Kluwer Academic Publishers,
101 Philip Drive, Norwell, MA 02061, U.S.A.

In all other countries, sold and distributed
by Kluwer Academic Publishers Group,
P.O. Box 322, 3300 AH Dordrecht, The Netherlands

printed on acid free paper

Printed in The Netherlands

Editor's preface

It is difficult to do justice to fracture mechanics in a textbook, for the subject encompasses so many disciplines. A general survey of the field would serve no purpose other than give a collection of references. The present book by Professor E. E. Gdoutos is refreshing because it does not fall into the esoteric tradition of outlining equations and results. Basic ideas and underlying principles are clearly explained as to how they are used in application. The presentations are concise and each topic can be understood by advanced undergraduates in material science and continuum mechanics. The book is highly recommended not only as a text in fracture mechanics but also as a reference to those interested in the general aspects of failure analysis.

In addition to providing an in-depth review of the analytical methods for evaluating the fundamental quantities used in linear elastic fracture mechanics, various criteria are discussed reflecting their limitations and applications. Particular emphases are given to predicting crack initiation, subcritical growth and the onset of rapid fracture from a single criterion. Those models in which it is assumed that the crack extends from tip to tip rely on the *specific* surface energy concept. The differences in the global and energy states before and after crack extension were associated with the energy required to create a unit area of crack surface. Applications were limited by the requirement of self-similar crack growth. Quantities such as energy release rate, stress intensity factor, path independent integral, etc. fall into this category. An alternative view based on physical observation is that crack growth is not continuous but is a discrete process of failure initiation, repeated many times, at a finite distance from the crack tip. Attention would then be focused on the energy stored in a unit volume of material reaching a critical state. Crack growth is considered as the loci of fracture initiation sites. Consistency can thus be achieved not only in assessing initiation, slow growth and rapid fracture but also in the simultaneous description of yielding and fracture. Macroplasticity off to the side of a macrocrack extension is distinguished from the microplasticity that may prevail ahead of a growing macrocrack. By the same token, microcracks may occur in regions of macroplasticity. The time and location at which energy is dissipated to deform and fracture at a specified size scale of material need to be identified. The book gives a comprehensive explanation of how the stationary values of the strain energy density function can be applied to locate the sites of yield and fracture initiation. Yielding and fracture at the macroscopic level are shown to occur at different locations. The former always precedes the latter. Many illustrative examples are presented and critiqued against the other criteria. Fatigue crack growth can be treated in the same way by considering the cumulation of the strain on volume energy density. When the disturbance becomes time dependent, inertia effects of material elements around the crack come into play. This can occur regardless of whether the crack is stationary or moving. Further material on dynamic fracture can be found in the references.

The important contribution on applying fracture mechanics principles is not the number of examples but the methodology for resolving the loading rate and specimen size effect. Straight line relations obtained from the constant rate change of strain energy density factor with crack growth permit linear interpolation of the results. Data on crack growth can thus be obtained with a limited number of experiments and can be used to predict situations other than those tested. A realistic application of fracture mechanics could not be made without a sound understanding of the fundamentals. To this end, the book has met the objective.

Bethlehem, Pennsylvania
December, 1989

G. C. SIH

Preface

The objective of engineering design is the determination of the geometry and dimensions of machine or structural elements and the selection of material in such a way that the elements perform their operating function in an efficient, safe and economic manner. For this reason the results of stress and displacement analysis are coupled with an appropriate failure criterion, which is basically a postulate predicting the event of failure itself. Traditional failure criteria cannot adequately explain the number of structural failures that occur at stress levels considerably lower than the ultimate strength of the material. Example problems include bridges, tanks, pipes, weapons, ships, railways and aerospace structures. On the other hand, experiments performed by Griffith in 1921 on glass fibers led to the conclusion that the strength of real materials is much smaller, typically by two orders of magnitude, than their theoretical strength. In an effort to explain these phenomena the discipline of fracture mechanics has been created. It is based on the realistic assumption that all materials contain crack-like defects which constitute the nuclei of failure initiation.

Defects can appear in a structure in three major different ways: first, they can exist in a material due to its composition, as second-phase particles, debonds in composites, etc.; second, they can be introduced into a structure during fabrication, as in welds; and, third, they can be created during the service life of a component, like fatigue, environment-assisted or creep cracks. Fracture mechanics studies the load-bearing capacity of structures in the presence of initial defects, where a dominant crack is usually assumed to exist.

A new design philosophy is therefore introduced by fracture mechanics as opposed to the use of the conventional failure criteria. As catastrophic fracture is a consequence of the unstable propagation of a crack from a pre-existing defect, we are faced with the question: 'Can fracture be prevented by constructing structures that have no defects?' The answer is 'no', on the grounds of practicality. Then, the safe design of structures should proceed along two lines: either the *safe operating load* should be determined when a crack of a prescribed size is assumed to exist in the structure; or, given the operating load, the *size of the crack that is created in the structure* should be determined.

Fracture mechanics is searching for parameters which characterize the propensity of a crack to extend. Such a parameter should be able to relate laboratory test results to structural performance, so that the response of a structure with cracks can be predicted from laboratory test data. This is determined as a function of material behavior, crack size, structural geometry and loading conditions. On the other hand, the critical value of this parameter – known as *fracture toughness*, a property of the material – is determined from laboratory tests. Fracture toughness expresses the ability of the material to resist fracture in the presence of cracks. By equating this parameter to its critical value a relation is obtained between applied load, crack and structure geometry which gives the necessary information for structural design. Fracture toughness is used to rank a material's ability to resist fracture within the framework of fracture mechanics, in the same way that yield or ultimate strength is used to rank a material's resistance to yield or fracture in the conventional design criteria. In selecting materials for structural applications a choice has to be made between materials with a high yield strength but comparatively low fracture toughness or a lower yield strength but higher fracture toughness.

The phenomenon of fracture of a solid is complicated and depends on a wide variety of factors, including the macroscopic effects, the microscopic phenomena which take place at the locations where the fracture nucleates or grows, and the composition of the material. The study of the fracture process depends on the scale level at which it is considered. At one extreme is the rupture of cohesive bonds in the solid, and the associated phenomena take place within distances of the order of 10^{-7} cm. For such studies the principles of quantum mechanics should be used. At the other extreme the material is considered as a homogeneous continuum and the phenomenon of fracture is studied within the framework of continuum mechanics and classical thermodynamics. Fracture studies that take place at scale levels between these two extremes concern movement of dislocations, formation of subgrain boundary precipitates and slip bands, grain inclusions and voids. Thus, the understanding of the phenomenon of fracture depends to a large extent on the successful integration of continuum mechanics with materials science, metallurgy, physics and chemistry. Due to the insurmountable difficulties encountered in an interdisciplinary approach the phenomenon of fracture is usually studied within only one of the three scale levels: namely, the atomic, the microscopic and the continuum. Attempts are under way to find a unified, interdisciplinary approach to the phenomenon of the failure of solids.

The purpose of this book is to present a clear, straightforward and unified interpretation of the basic problems of fracture mechanics with particular emphasis given to fracture mechanics criteria and their application in engineering design. The book is divided into nine chapters.

The first, introductory, chapter gives a brief account of some characteristic failures that could not be explained by the traditional failure criteria, and of Griffith's experiments which gave impetus to the development of a new philosophy in engineering design based on the discipline of fracture mechanics. The next two chapters deal with the determination of the stress and deformation fields in cracked bodies and provide the necessary prerequisite for the development of the criteria of fracture mechanics. More specifically, Chapter 2 covers the basic analytical, numerical and experimental methods for determining the linear elastic stress field in cracked bodies, with particular emphasis on the local behavior around the crack tip, and Chapter 3 is devoted to the determination of the elastic–plastic stress and displacement distribution around cracks for time-independent plasticity. Addressed in the fourth chapter is the theory of crack growth, based on the global energy balance of the entire system. The fifth chapter deals with the theoretical foundation of the path-independent J-integral and its use as a fracture criterion. Furthermore, a brief presentation of the crack opening displacement fracture criterion is given. Chapter 6 studies the underlying principles of the strain energy density theory and demonstrates its usefulness and versatility in solving a host of two- and three-dimensional problems of mixed-mode crack growth in brittle and ductile fracture. Chapter 7 presents in a concise form the basic concepts and the salient points of dynamic fracture mechanics. Addressed in Chapter 8 is the phenomenon of fatigue and environment-assisted crack growth which takes place within the framework of the macroscopic scale level. Finally, Chapter 9 presents the basic principles of engineering design based on the discipline of fracture mechanics and gives a number of example problems. The applicability of fracture mechanics to composites and concrete is also discussed. The chapter concludes with a brief description of the more widely used nondestructive testing methods for defect detection.

Particular care was taken throughout the book to give a clear, consistent, simple and straightforward presentation of the basic concepts of the discipline of fracture mechanics from a continuum mechanics viewpoint. The book is self-contained and can be used as a textbook in undergraduate and postgraduate courses and as a reference book by all those who are interested in developing design methodologies and promoting research to include the influence of initial defects or cracks.

The author wishes to express his gratitude to Professor G. C. Sih of Lehigh University for his pioneering work on fracture mechanics on which parts of the book were based, his very stimulating discussions and his comments and suggestions during the writing of the book. Thanks are also due to my secretary, Mrs L. Adamidou, for typing the manuscript. Finally, I wish to express my gratitude to my wife, Maria, not only for proofreading the manuscript, but for her understanding and patience during the writing of the book.

Xanthi, Greece, 1989 EMMANUEL E. GDOUTOS

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1

Introductory chapter

1.1. Conventional failure criteria

The mechanical design of engineering structures usually involves an analysis of the stress and displacement fields in conjunction with a postulate predicting the event of failure itself. Sophisticated methods for determining stress distributions in loaded structures are available today. Detailed theoretical analyses based on simplifying assumptions regarding material behavior and structural geometry are undertaken to obtain an accurate knowledge of the stress state. For complicated structure or loading situations experimental or numerical methods are preferable. Having performed the stress analysis, a suitable failure criterion is selected for an assessment of the strength and integrity of the structural component.

Conventional failure criteria have been developed to explain strength failures of load-bearing structures which can be classified roughly as ductile at one extreme and brittle at another. In the first case, breakage of a structure is preceded by large deformation which occurs over a relatively long time period and may be associated with yielding or plastic flow. The brittle failure, on the other hand, is preceded by small deformation and is usually sudden. Defects play a major role in the mechanism of both these types of failure and those associated with ductile failure differ significantly from those influencing brittle fracture. For ductile failures, which are dominated by yielding before breakage, the important defects (dislocations, grain boundary spacings, interstitial and out-of-size substitutional atoms, precipitates) tend to distort and warp the crystal lattice planes. Brittle fracture, however, which takes place before any appreciable plastic flow occurs, initiates at larger defects such as inclusions, sharp notches, surface scratches or cracks.

Materials that fail in a ductile manner undergo yielding before they ultimately fracture. Postulates for determining those macroscopic stress combinations that result in initial yielding of ductile materials have been developed and are known as yield criteria. At this point it should become clear that a material may behave in a ductile or brittle manner, depending on the temperature, rate of loading and other variables present. Thus, when we speak about ductile or brittle materials we actually mean the ductile or brittle *state* of materials. Although the onset of yielding is influenced by factors such as temperature, time, size effects, there is a wide range of circumstances where yielding is mainly determined by the stress state itself. Under such conditions for isotropic materials there is extensive evidence that yielding is a result of distortion and is mainly influenced by shear stresses. Hydrostatic stress states, however, play a minor role in the initial yielding of metals. Following these reasonings the Tresca and von Mises yield criteria have been developed [1.1].

The Tresca criterion states that a material element under a multiaxial stress state enters a state of yielding when the maximum shear stress becomes equal to the critical shear stress in a pure shear test at the point of yielding. The latter is a material parameter. Mathematically speaking, this criterion is expressed by [1.2, 1.3]

$$\frac{|\sigma_1 - \sigma_3|}{2} = k, \quad \sigma_1 > \sigma_2 > \sigma_3, \tag{1.1}$$

where σ_1, σ_2, σ_3 are the principal stresses and k is the yield stress in a pure shear test.

The von Mises criterion is based on the distortional energy and states that a material element initially yields when it absorbs a critical amount of distortional strain energy which is equal to the distortional energy in uniaxial tension at the point of yield. The yield condition is written in the form [1.4, 1.5]

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_y^2 \tag{1.2}$$

where σ_y is the yield stress in uniaxial tension.

However, for porous or granular materials as well as for some glassy polymers it was established that the yield condition is sensitive to hydrostatic stress states. For such materials the yield stress in simple tension is not equal in general to the yield stress in simple compression. A number of pressure-dependent yield criteria have been proposed in the literature.

On the other hand, brittle materials – or, more strictly, materials in the brittle state – fracture without appreciable plastic deformation. For such cases the maximum tensile stress and the Coulomb–Mohr [1.6, 1.7] criterion gained popularity. The latter criterion was mainly employed in rock and soil mechanics. The maximum tensile stress criterion assumes that rupture of a material occurs when the maximum tensile stress exceeds a specific stress which is a material parameter. The Coulomb–Mohr criterion states that fracture occurs when the shear stress τ on a given plane becomes equal to a critical value which depends on the normal stress σ on that plane. The fracture condition can be written as [1.6, 1.7]

$$|\tau| = F(\sigma), \tag{1.3}$$

where the curve $\tau = F(\sigma)$ on the σ–τ plane is determined experimentally and is considered as a material parameter.

The simplest form of the curve $\tau = F(\sigma)$ is the straight line, which is expressed by

$$\tau = c - \mu\sigma. \tag{1.4}$$

Under such conditions the Coulomb–Mohr fracture criterion is expressed by

$$\left(\frac{1+\sin\omega}{2c\cos\omega}\right)\sigma_1 - \left(\frac{1-\sin\omega}{2c\cos\omega}\right)\sigma_3 = 1, \tag{1.5}$$

where $\tan\omega = \mu$ and $\sigma_1 > \sigma_2 > \sigma_3$.

Equation (1.5) suggests that fracture is independent of the intermediate principal stress σ_2. Modifications to the Coulomb–Mohr criterion have been introduced to account for the influence of the intermediate principal stress on the fracture of pressure-dependent materials.

The above briefly outlined macroscopic failure criteria for describing the onset of yield in materials with ductile behavior or fracture in materials with brittle behavior have been used extensively in the design of engineering structures. In order to take into account uncertainties in the analysis of service loads, material or fabrication defects and high local or residual stresses, a safety factor is employed to limit the calculated critical equivalent yield or fracture stress to a portion of the nominal yield or fracture stress of the material. The latter quantities are determined experimentally. This procedure of design has been succesful for the majority of structures for many years.

However, it was early realized that there is a broad class of structures, especially those made of high-strength materials, whose failure could not be adequately explained by the conventional design criteria. On the other hand, Griffith [1.8, 1.9], from a series of experiments run on glass fibers, came to the conclusion that the strength of real materials is much smaller, typically by two orders of magnitude, than their theoretical strength. The theoretical strength is determined by the properties of a material's internal structure, and is defined as the highest stress level that the material can sustain. In the following two sections we shall give a brief account of some characteristic failures which could not be explained by the traditional failure criteria, and some of Griffith's experiments will be detailed. These were the major events that gave impetus to the development of a new philosophy in structural design based on the discipline of fracture mechanics.

1.2. Characteristic brittle failures

The phenomenon of brittle fracture is frequently encountered in many aspects of everyday life. It is involved, for example, in splitting logs with wedges, in the art of sculpture, in cleaving layers of mica, in machining materials and in many manufacturing and constructional processes. On the other hand, many catastrophic structural failures involving loss of life have occurred as a result of sudden, unexpected brittle fracture. The history of technology is full of such incidents. It is not the intent here to overwhelm the reader with the vast number of disasters involving failures of bridges, tanks, pipes, weapons, ships, railways and aerospace structures, but rather to present a few characteristic cases which substantially influenced the development of fracture mechanics.

Although brittle fractures have occurred in many structures over the centuries, the problem arose in acute form with the introduction of all-welded designs. In riveted structures, for example, fractures usually stopped at the riveted joints and did not propagate into adjoining plates. A welded structure, however, appears to be continuous and a crack growth may propagate from one plate to the next through the welds, resulting to global structural failure. Furthermore, welds may have defects of various kinds, including cracks, and usually introduce high-tensile residual stresses.

The most extensive and widely known massive failures are those that occurred in tankers and cargo ships that were built, mainly in the U.S.A., under the emergency shipbuilding programs of the Second World War [1.10–1.14]. Shortly after these ships were commissioned several serious fractures appeared in some of them. The fractures were usually sudden and were accompanied by a loud noise. Of approximately 5000 merchant ships built in U.S.A., more than one-fifth developed cracks before April 1946. Most of the ships were less than three years old. In the period between November 1942 and December 1952 more than 200 ships experienced serious failures. Ten tankers and three Liberty ships broke completely in two, while about 25 ships suffered complete fractures of the deck and bottom plating. The ships experienced more failures in heavy seas than in calm seas and a number of failures took place at stresses that were well below the yield stress of the material. A characteristic brittle fracture concerns the tanker *Schenectady*, which suddenly broke in two while in the harbor in cool weather after she had completed successful sea trials. The fracture occurred without warning, extended across the deck just aft of the bridge about midship, down both sides and around the bilges. It did not cross the bottom plating [1.15].

Extensive brittle fractures have also occurred in a variety of large steel structures. Shank [1.16], in a report published in 1954, covers over 60 major structural failures including bridges, pressure vessels, tanks and pipelines. Following Shank the earliest structural brittle failure on record is a riveted standpipe 250 ft high in Long Island that failed in 1886 during a hydrostatic acceptance test. After pumping water to a height of 227 ft, a 20 ft long vertical crack appeared in the bottom, accompanied by a sharp rending sound, and the tower collapsed. In 1938 a welded bridge of the Vierendeel truss type built across the Albert Canal in Belgium with a span of 245 ft collapsed into the canal in quite cold weather. Failure was accompanied by a sound like a shot and a crack appeared in the lower cord. The bridge was only about one year old. In 1940 two similar bridges over the Albert Canal suffered major structural failures. In 1962 the one-year-old King's Bridge in Melbourne, Australia, fractured after a span collapsed as a result of cracks that developed in a welded girder [1.17]. A spherical hydrogen welded tank of 38.5 ft diameter and 0.66 in. thickness in Schenectady, New York, failed in 1943 under an internal pressure of about 50 lb/in^2 and at ambient temperature of 10°F [1.16]. The tank burst catastrophically into 20 fragments with a total of 650 ft of herringboned brittle tears. Concerning early aircraft failures, two British de Havilland jet-propelled airliners known as Comets (the first jet airplane designed for commercial service) crashed near Elba and Naples in the Mediterranean in 1954 [1.18]. After these accidents, the entire fleet of these passenger aircraft was grounded. In order to shed light into the cause of the accident a water tank was built at Farnborough into which was placed a complete Comet aircraft. The fuselage was subjected to a cyclic pressurization, and the wings to air loads that simulated the corresponding loads during flight. The plane tested had already flown for 3500 hours. After tests giving a total lifetime equivalent to about 2.25 times the former flying time, the fuselage burst in a catastrophic manner after a fatigue crack appeared at a rivet hole attaching reinforcement around the forward escape hatch. For a survey and analysis of extensive brittle failures the interested reader is referred to reference [1.19] for large rotating machinery, to [1.20] for pressure vessels and piping, to [1.21] for ordnance structures and to [1.22] for airflight vehicles.

From a comprehensive investigation and analysis of the above structural failures , the following general remarks can be drawn.

(a) Most fractures were mainly brittle in the sense that they were accompanied by very little plastic deformation, although they were made of materials with ductile behavior at ambient temperatures.
(b) Most brittle failures occurred in low temperatures.
(c) Usually, the nominal stress in the structure was well below the yield stress of the material at the moment of failure.
(d) Most failures originated from structural discontinuities including holes, notches, re-entrant corners, etc.
(e) The origin of most failures, excluding those due to poor design, was pre-existing defects and flaws, such as cracks, accidentally introduced into the structure. In many cases the flaws that triggered fracture were clearly identified.
(f) The structures that were susceptible to brittle fracture were mostly made of high-strength materials which have low notch or crack toughness (ability of the material to resist loads in the presence of notches or cracks).
(g) Fractures usually propagated at high speeds which, for steel structures, were in the order of 1000 m/s. The observed crack speeds were a fraction of the longitudinal sound waves in the medium.

These findings were very essential for the development of a new philosophy in structural design based on the discipline of fracture mechanics.

1.3. Griffith's work

Long before 1921, when Griffith published his monumental theory on the rupture of solids, a number of pioneering results had appeared which gave evidence of the existence of a size effect on the strength of solids. These findings, which could be considered as a prelude to the Griffith theory, will now be briefly described. Leonardo da Vinci (1452–1519) ran tests to determine the strength of iron wires [1.23]. He found an inverse relationship between the strength and the length for wires of constant diameter. We quote from an authoritative translation of da Vinci's sketch book, according to reference [1.24], the passage:

Observe what the weight was that broke the wire, and in what part the wire

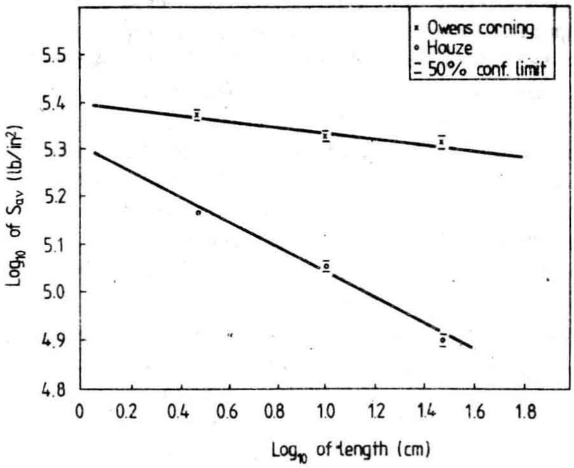

Figure 1.1. **Logarithm of average tensile strength versus logarithm of specimen length for carefully protected glass fibers (x) and fibers damaged by rough handling (o) [1.29].**

broke ... Then shorten this wire, at first by half, and see how much more weight it supports; and then make it one quarter of its original length, and so on, making various lengths and noting the weight that breaks each one and the place in which it breaks.

Todhunter and Pearson [1.25] refer to two experimental results analogous to those of da Vinci. According to [1.25], Lloyd (about 1830) found that the average strength of short iron bars is higher than that of long iron bars and Le Blanc (1839) established long iron wires to be weaker than short wires of the same diameter. Stanton and Batson [1.26] reported the results of tests conducted on notched-bar specimens at the National Physical Laboratory, Teddington, after the First World War. From a series of tests it was obtained that the work at fracture per unit volume was decreased as the specimen dimensions were increased. Analogous results were obtained by Docherty [1.27, 1.28] who found that the increase of the plastic work at fracture with the specimen size was smaller than that obtained from geometrical similarity of the strain patterns. This means that the specimens behaved in a more brittle fracture as their size was increased.

All these early results gave indication of the so-called size effect of the strength of solids, which is expressed by an increase in strength as the dimensions of the testpiece decrease. Newly derived results at the U.S. Naval Research Laboratory on the strength of glass fibers [1.29] corroborated the early findings of Leonardo da Vinci. Figure 1.1, taken from reference [1.29], shows a decrease of the logarithm of the average strength of glass as a function of the logarithm of the specimen length. The upper line refers to fibers for which precautions have been taken to prevent damage in handling. The lower line was obtained for fibers that