Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

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Rational Approximation and Interpolation

Proceedings, Tampa, Florida 1983

Edited by P. R. Graves-Morris, E. B. Saff and R. S. Varga

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Preface

This volume contains the proceedings of the Conference on Rational Approximation and Interpolation, which took place December 12-16, 1983 at the University of South Florida, Tampa, Florida. The conference was held under the auspices of the U.K. - U.S. Cooperative Science Program, an informal agreement between the U.S. National Science Foundation and the U.K. Science and Engineering Research Council to promote and support mutually beneficial scientific activities. The primary purpose of the conference was to bring together pure and applied mathematicians, physicists and engineers to exchange information and set objectives for future research efforts dealing with rational approximation and interpolation.

P.R. Graves-Morris and E.B. Saff were the primary organizers of the conference. There were 28 participants from the U.S., 14 from the U.K. and 14 others representing 11 additional countries. The number of conference members was kept limited so as to promote discussion among members with diverse backgrounds, in accordance with the aims of the U.K. - U.S. Cooperative Science Program.

The contributions to this volume include original research papers as well as a few survey articles. All of these papers were refereed and we are grateful to many advisors for their diligence. It is hoped that this volume reflects the breadth of interest in rational approximation and interpolation, and serves as a source of inspiration for further research.

We wish to thank the U.S. National Science Foundation and the U.K. Science and Engineering Research Council for sponsoring the participants from their respective countries. We are also indebted to the University of South Florida (USF) Division of Sponsored Research for the support of the other conference partcipants. The conference planning and activities were facilitated by the USF Center for Mathematical Services, the USF organizing committee consisting of Prof. M. Blake and Prof. J. Snader, and the conference co-host, Prof. R.S. Varga. The efforts of these individuals far exceeded the norm. The secretarial help provided by Mary Baroli is also deserving of accolade as were the efforts of several staff members and students in the USF Department of Mathematics. We are further indebted to several companies in the Tampa area for having provided additional support for the conference functions.

P.R.G.-M., E.B.S., R.S.V.

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THE FABER OPERATOR

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Abstract. The boundedness of the Faber operator T and its inverse T^{-1}, considered as mappings between various spaces of functions, is discussed. The relevance of this to problems of approximation, by polynomials or by rational functions, to functions defined on certain compact subsets of $\mathbb{C}$ is explained.

1. Introduction

Let D denote the closed unit disk $\{w: w \in \mathbb{C}, |w| \leq 1\}$ and $A(D)$ the well-known disk algebra of functions analytic in the interior D^o of D and continuous on D, with the supremum norm. The sets K we wish to consider are compact subsets of $\mathbb{C}$ whose interior K^o is a simply connected domain and whose boundary ∂K is a rectifiable Jordan curve. The corresponding algebra is denoted by $A(K)$.

Associated with K there is a sequence of polynomials $\{F_n(z)\}$ introduced first by Faber in his thesis [10] and subsequently known as Faber polynomials. These are defined as follows. Let $z = \psi(w)$ be the Riemann mapping function of $\mathbb{C}\setminus D$ onto $\mathbb{C}\setminus K$ with $\psi(\infty) = \infty$, of the form

$$z = \psi(w) = \alpha w + b_0 + \sum_{n=1}^{\infty} b_n w^{-n} .$$

The number $|\alpha|$ is called the transfinite diameter of K. It is strictly positive for the domains K we are considering. We shall

*The author thanks the Department of Mathematics of the University of California at San Diego for its kind hospitality while this report was being written.

assume, by scaling and rotation, if necessary, that $\alpha = 1$. The Faber polynomials $F_n(z)$ associated with K are defined by

$$\frac{\psi'(w)}{\psi(w)-z} = \sum_{n=0}^{\infty} F_n(z) w^{-(n+1)}, \quad |w| > 1,$$

or, alternatively, by

$$F_n(z) = \frac{1}{2\pi i} \int_{|w|=1} \frac{w^n \psi'(w)}{\psi(w)-z}\, dw .$$

To see that these are indeed polynomials we note that

$$\frac{d^k}{dz^k} F_n(z)\Big|_{z=0} = \frac{k!}{2\pi i} \int_{|w|=1} \frac{w^n \psi'(w)}{(\psi(w))^{k+1}}\, dw .$$

For $k = n$ the right side above is $n!$, since $\alpha = 1$ and for $k > n$ it is zero. Thus $F_n(z)$ is a monic polynomial,

$$F_n(z) = z^n + \text{lower order terms} .$$

Let $\Pi(n)$ denote the set of all polynomials of degree at most n and set $\Pi = \bigcup_{n=1}^{\infty} \Pi(n)$. The Faber operator is defined, initially on Π, by

$$(1.1) \quad (Tp)(z) = \frac{1}{2\pi i} \int_{|w|=1} \frac{p(w)\psi'(w)}{\psi(w)-z}\, dw .$$

Note that if $p(w) = w^n$ then $(Tp)(z) = F_n(z)$. Clearly, $T(\Pi(n)) = \Pi(n)$ and the mapping

$$T: \Pi(D) \longrightarrow \Pi(K)$$

is injective. Here $\Pi(D)$ and $\Pi(K)$ denote Π considered as a subspace of $A(D)$ or $A(K)$. Sets K (of the kind we are considering, of course) where the operator T is bounded are called Faber sets. For the moment we are considering only the supremum norms on $A(K)$ and $A(D)$ respectively and so T is bounded if and only if

$$\|Tp\|_\infty \le \|T\| \cdot \|p\|_\infty .$$

The bounded operator T, given by (1.1) can be extended to a bounded linear mapping of $A(D)$ into $A(K)$ given by

$$(1.2)\quad (Tf)(z) = \frac{1}{2\pi i}\int_{|w|=1} \frac{f(w)\psi'(w)}{\psi(w)-z}\,dw\ .$$

This operator, again denoted by T, is injective. This follows from the fact, established in [13], §3, that, if $f \in T(A(D))$, then we may associate with f a Faber expansion

$$f(z) = \sum_{k=0}^{\infty} a_k F_k(z)\ ,$$

with $f(z) \equiv 0$ if and only if $a_k = 0$ for all $k \geq 0$.

Care must be taken with formula (1.2) since in general $f(w)$ is defined only for $|w| \leq 1$ and $\psi'(w)$ only for $|w| \geq 1$. It is here, for example that we make use of the fact that ∂K is rectifiable so that $\psi' \in H^1$ ([8] Theorem 3.12). Since the boundedness or unboundedness of T depends only on its behavior on a dense set, namely on Π, we need consider (1.2) only for functions which are defined for $|w| > 1$ as well. This might permit the condition $\psi' \in H^1$ to be relaxed somewhat, by the use of Abel limits, say; but this seems of little interest.

When K is a Faber set the inverse T^{-1} of T, defined on the range of T, is given by

$$(T^{-1}f)(\zeta) = \frac{1}{2\pi i}\int_{|w|=1} \frac{(f\circ\psi)(w)}{w-\zeta}\,dw$$

for $\zeta \in D^o$. For $|\zeta| = 1$ this would be the Hilbert transform of the composite function $(f\circ\psi)(w)$. The mapping T^{-1} is bounded if and only if T is surjective and in that case T is an isomorphism between $A(K)$ and $A(D)$. Such a set K for which T and T^{-1} are bounded is called an inverse Faber set. Of course, T is a Banach space isomorphism and not an algebra isomorphism; products are not preserved.

2. Polynomial and Rational Approximation

We define, as usual, the best polynomial approximation $E_n(f)$ and the best rational approximation $r_n(f)$ to a function $f(z) \in A(K)$ by

$$E_n(f) = E_n(f,K) = \inf\{\|f-p_n\|_\infty,\ p_n \in \Pi(n)\}$$

$$r_n(f) = r_n(f,K) = \inf\{\|f-r_n\|_\infty,\ r_n \in R(n)\}\ .$$

Here $R(n) = R(n,K)$ denotes the set of rational functions of degree at most n with poles off K. The Faber operator (1.2) maps $R(n,D)$ onto $R(n,K)$. This follows from the elementary contour integral calculation

$$T\left(\frac{1}{w-w_k}\right)(z) = \frac{1}{2\pi i}\int_{|w|=R>1} \frac{\psi'(w)}{\psi(w)-z}\,\frac{dw}{w-w_k} = \frac{\psi'(w_k)}{z-\psi(w_k)} .$$

The following result follows immediately from the above considerations, but we state it as a theorem since it shows why inverse Faber sets are of interest.

Theorem 1. _Let_ K _be an inverse Faber set_. _Then_

$$E_n(f,K) \le c\, E_n(T^{-1}f,D) \le c\, E_n(f,K) ,$$

$$r_n(f,K) \le c\, r_n(T^{-1}f,D) \le c\, r_n(f,K) ,$$

for all $f \in A(K)$. _Here_ c _is a generic constant, not necessarily the same at each occurrence_.

If $g(w) \in A(D)$ has modulus of continuity $\omega(\delta)$ defined by

$$\omega(\delta,g) = \max\{|g(w_1)-g(w_2)|,\ w_1,w_2 \in D,\ |w_1-w_2| \le \delta\}$$

then, by a well-known theorem of Jackson,

$$E_n(g,D) \le c\,\omega\left(\frac{1}{n},g\right) .$$

Hence if K is an inverse Faber set, $E_n(f,K)$ is precisely of the order of $\omega\left(\frac{1}{n},T^{-1}f\right)$.

To get a reasonably good polynomial approximation to $g(w) = \sum_{k=0}^{\infty} a_k w^k$ in $A(D)$, Kövari has shown that we may take the de la Vallée-Poussin means, defined as follows. Set

$$S_n(w) = \sum_{k=0}^{n} a_k w^k , \qquad V_n(w) = \frac{1}{n}\sum_{k=n}^{2n-1} S_k(w) .$$

Note that $V_n(w)$ is a polynomial of degree at most $2n-1$. Then ([12], p.367)

$$\|g-V_n\|_\infty \le 4E_n(g,D) .$$

In most cases, for example when dealing with lip α problems, the

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