Elementary Algebra
Second Edition

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Preface

Elementary Algebra was originally designed as a text for the college student who had no previous training in algebra as well as for others who felt the need for an intensive review of the basic fundamentals of algebra. This revised edition has been developed for the same students.

The first chapter includes considerable material associated with general formulas that are familiar to the student. It is the author's belief that such an approach will give the student a more solid feeling of security as he starts the use of literal expressions.

This revised edition is somewhat more rigorous than the original text. The use of postulates, herein called laws, with undefined and defined terms, characterizes the general development of topics.

The use of set theory has been introduced as an integral part of the understanding of algebra. Its application to various topics has been made in numerous parts of the text.

Terms are defined throughout the book as they are introduced. The theory is presented in a simple style to assist the student in his understanding of algebra. Following the development of each topic, illustrative examples are solved to assist the student in the application of theory to the practical problem. Considerable care has been taken to develop these examples in a manner that will give the student full comprehension of what has been done as well as the reasons for each step in the problem.

An entire chapter has been devoted to the use of parentheses, brackets, and braces. Students often learn quickly and adequately how to remove such symbols, but they frequently fail to see the application of the same principles when it is necessary to insert the symbols in a problem. This book provides substantial drill for inserting symbols as the need arises.

Another chapter has been directed toward changing the subject of formulas. The emphasis on this unit of study is designed to assist the student with his study of physics and chemistry.

All students, even the superior ones, find that the converting of statements into algebraic language is difficult. In an attempt to assist the student of algebra in this phase of the course, the author has included, from the very beginning of the book, exercises for training in this respect.

Drill exercises are provided immediately following the development of each integral part of the course. Near the close of each chapter a drill exercise is found that is composed of all types of problems found in the chapter. Review tests which cover the theory presented in various chapters are included. These may be used strictly as tests and examinations or as additional drill exercises.

Answers to the odd-numbered problems are included in the text. The answers to the even-numbered problems are available from the publisher.

The author is indebted to his colleagues for their numerous suggestions and inspiration in the preparation of the manuscript for this revision. To the editorial staff of Allyn and Bacon, Inc. is extended appreciation for the efforts they have made in making this publication a reality. The author is especially grateful to Miss Sarah B. Grant, of the editorial staff, for her assistance in the final preparations for the publication of the book.

D. S. R.

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From Arithmetic to Algebra 1

1. INTRODUCTION

A great many years ago mankind found it necessary to invent some system to represent definite values of size and quantity. It may have been that one of man's first needs for numbers was to count the sheep that he watched over as they grazed. The use of pebbles or sticks or the fingers on his hand may have been adequate at first. As civilization progressed, however, certain types of calculations and solutions to problem situations became more complex. The number symbols that were used were difficult to manipulate in those more complex situations. Algebra was developed to assist in the simplification of those cases. In the study of algebra we use letters of the alphabet to represent value, size, and quantity. Letters can represent different values in different situations. The fact that a letter is not a definite or constant value provides us with a system whereby we can derive results that are general; we can then use that system or principle in specific numerical cases identical to the one in which the letters were used.

In arithmetic it was found that the perimeter of a square was equal to the sum of the four sides or four times the length of one side. This fact can be stated as follows:

The perimeter of a square equals four times one side, or

$$p = 4s$$

This formula is true for any square.

The letters we use are often referred to as *unknowns* or *variables*.

In arithmetic certain formulas for finding length, area, volume, and distance were used. Those formulas made use of letters which represented certain values. A *formula* is the statement of a rule to be used in determining a numerical quantity by using letters (or numerical figures and letters).

In the above formula for finding the perimeter of a square our statement is:

The perimeter of a square is equal to four times the length of one side of the square.

This sentence may be simplified as follows:

Perimeter equals four times one side.

The statement may be simplified further by substituting p (the first letter in the word) for the number of units in the perimeter of the square and s for the number of units in one side. This gives us the expression

$$p = 4s$$

and is read: "The perimeter is equal to four times the length of one side."

The following formulas were used in the study of arithmetic:

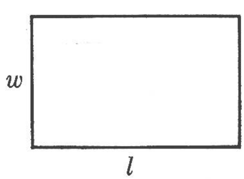

Figure 1

1. Perimeter of a rectangle

$P = 2l + 2w$ (Where l equals the number of units in the length and w equals the number of units in the width)

This formula would read: "The perimeter of a rectangle is equal to two times the number of units in the length plus two times the number of units in the width."

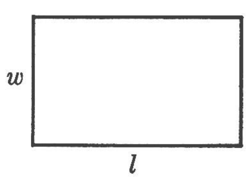

Figure 2

2. Area of a rectangle

$A = lw$ (Where l equals the number of units in the length and w equals the number of units in the width)

The formula would read: "The area of a rectangle is equal to the number of units in the length multiplied by (or times) the number of units in the width."

3. Area of a triangle

$A = \frac{1}{2}bh$ (Where b equals the number of units in the base and h equals the number of units in the height or altitude)

The formula would read: "The area of a triangle is equal to one-half the product obtained by multiplying the number of units in the base by the number of units in the altitude."

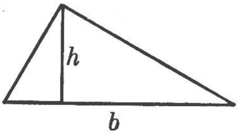

Figure 3

4. Area of a square

$A = s^2$ (Where s equals the number of units in one side of the square)

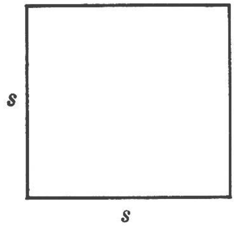

Figure 4

The formula would read: "The area of a square is equal to the square of one side." (To square a quantity means to multiply that value by itself.)

5. Area of a circle

$A = \pi r^2$ (Where r equals the radius of the circle and π equals 3.1416 or approximately $\frac{22}{7}$)

This formula would read: "The area of a circle is equal to the product obtained by multiplying pi by the square of the radius."

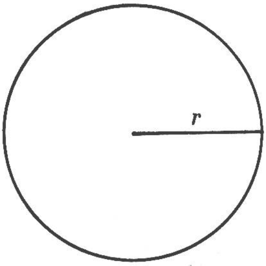

Figure 5

6. Circumference of a circle

$C = 2\pi r$ (Where π is the same value given above and r is the radius of the circle)

The formula would read: "The circumference (distance around) a circle is equal to twice the product obtained by multiplying pi by the radius of the circle."

The following is another formula frequently used for finding the circumference of a circle:

$C = \pi d$ ($2r = d$, since by definition, the diameter of a circle is equal to two times the radius)

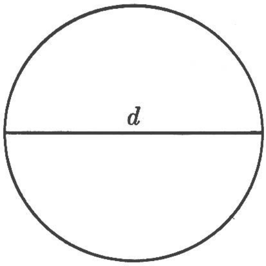

Figure 6

This formula would read: "The circumference of a circle is equal to the product obtained by multiplying the diameter by pi."

7. Volume of a cube

$V = s^3$ (Where s or e is equal to one side or edge of the cube)

or

$V = e^3$

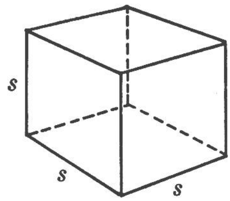

Figure 7

This formula would read: "The volume of a cube is equal to the cube of one side or edge of the cube." (The cube of a number means the product obtained when that number is used as a factor three times, as: $2^3 = 2 \cdot 2 \cdot 2 = 8$.)

8. Volume of a rectangular solid

$V = lwh$ (Where l equals the length, w equals the width and h equals the height)

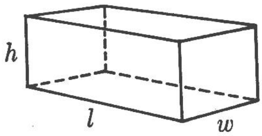

Figure 8

This formula would read: "The volume of a rectangular solid is equal to the product obtained by multiplying together the number of units in the length, the width, and the height."

9. The volume of a cylinder

$V = \pi r^2 h$ (Where r equals the radius of the base and h equals the height of the cylinder)

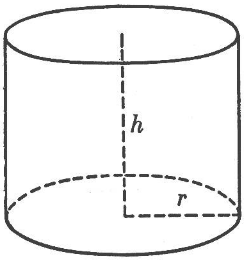

Figure 9

The formula would read: "The volume of a cylinder is equal to the product obtained by multiplying together pi, the square of the number of units in the radius of the base, and the number of units in the height of the cylinder.

Another formula frequently used for finding the volume of a cylinder is:

$V = Bh$ (Where B is equal to the area of the base and h equals the height of the cylinder)

It can be observed that the base of a cylinder is a circle. Thus, the area of that base is found by using the formula for finding the area of a circle (Formula 5, above).

10. Volume of a cone

$V = \frac{1}{3}\pi r^2 h$ (Where r is the radius of the circular base and h is the perpendicular distance from the tip of the cone to its base)

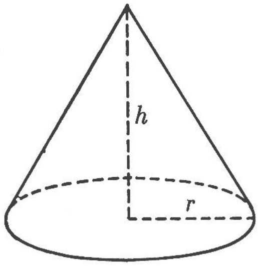

Figure 10

This formula would read: "The volume of a cone is equal to one-third the product obtained by multiplying together the square of the radius of the base, pi, and the height of the cone."

Example 1. Find the perimeter of a square if one side is 5 inches.

$$P = 4s$$
$$P = (4)(5) = 20 \text{ inches}$$

Example 2. Find the area of a circle if the diameter is 14 inches.

$$A = \pi r^2$$

$$A = \left(\frac{22}{7}\right)(7)^2$$ (If the diameter is 14 inches, the radius is 7 inches)

$$A = \left(\frac{22}{\cancel{7}}\right)\overset{7}{\cancel{(49)}} = 154 \text{ square inches}$$

Example 3. Find the volume of a cube whose edge is 3 inches.

$$V = e^3$$
$$V = (3)^3 = (3)(3)(3) = 27 \text{ cubic inches}$$

The following formulas frequently are used in the study of physics:

1. Converting centigrade temperature to Fahrenheit temperature: The Fahrenheit reading is equal to nine-fifths of the centigrade reading plus 32°.

$$F = \tfrac{9}{5}C + 32°$$

2. Converting Fahrenheit temperature to centigrade temperature: The centigrade reading is equal to five-ninths of the remainder when 32° is subtracted from the Fahrenheit reading.

$$C = \tfrac{5}{9}(F - 32°)$$

3. Ohm's Law states that the intensity of an electric current (amperes of the current) in a given electrical circuit is equal to the electromotive force (volts) divided by the resistance (R) to the flow of the current. (In this formula, E is used to denote the volts, I the intensity or amperes, and R the resistance, in ohms.)

$$I = \frac{E}{R}$$

4. The distance that an object travels in t hours at r miles per hour is

$$d = rt$$

5. The description of a "freely falling body starting from rest" means that, as an object falls through space, nothing interferes with its moving