NEW DIRECTIONS IN
MATHEMATICS EDUCATION

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LIST OF TABLES

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PREFACE

The standard of mathematics teaching in schools is currently the focus of unprecedented concern amongst educators worldwide, nowhere more so than in the United Kingdom where it is estimated that as many as 7 million people may be innumerate, illiterate, or both (The Times, 3 February 1987). The blame is laid variously upon teachers, politicians, parents, examiners, curriculum planners, teacher-training colleges, and universities. However, despite numerous attempts to improve matters, the problem seems to remain.

In 1978 the Department of Education and Science commissioned a detailed review of mathematics education in the United Kingdom under the chairmanship of Sir Wilfred Cockcroft. The findings were published in 1982 in a 300-page document which has since become known as the 'Cockcroft Report'. Amongst the conclusions of this report was a proposal for a so-called 'bottom-up' approach to mathematics in schools, whereby curricula should be seen to reflect the needs of the majority of pupils who would wish simply to use mathematics in their everyday lives, rather than the tiny minority of pupils who would be likely to obtain advanced qualifications in mathematics and pursue careers in cognate disciplines.

In addition, recent government concern over standards has led to the most radical proposals for change to the educational system, arguably since the introduction of state education itself. These include an agreed national curriculum, the introduction of a new examination (GCSE), testing at ages 7, 11, 14, and 16, and the drawing up of a 'league table' of schools based on performance on the national tests. In addition, the Inner London Education Authority has been abolished and schools throughout the country are now free to opt out of local authority control. These proposals serve to underline the unprecedented level of concern over educational standards generally and, in the wake of Cockcroft, over standards in mathematics teaching specifically.

Recently, educationalists have become increasingly aware of the need for a multidisciplinary approach to mathematics teaching, with teachers, educational researchers, psychologists, and mathematicians co-ordinating

efforts to tackle fundamental problems. In addition, educators have come to acknowledge the value of learning from the experiences of other countries, and of fostering international communication on interdisciplinary aspects of mathematics education at all levels, an aim embodied in the activities of the International Group for the Psychology of Mathematics Education, the thirteenth annual meeting of which will be held in 1989.

It was with these ideas in mind that we (the editors) organized a conference entitled 'Developments in Teaching Mathematics'. This conference, held under the aegis of the Northern Ireland Office of the British Psychological Society, drew together teachers, educational administrators, psychologists, mathematicians, and educational researchers. In addition to this multidisciplinary theme, the conference also had a distinctly international flavour, with contributors from the United States, Eire, Belgium, and England, as well as Northern Ireland.

The present volume seeks to build on the achievements of 'Developments in Teaching Mathematics'. Five of the chapters have their origins in the conference itself and the remaining chapters were commissioned to extend and develop ideas in keeping with the themes of the conference.

The book seeks to address several recent important developments in mathematics education. It has two general aims - first, to examine interdisciplinary aspects of mathematics teaching using a variety of practical and theoretical approaches to learning and instruction, and, second, to view problems in mathematics education from an international perspective. In addition, mathematics teaching in both the primary and post-primary sectors is considered.

Chapters are organized under four main headings reflecting these aims. Part I considers how psychological insights may contribute to mathematics teaching. Part II reviews specific problems in teaching and assessing basic arithmetical operations. Part III addresses several recent curriculum developments in the United Kingdom, and Part IV considers curriculum developments from an international perspective.

It is hoped that this book will prove a valuable source of debate and ideas for mathematics teachers (primary and post-primary), psychologists, educational researchers, administrators, school inspectors, and

mathematicians with an interest in the teaching of their subject.

We are grateful to many people for their assistance with this project. In particular, we should like to thank Ronnie Wilson and Joan Harbison of the British Psychological Society for their support in organizing the conference. We also wish to acknowledge the help of the Department of Education, Northern Ireland and the staff of Stranmillis College, Belfast. In addition, we owe debts of gratitude to Ken Travers, Martin Fitzpatrick, Paul Quigley, Maire Carville, and Karen Latimer. And a special note of thanks is due to Sue Joshua of Routledge who has been an enthusiastic supporter of the book from the outset.

POSTSCRIPT (JANUARY 1989)

As this book goes to press, changes continue at breakneck pace with no apparent cohesive philosophy behind them. The mathematics Working Group's proposal on Profile Components has been altered out of recognition in the face of overwhelming opposition from the individuals and organizations who expressed views during consultations; the long division algorithm has been reprieved. The sophisticated model for national assessment developed by the Task Group on Assessment and Testing still has to be proved in practice. There is a glaring lack of evaluation of the changes being made. The supply and support of enough suitably qualified teachers remains to be tackled. In summary, the unease and scepticism expressed in the final chapter over current developments in the United Kingdom remain undiminished.

Brian Greer

Gerry Mulhern

FOREWORD

All over the world mathematics educators are expressing concern about the degree of mathematical understanding displayed by children in school. It is time we listed our priorities and sought evidence for the effectiveness of suggested solutions. This book contains a collection of views contributed by people interested in mathematics education from a variety of perspectives. These views provide a basis for discussion on crucial issues in the field, for teachers, teacher-trainers, and students in training. The authors have invited me, in this foreword, to contribute some of my own views to the debate.

Mathematics education is concerned with the learning and teaching of mathematics and, to a large extent, the focus of attention is the age range 4 to 16 years. Mathematics educators tend to be mathematicians, teachers of mathematics, and psychologists. They are often involved with the training of future teachers and sometimes they do research. Those who come to the subject from or through mathematics can usually be relied upon to have a healthy respect and a veneration for the subject of mathematics.

Mathematics is part of universal education and the subject appears in the curriculum for all children. The type of mathematics taught, however, varies according to the perceived ability of the pupil. Children of the same age can display vastly different levels of attainment. The reasons for low attainment vary and in some cases it may be caused simply by a gap in knowledge or by a misconception which once discovered can be cured. A dangerous opinion is that which labels a group of children as incapable of learning mathematics and so relegates them to less and less content in the subject. A view of mathematics that is held by many is that it should be 'real world', with its immediate relevance and usefulness apparent to the child. In many cases it is the world as invented by the adult for the child. It cannot be the life the child will lead as an adult since nobody can know what this will be, and too often it is not the child's present life, either.

Actuality involves many variables which intrude and so in our 'real' examples we avoid them - we change the

newspaper article, the sign in the bank, or the score card, because they are too rich in information. Surely one of the advantages of mathematics is that one does not have these extra variables. This does not mean that we should not bring our mathematical knowledge to a problem, but if the best solution to a problem is to obtain an expert, then according to the rules of the 'real life' game it should be allowed. Do we as teachers play the game fairly or do we just pretend?

Teachers of mathematics are in schools to teach mathematics and when children do not learn they should very seriously seek to discover the reason. The answer may be in the level of difficulty of the topic we present, the lack of pre-requisite knowledge held by the child, the manner of presentation, the leaps between one part of the work and the next, or the fact that the child has not paid sufficient attention to the task. Most of these points belong within the teacher's part of the interaction that leads to learning. Does the teacher plan, analyse, dissect, and test the effectiveness of a lesson? Surely such self-analysis is part of being professional, the sensible way to improvement. Do educational authorities see that time is needed for these activities?

There is very little mathematics education research being carried out in the United Kingdom, and little evidence of what produces efficient teaching. Translating what is known into curriculum materials is also fraught with difficulties. In this volume, Mangan and Hoffman have addressed some of these problems, and de Corte and Verschaffel discuss research on the effectiveness of Logo - which is seen by many as an effective teaching medium.

Mathematics educators can be identified by their enthusiasm for mathematics and for the abundance of their 'good ideas' on how to make others like/use/succeed in mathematics. This enthusiasm, without evidence on effectiveness may be a disservice to teachers and children. We need copious research in classrooms to validate what are now firmly and fervently held beliefs before we start putting new 'good ideas' into the schools. It may be a good idea for a thinker to suggest a particular approach to the teaching of a topic. That same thinker may try it out with a group of children. It may be explained at length in a book or article BUT, when in the hands of a classroom teacher, is it robust?

An illustration of difficulties which might occur

when a theory is translated into classroom practice can be seen in the use of concrete materials to introduce some fairly formal pieces of mathematics. For many years teachers have been advised to make their mathematics more concrete or more practical and the theory behind this is often an adaptation of the works of Piaget.

The two research projects based at King's College, 'Children's Mathematical Frameworks' (CMF) and the 'Nuffield Research Project', have been monitoring the transition made by teachers and children as their classroom mathematics moves from concrete experiences to more formal, generalized, and often symbolic mathematics. The children are aged 8 to 14 years. The research is carried out in classrooms, through (a) one-to-one interviews with children, and (b) tape recording and observing lessons. In CMF there is no intervention on the part of the researcher and only in the second phase of the present research is the researcher involved in planning. Teachers are engaged in providing practical experience, the synthesis of which is a mathematical formalization. Some points which emerge from the observation of this activity are:

(a) Often the teacher seems to forget that the mathematics is new to the child and so makes large leaps between points.
(b) Sometimes the teacher does not take the experience seriously and so ignores the very physical nature of the concrete material. For example, 'Let us pretend this rod is 15 in length', when the rod already has a length!
(c) The teacher does not say why the transition is being encouraged, so the power of the generalization being urged on the class is lost. No wonder the children do not see a common thread in questions which require them to state the appropriate operation (see Chapter 2 for further illustration of this point in the work of Greer).
(d) The pre-requisites for a topic have not been assimilated. For example, is there any point in teaching equivalent fractions, using regions, to children who cannot give a fractional name to a region?

Planning how to use material, finding what is available, researching the theory and its implications, takes time. Listening to a tape-recording of what one said in class, analysing it and planning for improvement takes time.

Listening to a child explain what he or she thought the lesson was about is vital but time-consuming.

Chapter 11 in this book describes the success of Japanese children when studying mathematics. The recently published results of the Second International Mathematics Survey (SIMS) clearly demonstrate the superiority of the Japanese performance on questions attempted by children all over the world. As the chapter makes clear, there are several differences between the mathematics taught in the United Kingdom and that taught in Japan, not least being the amount of time spent on the subject. A set of data from the International Study concerns the attitute of teachers to the mathematics they teach. Opinions on how easy it is to teach the subject vary from country to country. McKnight and colleagues (1987) writing on the SIMS results from an American viewpoint say:

> Taken together, these data suggest that Japanese teachers perceived teaching mathematics as a difficult, demanding enterprise, the success of which had considerable impact on the achievement of their students. By contrast, US teachers seemed to see teaching mathematics as less demanding and to view the learning of mathematics as an enterprise over which they had relatively little control. There were also some indications, although less direct, that teachers in the US and Japan were given somewhat differing status.
>
> (McKnight et al. 1987)

In Chapter 12, Greer and Mulhern discuss the future of mathematics teaching within the new national curriculum and the required attainment targets. By law, children will have to be assessed on their progress regularly throughout their school career. The intention is to improve the quality of education. It is worth considering whether the changes will bring this about without attempting to provide teachers with the time and opportunity to analyse and eventually improve their own performance. Failure on assessments may tell us something is wrong but there will be no improvement without scrutiny of where the cause may be.

Brian Greer and Gerry Mulhern have provided statements from psychology, research, the laboratory,

other countries, the innovator and the classroom teacher. All of these add to the debate of what never has been an easy problem with an easy solution.

Kathleen Hart
July, 1988

CONTENTS