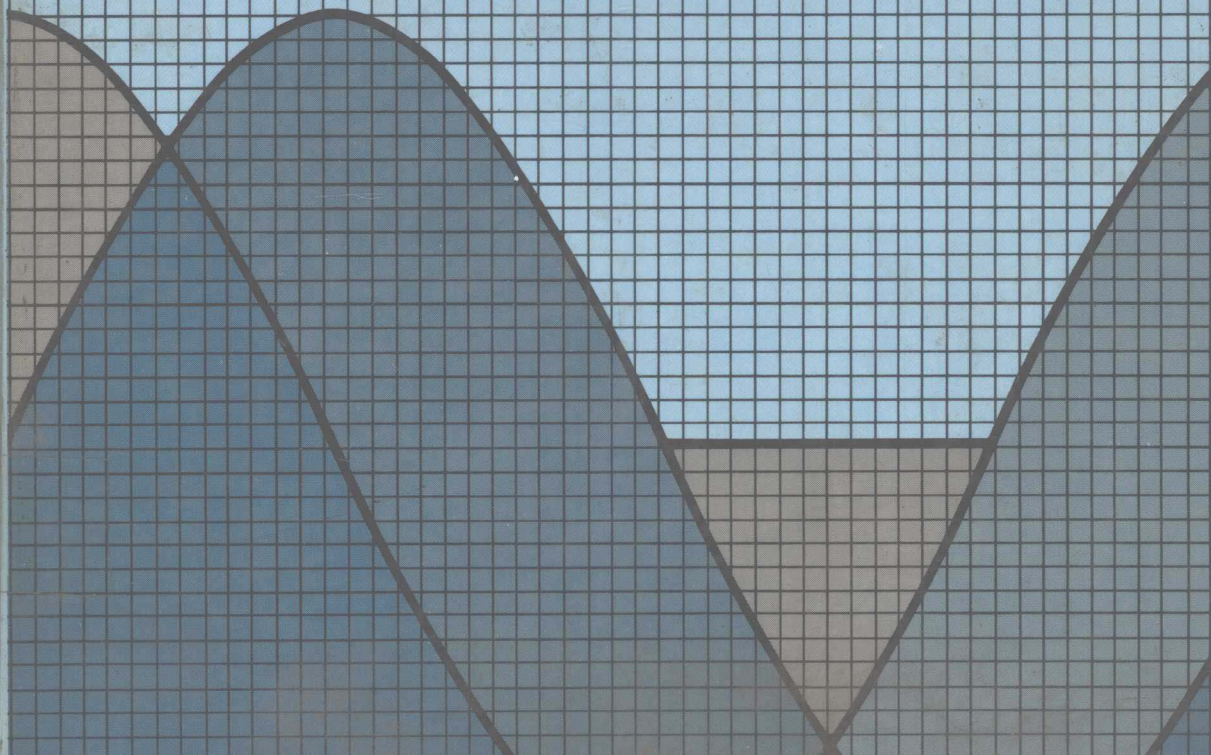
MALCOLM W. POWNALL

Functions and graphs

calculus preparatory mathematics



Functions and graphs

calculus preparatory mathematics

MALCOLM W. POWNALL
Professor of Mathematics
COLGATE UNIVERSITY, HAMILTON, NEW YORK

Prentice-Hall, Inc., Englewood Cliffs, New Jersey 07632

Library of Congress Cataloging in Publication Data

Pownall, Malcolm W.
Functions and graphs.

Includes Index.
1. Functions. 2. Algebra—Graphic methods. I. Title.
QA331.P87 1983 515 82-11304
ISBN 0-13-332304-8

Editorial and production supervision by Maria McKinnon
Interior design by Maureen Eide and Maria McKinnon
Cover design by Maureen Eide
Manufacturing buyer: John Hall

ISBN 0-13-332304-8

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Prentice-Hall International, Inc., *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*
Prentice-Hall of Canada, Ltd., *Toronto*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Prentice-Hall of Southeast Asia Pte. Ltd., *Singapore*
Whitehall Books Limited, *Wellington, New Zealand*

Functions and graphs

For Joe, Betsy, Kathy, and Tom

Preface

This text is intended to prepare students for a college level course in calculus. It has developed from notes prepared for a summer course at Colgate for prefreshmen enrolled in the University Scholars Program. This program includes students with diverse secondary school backgrounds, and widely differing educational objectives; a common purpose, however, has been their intention to pursue college mathematics at the calculus level.

It seems to me that good preparation for calculus demands attention on two fronts: (1) developing specific mathematical skills and knowledge that many students do not acquire in secondary school; and (2) gaining some insight and familiarity with the kinds of questions that arise in calculus. I have tried to address both these needs.

Topics from algebra, trigonometry, and coordinate geometry form the syllabus of the text, which is centered about the unifying concept of function. I have been selective in the choice of topics because most students (and instructors) are anxious to get on with calculus. I believe the material contained in this book provides very solid background for that purpose and have attempted to present these topics clearly at a level suitable for college students.

I have tried to set the stage for calculus by emphasizing functions, graphs, and word problems of the sort encountered in a traditional calculus course. Limits and continuity are done informally. I have treated rate of change, linearity, and linear interpolation in more detail than is common in a course of this type and have used interpolation repeatedly throughout the remainder of the book. Change of variables, substitution, composition and decomposition of functions are valuable tools and I have given considerable attention to them.

Inverses of functions are discussed in general and then applied in several special cases. The approach to trigonometry is analytic, as I believe it should be for calculus-bound students; those seeing trigonometry for the first time will not find this approach any more difficult, while students seeing it again are more likely to find this approach new.

The book is divided into two parts. Part I consists of material that most college students have studied in high school: the laws of algabra, integral exponents, the number line, inequalities, coordinate geometry, and the elements of graphing equations. However, for may students the initial exposure to these topics has not "taken", and so Part I offers an opportunity for review at the college level.

The main part of the text is contained in Part II, which deals with elementary functions and graphs. The instructor will find most of the standard topics included, with particular emphasis on those topics that are actually needed in calculus. Chapter 4 on functions, and Chapter 7 on composition and inverses, are especially detailed.

There is enough material in Parts I and II for 50 to 60 class periods (excluding tests and review periods). Accordingly, some omissions will have to be made in a brief course. Here are some possible shortcuts:

1. For well-prepared students, Part I may be covered quickly. This would save up to 6 class periods.
2. In Chapter 5, Sections 5-8 and 5-9 on asymptotes of rational functions could be deleted, since these topics are often included in the calculus syllabus. (The concept of an asymptote does recur in later chapters, and so the descriptive "definition" would have to be added at that time.) This omission would save about 3 class periods.
3. Chapter Six on complex zeros of polynomials is entirely optional. Its omission would save 3 or 4 class periods.
4. Many calculus courses include conic sections and polar coordinates. Omission of Section 10-5 and Chapter 11 would save 6 or 7 class periods.

I would like to thank Gertrude Pownall and Thomas Tucker, who made valuable suggestions after teaching from an early version of the manuscript. I am also indebted to the editorial staff of Prentice-Hall, Inc., and their reviewers for excellent advice and constructive criticism. I have gratefully accepted many of their helpful recommendations.

Finally I wish to express my heartfelt thanks to Lorraine Aveni who patiently and expertly typed many versions of the manuscript.

Hamilton, New York — Malcolm W. Pownall

A word to the student

The purpose of this book is to prepare you for a college level course in calculus. Calculus is one of the most important branches of mathematics, because it helps us to answer fundamental questions that arise in fields as diverse as mechanical engineering, health sciences, theoretical economics, and business adminsitration. Calculus has traditionally been a required course for many branches of science, and is often required today for entry into professional programs such as medicine and business administration.

Calculus is exciting and fun, but it is also demanding. In this book we will concentrate on reviewing and developing the knowledge and skills you will need if you are to succeed and get the most pleasure out of your calculus experience. Now and then we will glimpse ahead, to get some idea of what calculus is about, but we will not deal with calculus itself.

The main concept in our work is that of function. We will make a careful study of the linear and quadratic functions; the polynomial and rational functions; exponential, logarithmic, and trigonometric functions. You have undoubtedly encountered some of these functions in previous courses. In addition to their algebraic properties we will be strongly interested in discussing their graphs, because skill in graphing is one of the most important tools a student can bring to a calculus course. The functions we study have important applications, many of which we will study here; other applications will come later when you have the additional tools of calculus to work with.

Before we get underway with the study of functions and their graphs in Part II, we present some background material on the algebra and geometry of

real numbers, and on the basics of graphing. Some of the topics covered in Part I may be familiar from high school, but a review of these items will help you to sharpen your skills. Other topics such as solving inequalities, and using symmetry to draw graphs, are less likely to be familiar but will prove to be valuable tools in this course and in calculus. I have written this book with the hope that you, the student, will be able to make good use of it in learning mathematics. I have tried to state ideas clearly and to follow them with examples that illustrate these ideas. When reading the text and studying examples you should have pencil and paper at hand so that you can follow through any details not completely carried out. Some details are left out in order to make you think actively while you read.

The book is not intended to replace your instructor—so when you have questions about a reading assignment that you cannot answer after a reasonable amount of thought and effort, be sure to ask.

Mathematics is not a spectator sport—participate! Your instructor will encourage your participation by assigning homework exercises. You should do them as carefully and completely as you can since your ability to do the exercises is the best test of whether you are learning the material. To help you know how well you are doing, answers to most odd numbered problems are in the back of of the book. Of course, you should not look at an answer until you have completed your own solution. Many students will not be able to do all of the problems assigned, and your instructor will be glad to have your ask questions after you have made a reasonable effort. Chances are that some others in the class will have the same questions.

Some of the exercises are labeled "FOR THE ENTHUSIASTS." Do not be scared by this heading. These are not necessarily harder problems. I have put them there for students who enjoy doing mathematics and have time for more than the standard fare, and for instructors who want to enrich their courses. Frequently these problems deal with more theoretical aspects of a topic, other times they simply offer interesting digressions, or perhaps they just give more detail on the material of the section. If you or your instructor are enthusiasts, please consider these problems an invitation.

Each chapter is followed by a summary and a set of review exercises. These are intended to assist you in reviewing the chapter or in preparing for tests. Warning: You should not use these as your *only* means of preparation, for no two-page summary can adequately cover the material of many pages! I suggest you *first* thoroughly review the sections of the chapter and *then* look at the summary, which lists some things you should know, and some things you should be able to do. If your initial review was thorough you should feel comfortable about most of these things and you should be able to do the review problems easily without referring back to the sections. If you find things you do not remember clearly, or problems you cannot do, then the corresponding section or sections should be thoroughly restudied.

Students in the Colgate University Program of University Scholars have studied from this text as it developed over a number of years. I have found the most conspicuous characteristic of these students to be a high degree of motivation, and I have been pleased at their diligence, enthusiasm, and success. I hope that you too will be diligent, that your experiences in this course will whet your appetite for more mathematics, and that you will be very successful!

Contents

Fundamentals

ONE

1 The algebra and geometry of real numbers

1-1 THE ALGEBRA OF REAL NUMBERS (A BRIEF REVIEW)

In this book we will do most of our work within the system of real numbers. This system includes the integers (whole numbers) 0, ± 1, ± 2, ± 3, and so on. It also includes numbers like $-8/3$, 3.651, π, $\sqrt{5}$, and $(-1 + \sqrt{5})/3$. The system of real numbers does *not* include complex or "imaginary" numbers, such as $i = \sqrt{-1}$ and $2 - 3i$.

Here we have listed for review and reference some of the algebraic laws governing the system of real numbers. You should be familiar from elementary algebra with the way in which these laws are used in working with algebraic expressions and equations.

Basic Properties of the Real Number System

The fundamental operations of addition and multiplication can always be carried out within the system of real numbers. These operations obey the following laws.

Closure Laws.

1. For addition. If a and b are any real numbers, then their **sum** $a + b$ is a real number; a and b are called **terms** in this sum.
2. For multiplication. If a and b are any real numbers, then their **product** ab is a real number; a and b are called **factors** in this product. The product ab is sometimes written $a \times b$ or $a \cdot b$.