FUNDAMENTAL
CONSTANTS
OF PHYSICS

THE FUNDAMENTAL CONSTANTS
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Preface

Enormous strides have been made in the first half of the twentieth century toward the unification of many branches of physics and chemistry, once regarded as quite unrelated and distinct, into one body of well-organized knowledge. This unification is probably nowhere better reflected than in the history of the progress made toward improving the precision and consistency of our knowledge regarding the values of the fundamental physical constants and conversion factors. The number and the precision of the established theoretical relationships connecting these constants have also increased phenomenally. In fact a history of progress on the constants during the first half of the twentieth century is not very different from a history of physics itself during that period.

This book has been written with two primary objectives: (a) to give a complete survey of the present status of our knowledge of the fundamental constants and conversion factors of the physical sciences and to describe the modern precision experiments from which that knowledge is at present directly drawn, and (b) to present an account of the evolution of our knowledge of this subject through various degrees of experimental precision and theoretical sophistication in sufficient detail to permit the reader to appreciate the breadth and firmness of the entire experimental foundation. A substantial fraction of the world-wide research effort of physicists and chemists during the last half century has in fact been required to build that foundation. Most of the earlier and less precise experiments must of course be reviewed rather briefly. Care, however, has been taken to provide ample reference to the original sources so that the reader can look up omitted details for himself. The book has been written both for physics students at the graduate level and for professional physicists. No mathematical preparation beyond that required for graduate work in physics is presupposed.

The analysis of the data on the values of the atomic constants has relied heavily on the theory of least squares. Many physicists feel

quite unfriendly to the application of this branch of statistical theory
to physical data. The famous Dutch physicist, H. A. Kramers, once
said, "The theory of errors and least squares is like love—one cross
word can spoil it all!" Nevertheless, we see no escape from it which
is not subject to even worse objections. Actually most of the mistrust
of least squares and error statistics, if carefully examined, is in reality
mistrust of the data to which these doctrines are applied rather than
mistrust of the doctrines themselves. Two examples will illustrate this:

It is related that a certain statistician sent 10,000 questionnaires
to a randomly selected list of American citizens, requesting of each
one their best estimate of the height of the Queen of Siam. Each report
was also to be accompanied with the estimated standard deviation
of the data furnished. Many of the answers attached error measures to
the individual estimates as large as ±50 per cent, but the statistician
conscientiously took a weighted average of these data using, in accord
with the accepted principles of error statistics, weights made inversely
proportional to the square of the associated error estimate of each
datum. Because of the very large number of independent estimates
he came out with a height good to ±0.5 per cent! There was only one
trouble with the whole procedure—Siam didn't have a queen.

The second example concerns a certain retired sea captain who made
his home in a secluded spot on the island of Zanzibar. As a sentimen-
tal reminder of his seafaring career he still had his ship's chronometer
and religiously kept it wound and in good operating condition. Every
day exactly at noon, as indicated on his chronometer, he observed the
ritual of firing off a volley from a small cannon. On one rare occasion
he received a visit from an old friend who inquired how the captain
verified the correctness of his chronometer. "Oh," he replied, "there is
a horologist over there in the town of Zanzibar where I go whenever I
lay in supplies. He has very reliable time and as I have fairly frequent
occasion to go that way I almost always walk past his window and
check my time against his." After his visit was over the visitor
dropped into the horologist's shop and inquired how the horologist
checked his time. "Oh," replied he, "there's an old sea captain over
on the other end of the island who, I am told, is quite a fanatic about
accurate time and who shoots off a gun every day exactly at noon, so
I always check my time and correct it by his."

We are indebted for this last story to Professor George Harrison
of the Massachusetts Institute of Technology. Beyond the obvious

moral contained in both stories that no amount of statistics or replicated observations will yield reliable results from unreliable data, we believe the second yarn has a further value in that it may conceivably add a new word to our terminology, namely, "the Zanzibar." To commit a "Zanzibar" should henceforth convey a clear meaning to the reader of this book. It is a kind of error which he may be sure we have earnestly sought to avoid.

In an analysis of data, which may be based on least squares or any other analytic criterion, it is of utmost importance to assign weight to each datum in a consistent and equitable manner. The error measure associated with the result of an experiment is therefore just as important as the result itself. Furthermore the basis for the quotation of error measures must be uniform throughout. There are of course several such measures; e.g., probably error, "confidence limits" of error, standard error, mean absolute error. Because the root-mean-square, or standard error has a simple law of propagation which is not shared by other error measures except for the Gaussian, or normal, distribution, we have adopted it throughout this book. The meson data of Chapter 4 are an exception to this rule. Particularly with regard to meson lifetimes the literature is not clear whether probable errors or standard deviations are quoted, and the experimental data do not justify considering the assigned error measures as more than qualitative indications of accuracy. The choice that we shall use the standard error is, in some part, distinct from the question of how the accuracy of a measurement is to be determined. Standard errors either can be estimated from an analysis of the experimental situation and knowledge of how each component contributes to the uncertainty of the resultant measurement, or can be determined empirically by repeating the experiment several times and observing the distribution of the results. This latter method can, however, only detect the random errors of the experiment but cannot identify systematic errors. In practice both methods are combined, and the final assigned error is often strongly influenced by the experimenter's considered opinion of the results. If the experimenter is honest and unbiased there is no objection to this. In general each experiment must be finally reviewed by the analyst, who adds his considered opinion in an effort to place all the experiments on a more nearly uniform basis.

Whereas the precision of most of the experiments considered in this book is measured in parts per hundred thousand or parts per million,

the subject of the masses and lifetimes of mesons (Chapter 4) is hardly out of the early survey stage. The modern concepts of mesons and the recognition of the true complexity of the mass spectrum of fundamental particles is less than ten years old. The discussion of meson masses is therefore a subject unto itself. The errors involved are a factor of one thousand times larger than those associated with the "classical" atomic constants, and in many of the experiments there are certainly unidentified systematic errors. The situation is much less certain and much more fluid than the remainder of the book and is subject to alteration and modification much more rapidly. The field is at present so active that a new analysis would be required with each new issue of the *Physical Review!* The data are presented here as a tabulation of current results and as a possible source of material for future more detailed analysis. The table of Least-Squares Adjusted Values (Table 8-7) therefore omits all reference to mesons and hyperons in order to avoid giving an unwarranted aura of finality to the data. The data presented in Chapter 4 are summarized in Table 4–17.

Some readers accustomed to the MKS system of units may be disappointed to discover that our table of output values is not expressed in these units. All the disciplines of physics and chemistry meet and are represented in the fundamental constants and the truth is that the MKS has not been generally adopted in all these domains. One hesitates to express the normal mole volume of an ideal gas in cubic meters per kilogram molecule. Kilogram-molecule units for the gas constant, Avogadro's number, or the Faraday constant would seem equally unfamiliar and likely to be misunderstood. The MKS system is primarily an engineering system which may soon completely replace the FPS (or English) system. Its primary utility lies in the field of electrical engineering and classical electromagnetic theory. We have retained the cgs units because of their wide use in all fields of physics and chemistry. Those who prefer other systems of units may easily convert.

We wish to express our gratitude to the numerous research workers in many parts of the world (far too large a number to list here) who have been so kind as to keep us informed of their newly obtained results at the earliest possible moment. We are much indebted to Professor John Tukey of Princeton for much valuable advice and discussion on the topics of error statistics and least squares. We owe a great debt to Raymond T. Birge, whose pioneer work in this field of en-

deavor laid the foundation on which we have subsequently built. He was one of the first men to perceive the need for making from time to time broad general surveys and assessments of the state of our knowledge of the physical constants. His first paper of this kind published in 1929 set a noteworthy example of painstaking, careful research, based on a great many sources of information and carried out over a very broad range of research fields in physics and chemistry. The task was a severe one not only because of its sheer size but because there were huge discrepancies between sources of information and an enormous amount of patient effort was required to distinguish truth from error and to ferret out mistakes and oversights in this chaos. His kindly and constructive criticism and encouragement have been of inestimable value over a long period of years. It therefore is a pleasure to be able to express our homage and gratitude by dedicating this book to Professor Raymond T. Birge.

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Introduction

1.1 OUTLINE AND OBJECTIVES

Our knowledge of the constants of physics has been greatly improved in the 25 years between 1930 and 1955, partly by reason of the development of new methods of measurement and the perfecting of new physical techniques and partly by the important advances in our understanding of the physical world achieved through research. Microwave resonance methods and atomic beam techniques are examples of important technical improvements which have contributed much since 1940. Our understanding of atomic and nuclear physics by reason of the development of the quantum theory through successive stages of refinement has greatly clarified relationships having to do with the interaction of electrons and radiation. Much stimulus for improvement has come during this period from critical studies of the physical constants undertaken by various authors who at various times have reviewed the consistency of the many experimental results in an effort to establish the most reliable values. The first of these reviews was a remarkable paper (1) by Raymond T. Birge in 1929 which opened this era of critical examination and improvement.

The accuracy of our knowledge of such constants as Avogadro's number, the electronic charge, Planck's constant, the velocity of light, or the fine structure constant of Sommerfeld has been greatly improved by new techniques which bear little relationship to the earlier and perhaps intuitively more direct measurements such as Millikan's oil-drop experiment or the studies of Brownian motion. Such earlier methods had great value for the progress of science and still possess historical importance for their role in the discovery and logical development of the concepts of atomic physics. They will be reviewed here because we cannot have faith in what we now know unless we see the path by which that knowledge was won.

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But the earlier pioneer methods are now so greatly surpassed in accuracy by the more modern though frequently less direct methods that, although the earlier results are not inconsistent with the later ones, these alone are significant today in determining the numerical values of the physical constants.

Because of this situation this tract will concern itself with three objectives: (1) a brief introductory outline of the system of physical units and the standards set up to maintain them; (2) a sketch of the history of the progress of our knowledge of the general physical constants with special emphasis on the atomic constants, giving brief descriptions of both the earlier methods employed and the more modern methods now in use for their evaluation; (3) a detailed study as of 1955 of the status of our knowledge of the physical constants and a least-squares evaluation to obtain the best compromise values consistent with the entire body of present knowledge. These final output results are tabulated at the end of this text.

1.2 PHYSICAL CONSTANTS AS UNITS

When the science of physics was in its infancy arbitrary units were chosen, such as the kilogram, the meter, and the second; and standards were set up to maintain them. Although the motivation was commercial as well as purely scientific, no science of physical measurement could have been developed without this step. One of the important fruits of physical research has been the discovery that Nature herself has fundamental units such as the charge on the electron, the rest mass of the electron, Planck's constant of action, the speed of light.

In this connection it may be well to restate a postulate which R. C. Tolman clearly recognized but which is frequently overlooked or tacitly assumed. We shall call this the "postulate of the reproducibility of proper quantities." By this is meant that when a quantitative property of some fundamental system, such as the mass of a particular atom or one of its characteristic wavelengths, is observed in a Lorentz frame in which that system is at rest (called the "proper frame" of the system), we postulate that the same result will always be obtained. Nothing in our framework of well-established physical observations seems at variance with this postulate, and we shall therefore make it.

The thought lies close at hand that since Nature supplies us with

reliable and invariable units it might be well to abandon the earlier and more arbitrary artificial units completely. The convenience, the accessibility, and in many cases the superior precision of the arbitrary units make their complete abandonment extremely improbable in the foreseeable future. No one will care to check the accuracy of an ammeter by counting the number of electrons which flow in unit time through its windings. In spite of the very satisfactory accuracy which has recently been attained in measuring many of the natural constants, the artificial arbitrary standards of the cgs system are still superior in accuracy. The statement is true save perhaps for one exception—the meter bar, which, since the supplementary definition of the meter in terms of the wavelength of a certain line in the spectrum of cadmium in 1927, is no longer to be considered as a primary standard of length.

Even if such important practical considerations were ignored, we do not as yet know enough to select wisely the natural constants on which to base such a system. In the present state of our knowledge of physics there is in fact a great embarrassment of choice as regards natural constants that might be adopted as units of measurement. The mass of any one of a dozen or more "fundamental particles" might be chosen to replace the kilogram. As units of length any one of the famous hierarchy of four present themselves: The Rydberg wavelength, $R_\infty = 4\pi a_0/\alpha$; the Bohr radius, $a_0 = h^2/(4\pi^2 me^2)$; the Compton wavelength, $h/(mc) = 2\pi\alpha a_0$; and the Lorentz radius of the electron, $r_0 = \alpha^2 a_0$. The speed of light could be combined with any one of these fundamental lengths to furnish a fundamental unit of time. Almost the only physical magnitudes about which there would be little doubt as to choice are the unit of electrical charge and the unit of velocity. For these the electronic charge and the velocity of light would certainly be indicated. Our present arbitrary standards, however, still afford a precision for defining velocity and charge many orders of magnitude superior to the accuracy with which the velocity of light and the electronic charge have been determined.

It is proper then to begin by outlining the base of our system of physical measurements in terms of its units, standards, and fundamental constants.

REFERENCE

1. R. T. Birge, *Rev. Mod. Phys.*, **1**, 1 (1929).

Arbitrarily Defined Physical Units and Standards

2.1 LENGTH, MASS, AND VOLUME

In its original conception the meter was intended to be defined as one ten-millionth of the earth's quadrant on the meridian through Paris, and all units of volume were to be derived from it. It was planned that the unit of mass, the kilogram, should be identical to the mass of a cubic decimeter of water at its maximum density. The units of length and mass are, however, now defined independently of these conceptions.

The meter is defined as the distance between two engraved lines on a certain platinum-iridium bar, kept at the International Bureau of Weights and Measures at Sèvres near Paris, France, when the bar is supported in a definitely specified manner at the temperature of melting ice and at standard atmospheric pressure (760 millimeters of mercury). A supplementary definition of the meter in terms of the wavelength of a certain line in the spectrum of red cadmium light was adopted in 1927 by the Seventh General International Conference on Weights and Measures. The definition was based on the work (1) of Benoit, Fabry, and Perot in 1913 and is as follows:

$$1 \text{ meter } = 1{,}553{,}164.13 \text{ wavelengths} \qquad (2\text{-}1)$$

The conditions of temperature, pressure, and humidity together with many specifications regarding the light source which emits the line must be fulfilled for the definition of equation (2-1) to hold accurately.

Several spectral lines emitted by various isotopically pure substances, Hg^{198}, Kr^{36}, Xe^{136}, or Cd^{114} have been studied and are recommended by different proponents as preferable to the cadmium red line of the present definition. However, as of 1955 no international action had yet been taken to replace that standard, since the best choice was not entirely clear (3).

The kilogram is independently defined as the mass of a definite platinum-iridium standard, the International Prototype Kilogram, also kept at the International Bureau of Weights and Measures. The liter is defined as the volume of a kilogram of water, at standard atmospheric pressure and at the temperature of its maximum density, approximately 4°C. On a level of sufficient precision this is an ambiguous definition, since it does not specify the isotopic constitution of the water. The meter is the fundamental cgs unit of length and area and of such volumes as are based on linear measurements. Note, however, that, since the liter is defined independently of the meter, a conversion factor relating the cubic decimeter to the liter must be established by physical measurement. Henning and Jaeger (2) have obtained a result which Guillaume (2a) has corrected upward 1 ppm in 1927. We adopt this value with somewhat increased standard error* because of the ambiguity of unspecified isotopic constitution.

$$1 \text{ liter } = 1000.028 \pm 0.004 \text{ cm}^3 \qquad (2\text{-}2)$$

Thus the maximum density of water is

$$\delta_m = (1.000028)^{-1} = 0.999972 \pm 0.000004 \text{ kg} \cdot \text{dm}^{-3} \qquad (2\text{-}3)$$

It was once customary to define 1 cc as liter/1000, whereas 1 cm³ is liter/1000.028, but because of the frequent confusion of cc with cm³, the Joint Committee for the Standardization of Scientific Glassware recommended in 1924 that the designation ml (for milliliter) be used in place of cc.

2.2 TIME

The second is the fundamental unit of time. To quote a recent report (3a) by E. C. Crittenden on the actions taken at the Tenth General Conference on Weights and Measures in Paris, October 1954, "the second will presumably be defined [by the standing International Committee without waiting for another conference] as the fraction, 1/31,556,925.975 of the tropical year 1900.0." Techniques now exist for controlling the frequencies of electrical oscillations by means of the natural frequencies of vibration of cer-

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* Throughout this book we shall quote uncertainties in terms of the root-mean-square or "standard error." This, for reasons which we shall explain in Section 7.3, is preferred rather than the "probable error" or "mean absolute error."

tain molecules of which the ammonia molecule is one example. The three hydrogen atoms of the ammonia molecule form an equilateral triangle; the nitrogen atom is situated equidistant from the three hydrogen atoms but not in the same plane, so that the molecule has a tetrahedral structure. There are, however, two stable positions for the nitrogen atom, one on either side of the plane of the hydrogen atoms. Under suitable conditions the nitrogen atom may be made to oscillate back and forth between these two positions and the frequency of this oscillation may be very accurately measured and maintained.

A similar type of atomic clock makes use of the transitions in caesium between the hyperfine structure energy levels caused by the interaction between the electronic and nuclear spins. This hyperfine structure is the same phenomenon as that discussed in Section 6.7 in connection with the measurements of hyperfine structure splitting in hydrogen. The measured frequency of the central line of the hyperfine Zeeman pattern extrapolated to zero field is (3b)

$$\nu_0 = 9192631830 \pm 10 \text{ cycles sec}^{-1}$$

This represents an accuracy of one part in a thousand million and is by far the most accurate physical magnitude which has been measured.

A time standard based on this caesium resonance was suggested by H. Lyons (3c) and an early model was built at the United States National Bureau of Standards. A somewhat different type has been described by Zacharias, Yates, and Haun (3d). An improved resonance clock has recently been constructed at the National Physical Laboratory, Teddington, England, by Essen and Parry with which the above-quoted resonance frequency was determined. A photograph of this remarkable equipment appears as the frontispiece of this text.

These atomic clocks can furnish a more uniform standard of time than can be obtained from the rotation of the earth. The earth's rotation is known to have variations in rate of the order of 1 part in 10^8 by means of comparisons with sidereal time standards. The fact that the second is defined in terms of the tropical year *at the epoch 1900.0* indicates implicitly that the International Committee at the Tenth General Conference of 1954 was aware of the variability

of the year and that techniques exist whereby this variation can be measured. Hence the existence of time standards more accurate than the revolution of the earth in its orbit is implied. Without such standards it would be meaningless to refer the year to the epoch 1900.0 in preference to a current determination (i.e., epoch 1950.0 or 1956.0). There is another, and perhaps more compelling, reason for using atomic standards in place of astronomical ones. The unit of time can be determined by astronomical observation only after several years and hence we have our standards only in retrospect. The advantage of an atomic standard is that it enables frequency standardization to be effected in a few minutes, and the information can then be made available throughout the world almost instantaneously by utilizing the network of standard frequency transmission. At present the "Atomic Clocks" are used only to maintain a standard which is defined in terms of astronomical data; ultimately the unit of time will probably be defined directly in terms of the frequency of such atomic oscillations, but as yet (1955) no such action has been taken.

2.3 TEMPERATURE

The history of the measurement of temperature and the refinement of this concept from the early subjective stage based on sensations of hot and cold to the present definition of the absolute Kelvin or thermodynamic scale of temperature is outlined in many texts on thermodynamics. As an outgrowth of this history and the practical requirement of defining a number of convenient fixed points in different ranges of temperature for the purpose of calibrating thermometers we have two scales of temperature, namely, the International and the Kelvin, absolute, or thermodynamic scales. The reader should refer to the Smithsonian Physical Tables (4) or to the *Journal of Research of the National Bureau of Standards* (5) for the details of the definition of the 1948 International Temperature Scale. This is defined in terms of the platinum resistance thermometer and the electromotive force of a thermocouple over specified lower temperature ranges and in terms of Planck's black body law of radiation and certain fixed points at higher temperatures. The justification for such a scale is its convenience and reproducibility. The effort has been to define the International Temperature Scale in such a way as to make it as nearly as possible proportional

to Kelvin's absolute thermodynamic temperature scale. That temperature scale is the one which would result if the measurements could be made with a gas thermometer employing an ideal gas. It is the fundamentally significant physical scale and is independent of the properties of any particular substance. Kelvin himself urged that this absolute scale of temperature be defined by assigning a numerical value to the temperature of a single fixed point instead of the earlier method which defined two fixed points—the freezing and boiling points of water. With these earlier definitions the temperature of absolute zero is a matter of experimental determination, as in the centigrade or Fahrenheit scales. The Tenth General Conference on Weights and Measures in October 1954 decided (3a) "to define the thermodynamic scale of temperature by means of the triple point of water as fixed fundamental point, by assigning to it the temperature 273.16° Kelvin exactly." This yields a new thermodynamic scale of temperature numerically slightly different from the earlier one such that the best value of the ice point is 273.1500 ± 0.0002° K instead of 273.16° ± 0.01° K as formerly. The reason for the choice of the triple point of water rather than the ice point is the fact that it is uniquely defined without need for specifying a pressure. The reason for the change in the numerical definition is asserted to be that it achieves a better coincidence between the thermodynamic and International scales. As a result of this change, however, the centigrade degree is no longer exactly equal to the Kelvin degree and we have 1° (Centigrade) = 0.999964 ± 0.000036° (Kelvin), and the boiling point of water is 100°C = 373.1464 ± 0.0036°K.

Studies of the absolute temperature of the ice point or the triple point of water have been made by Joule and Thomson (6) using their porous plug method, by Chappuis (7a), Berthelot (7b), Henning and Heuse (7c), and Roebuck (7d). R. T. Birge (see reference 1, Chapter 1) has summarized and critically examined these experiments in his 1929 review. More recently Roebuck and Murrill (8a), Beattie (8b), and Keesom and Tuyn (8c) have reviewed the entire situation including their own work on the temperature of the ice point. All such measurements involve the determination of empirical constants to describe the departure in the behavior of the working gas in a gas thermometer from that of an ideal gas in terms of which the thermodynamic scale is conceived.