The Nonlinear Theory of Shells through Variational Principles

From Elementary Algebra to Differential Geometry

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Foreword

Shell analysis became a subject of central interest to structural engineering only in the middle of this century when both aircraft and civil engineers discovered the economy of these forms for construction. At the time, very few consistent differential equations existed to describe the shell behaviour, and indeed the solution of most equation systems then available was almost impossible for general cases.

Since those early days, much development has ensued. On the one hand the computer, coupled with the Finite Element Method, allowed the solution of very complex problems of shell behaviour, even without invoking the full Shell theory. On the other hand the theory itself was made much more elaborate and consistent omitting many over-simplifying assumptions.

The book by Professor Valid concentrates and develops the modern theory within a variational framework important for Finite Element Method applications. The approach presented is novel and in the second part of the book dealing with differential geometry the reader is introduced to an exciting and useful branch of mathematics.

Professor Valid, whom I have known for over twenty years now as a close friend and as a mathematical scientist, has a distinguished career and recently has put in many years of work into this 'Magnum Opus'. I believe the treatise will be a success and many mathematicians and scientists — as well as engineers — will benefit from the generality and the deep insights it provides. I wish the author and his book the greatest success.

O.C. Zienkiewicz

Preface

It is now widely accepted that variational principles constitute the most important tool for the analysis of mechanical structures. This computational tool is also suitable for different types of theoretical research, as has been shown by many remarkable treatises on Functional or Numerical Analysis. A word of caution: numerous variants have been proposed, but it is indisputable that their efficiency has been impaired because of the difficulties encountered in applying them.

The case of shells, and particularly of nonlinear shells, was selected as a sphere for this branch of research. Studies in this field have long been inhibited by the undue conventionality, obscurity and general stodginess of the available works of reference.

The principal feature of this treatise — which is neither state-of-the art, nor simply a textbook — lies specifically in the utilization of an intrinsic geometrical and coordinate-free method. This method uses linear mappings defined by tangent planes of a bidimensional manifold, whose interpretation by tensors is straightforward, but whose geometrical meaning is much more comprehensible.

Let us say at the outset that the standard techniques of differential geometry worked out on a curved manifold (here denoting the middle surface of the shell) are little on the intricate side simply because of the introduction of covariant derivatives and the external differential calculus on the manifold, even if the given manifold is immersed in a three-dimensional linear space. This is, in fact, due to the exclusive variations, say of tangent differentials, on a Riemannian manifold.

Thus a new mathematical technique born of a clear, geometrical representation appears to be indispensible, as this book shows.

It should be noted in this respect that, leaving aside the decomposition into tangent and normal variations on an immersed manifold, hence the use of Riemannian manifolds, another idea using classical referentials and classical differentials in a linear embedding space, has been proposed recently by J.-M. Souriau[1]. This approach lies in the description, and the study, of forces which act on an immersed manifold; but, in this approach, the classical representation by indices is unavoidable. A 'new' type of Mechanics could then be constructed by this author from the action-reaction principle, together with the consideration of geometrical properties of the entire space itself, with the concept of homology-cohomololgy applied to any mechanical system.

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[1] J.M. Souriau (1992) Mecanique des états condensés de la matiére. *Fédération de Mecanique de Grenoble, 1st Int. Seminar, 18-21 May.*

It is clear that extreme care must be taken, particularly in certain instances, in the interpretation implied by these representations, but the result of using this method will be a great simplification of calculus[2].

Nevertheless, remaining faithful to the decomposition — conventional itself — into tangent and normal components of a Riemannian manifold, and also to our geometrical intrinsic technique (already used in our previous book[3]), we considered it essential to include a sufficient mathematical appendix. The reader will find in that appendix, entitled 'From Elementary Alegrba to Differential Geometry', all the elements necessary for the comprehension of this book.

This being said, the book comprises in essence two main parts: the first four chapters deal with shells properly speaking, the four following chapters with their stability theory.

In Chapter 1, there is an original presentation of the primal formulation. A statement of classical mixed principles follows in Chapter 2, and static and dynamic formulations devoted to dual principles are the subjects of Chapters 3 and 4.

The second part, Chapters 5 to 8, deals with static and dynamic stability. Classical theories are recalled, albeit augmented by some new methods, among which are some original developments regarding mixed or dual principles.

In the latter section, the recall of the pioneering variational theories of Koiter should be noted. Also, the name of Budiansky, in the elastic post-buckling range, and the names of Hill, Hutchinson and Nguyen Quoc Son, in the plastic post-buckling range, and again those of Bolotin and Hsu in dynamic stability, could not be omitted. The original dual principles, introduced in the preceding chapters, raise the possibility of proposing some new ideas about dynamic stability. Here, nonlinear problems need, we believe, a permanent research effort, in particular for complex structures or large systems. More explicit detail is given in the 'Contents' section.

These somewhat involved developments regarding shells, or any system stability in general, are far from constituting an exhaustive statement. However, they do reflect the deep convictions of the author, for whom the concept of stability, and even more of instability, of nonlinear or equivalent mechanical systems, does appear to be a most fruitful area. In fact, the instability concept, with all its possibilities of bifurcated evolutions, appears to be the most common fate of all natural phenomena, stable states being no more than specious and provisional.

What we mean indeed by natural phenomena are all biological, sociological and economic phenomena — in a word, all 'human' phenomena — as well as astronomical and even cosmological phenomena.

Can we not envisage, for instance, that any complexification of a non-isolated system, in particular in biology, would generate bifurcations — and an almost unending sequence of myriads of bifurcations — by a mechanism which remains yet to be discovered? Our personal conviction would be to go so far as to see in this phenomenon the very (material) origin of life, be it animal or vegetable. Are we being more audacious than Prigogine[4], who asks: "... the dynamic instability, which allows to characterize some aspects of the behaviour of the brain could it be a product of the biological evolution?". We believe that

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[2] It could be observed that this approach is quite conformable to a recent technique in Numerical Analysis called "method of the fictitious domain".

[3] R. Valid (1981) *Mechanies of continuous Media and Analysis of Structures*. Amsterdam, North-Holland.

[4] L. Prigogine and I. Stengers (1986) La Nouvelle Alliance. Paris, Gallimand.

it is original biological evolution which could be a product of the dynamic instability. This author having not added regarding the brain: when we say this system is characterized by its intrinsic dynamic behaviour, by the behaviour instability that it generates?

Economic phenomena provide a striking example of this type of behaviour. Unpredicable, seemingly even whimsical, evolutions plunge specialists into deep perplexity. Phenomena — more or less local, under the very definition of the system under consideration — are submitted to an extraordinary degree to all kinds of influences, including those of a psychosocial nature, and which know only unstable equilibria. Ideas of Chaos Theory soon come to mind.

Amongst the many scientists who have risen to eminence in thermodynamics itself is P. Germain who used to distinguish between the 'prudent' and the 'audacious'[5]. It offers spectacular examples of instabilty which are also enhanced by the concept of chaos. Here Prigogine does not hesitate to declare: "It will be now on around the themes of stability and instability that our description of the world will be organized, and not around the opposition between hazard and necessity."

It need hardly be mentioned that the links between statistics and chaos, elementary systems and populations, between thermodynamics stability and chaos, and hence between all life phenomena, are most intimately bound up with one another. Indeed, we may go on to posit the premises of a thermodynamical theory of sexuality, whose explanation would dovetail at a deeper level with the second principle (of thermodynamics), sexuality having at its origin a tendency to disorder (no malice intended!), and with its multiple bifurcations, this in contrast to the order inherent in the reproduction by mitosis, that ultimate unstability event.

The conviction that lies behind the ideas advanced here should not be allowed to obscure a constant, human hesitation, and also that, amongst evident natural instabilities, it is undeniable that there are many self-regulated phenomena. Is it a question of degree, of limits, of chaos, or what? Or does it all come down to subjective belief?

These few reflections, more scientific than philosophic, as it might seem, do not aim by any means at the 'founding of an ethic on stability and instability'[6], but rather they aim to establish the prominent claims in this field of Mechanics. Through the intimate way Mechanics is bound up with reality, reflection and imagination fertilize one another in several areas. It is not merely the fact that the science of numerical analysis draws heavily on research in Mechanics, the extent of this dependence extends so far that the very name, Computational Mechanics, means the whole of computational science in the entire field of physics.

This book is thus no more than a modest contribution to research in what is a truly vast field.

Returning to the so-called intrinsic form of the presentation, let us recall, after the great Poincare and Hilbert, the opinion of L. Solomon in the preface to his excellent book on Linear Elasticity[7]:

"The state of things shows... that the contemporary research in theoretical elasticity is often slowed up by the insufficiency of the mathematical apparatus at its disposal. And

[5] P. Germain (1973) Thermodynamique des milieux continus, *Congrès Français de Mecanique, Poutiers, 17-20 September*.

[6] In answer to a remark from P. Germain (1989, private communication).

[7] L. Solomon (1968) *Elastiaté Lineaire*. Paris, Masson.

precisely the efforts which tend to improve and adapt this apparatus are frequently sources of continual and moving on progresses in this theory whose past is more than secular."
Is this viewpoint acceptable with regard to our subject?

In the overall framework of this topic, the reader might begin to feel a certain legitimate frustration at the dearth of practical applications of this treatise. However, the final chapter, 9, cites, alongside some computational strategies, a large number of practical contributions, with care being taken to show both their variety, and their excellence, but never forgetting just how paved with pitfalls is the road ahead that remains to be taken.

Abbreviations and Symbols

Adm.	Admissible
BC	Boundary conditions
BIE	Boundary integral equations
DM	Direct method
DMV	Donnel-Mushtari-Vlasov
DOF	Degree of freedom
FEM	Finite element method
IC	Initial conditions
IM	Indirect method
IS	Imperfect shell
KA	Kinematically admissible
KL	Kirchhoff-Love
LBB	Ladyzhenskaya-Babuska-Brezzi
LS	Linear space
MCM	Mechanics of continuous media
NC	Necessary condition
NVS	Normed vector space
PK	Piola-Kirchhoff
PL	Piola-Lagrange
PS	Perfect shell
PVW	Principle of virtual work
SA	Statically admissible
SC	Sufficient condition
s.t.	such that
3D	Three-dimensional
TL	Total lagrangian
TS	Transverse shear
UDL	Updated lagrangian
VS	Vector space
$\tilde{.}$	Transposition
$\lvert . \rvert$ or $\lVert . \rVert$	Norm
$\mathcal{L}(.,.)$	Set of linear mappings from . into .

$\mathcal{L}_s(.,.)$	Set of symmetric linear mappings from . into .
$H(\text{div}, \Omega)$	Hilbert space H of mappings A defined on Ω s.t. A and $\text{div}\, A \in L^2(\Omega)$
$L^2(\Omega)$	Functional space of square-summable quantities defined on domain Ω
$\underset{\sim}{A}(X)$	means A is a "function" of X

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