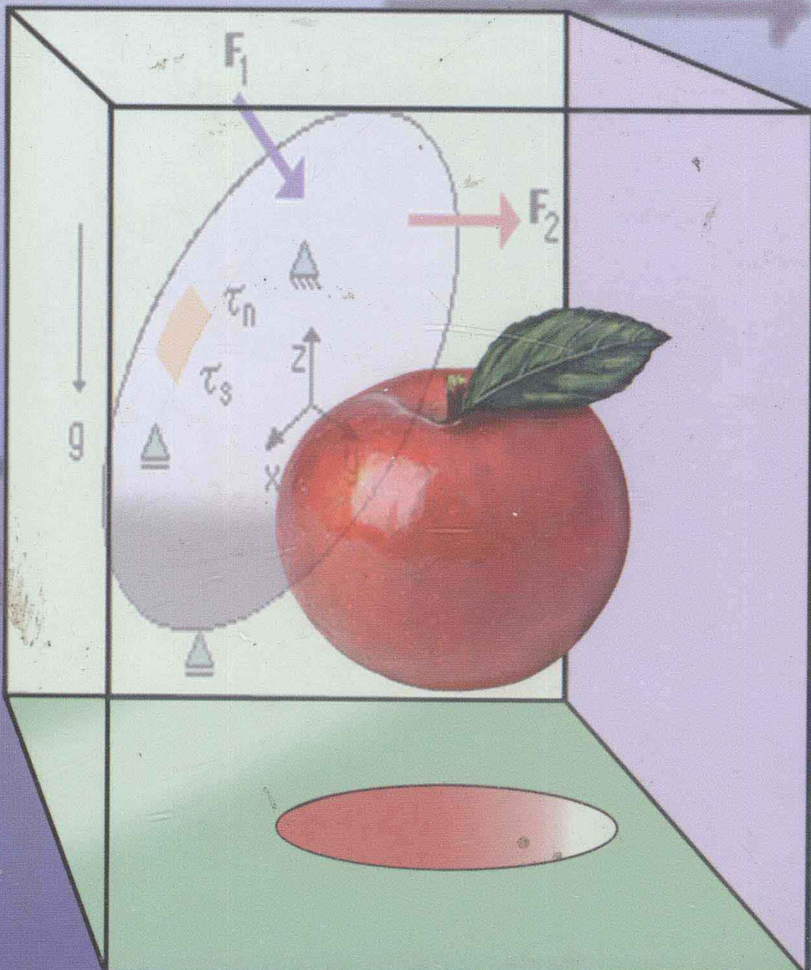

MECHANICS

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PREFACE

The book has been designed to serve as a textbook in Mechanics for B. Sc. (General and Hons.) students of physics of Indian and other Universities, keeping in mind the latest proposed syllabus by University Grants Commission.

The subject matter has been selected and developed in such a manner so as to provide a bridge between introductory and advanced level courses in physics. Care has been taken to keep the level of the text in accordance with understanding of wider group of undergraduate students of physics. The salient features of the book are:

- Basic concepts and fundamental principles are explained in simple and lucid language.
- A large number of solved typical problems of different types have been given in each chapter. These solved problems illustrate or supplement the text.
- A large number of self explanatory accurate diagrams and tables have been used to supplement the text.
- At the end of each chapter, good number of review questions and problems are given.
- A large number of *short question answers and objective type questions* with answers have been given at the end of each chapter to make the book useful for various competitive entrance examinations, e.g. UGC, CSIR, NET-SLET, etc.
- Two special chapters on Analytical Mechanics and Non-linear Oscillations, Chaos and Fractals are given so that students may follow advance courses in Mechanics easily.

It is hoped that with these unique features the book will fulfill the genuine requirement of the students and teachers.

Much originality cannot be claimed in a book of this kind. The authors take this opportunity to place on record their indebtedness to the large number of books and journals that they have freely consulted in the preparation of this book. The authors heartily thank the publishers, M/s Viva Books, for taking keen interest in getting the book printed well in time.

In spite of all precautions and care taken to avoid errors and misprints, there might have crept some due to oversight and the authors will feel highly obliged to those fellow teachers and students who will bring them to their notice. Any suggestions for improvement will be thankfully acknowledged.

Authors

CONTENTS

Chapter 1

PHYSICAL FUNDAMENTALS OF MECHANICS

1.1 INTRODUCTION

Physics is one of the fundamental natural sciences studying the laws of inanimate nature. Obviously, studying physics is an exciting and challenging adventure. The word *physics* comes from a Greek term meaning nature; and, therefore, physics should be a science dedicated to the study of all natural phenomena.

The nature or say universe consists of material bodies which are in permanent interaction and motion. All the observable natural phenomena are governed by certain laws. The study and the explanation of the laws governing the connections between various processes and phenomena are the fundamental aim of every branch of physics and in general of science. The analysis of the interaction of material bodies and of the laws of electromagnetic phenomena is the aim of physics.

The motion of the substance has various forms: mechanical, electromagnetic, thermal, etc. The mechanical form of motion of the matter is the simplest one. It consists in the movement of bodies or their parts relative to one another. We can see movements of bodies everywhere in our daily life. The laws of mechanical motion are studied in '*mechanics*'. The other branches of physics cannot be studied without mechanics since various displacements are observed almost in all physical phenomena.

Mechanics is generally subdivided into three parts: *Kinematics*, *Statics*, and *Dynamics*. In kinematics the motion of the bodies is considered irrelative to the factors causing the motion or changing its characters. Statics deals with the laws of equilibrium of the bodies and dynamics studies the laws of motion and the causes producing the motion and changing it. On the otherhand, if the laws of motion are known it is possible to derive the laws of equilibrium as a special case of the former. Therefore in physics, usually the laws of statics are not considered separately and are studied in connection with the general laws of dynamics.

Bodies are macroscopic systems consisting of a very large number of molecules or atoms, so that the sizes of these systems are many times larger than the intermolecular distances. We may note that a *material point* or *particle* is a body whose size and shape are of no consequence in the problem under consideration. For example, in studying the motion of the planets around the sun, they can be regarded as particles since the distances of the planets from the sun are many times greater than their sizes. *Classical*, or *Newtonian*, mechanics deals with the motion of bodies travelling at velocities that are very much less than that of light in a vacuum. The investigation of the motion of bodies travelling at velocities commensurate wih the velocity of light is taken up in *relativistic mechanics* which is based on the theory of relativity. Specific features of the motion of microparticles are dealt with in *quantum* (wave) *mechanics*. Microparticles are particles whose rest mass is commensurate with, or smaller than, the rest mass of atoms.

Problems of the internal structure of bodies, as well as the nature and laws of their interaction are beyond the scope of mechanics, and constitute the content of other branches of physics.

Motion of a body or a particle occurs both in space and in time (space and time are in alienable forms of existence of matter). According to Newton, time is absolute and flows on without relation to the presence of any physical object or event. In this absolute space and time concept of Newton, motion of an object is characterized by change of its position in space as time evolves. Today, we know that in fact there is no object (in the universe) at rest in this 'absolute space', with respect to which one can define the position of another object. Obviously, at any instant of time, the position of an object is always defined by the '*observer*' with respect to himself. An observer, always, first creates a local space, in which positions of other objects are then described. This local space, to which is attached the observer, is called a *reference frame* or *frame of reference*. A frame of reference is a real or conditionallly rigid body with respect to which the motion of the body being studied is to be considered. Rigidly fixed in the frame of reference is some kind of coordinate system so that the position of each point of a moving body can be uniquely determined by the three co-ordinates of the point.

We may note that the concept of rest and motion are relative, i.e., these are in relation to a frame of reference or observer. Absolute motion is a meaningless conception.

1.2 FRAME OF REFERENCE AND COORDINATE SYSTEMS

We have seen that both rest and motion are relative concepts; i.e., they depend on the condition of the object relative to the body that serves as reference. A tree and a house are at rest relative to the earth, but in motion relative to the sun.

To describe motion, therefore, the observer must define a *frame of reference* relative to which the motion is analyzed. The best that can be done is to define a particular reference system at rest relative to a particular set of stars, called the *fixed stars*. All other frames of reference can be defined relative to this particular frame of reference.

A frame of reference is a real or conditionally *rigid body* with respect to which the motion of the body being studied is to be considered. Rigidly fixed in the frame of reference is some kind of coordinate system so that the position of each point of a moving body can be uniquely determined by the three coordinates of the point. The following systems of coordinates are most frequently employed in mechanics:

(*i*) Rectangular or Cartesian coordinates: (x, y, z),
(*ii*) Spherical-polar coordinates: (r, θ, φ), and
(*iii*) Cylindrical coordinates: (ρ, ϕ, z).

(i) Rectangular Cartesian Coordinate System

In this coordinates system, the three dimensions are represented by three axes x, y and z perpendicular to each other. The coordinates of any point in space are taken as distances from the origin along the three axes x, y and z and written as (x, y, z). This system is generally used where independent rectangular coordinate systems possible in space: (a) *the left handed* and (b) *the right handed* which cannot be made to coincide with each other by any means of translation or rotation. Cartesian coordinate system by convention is chosen to be right handed (Fig. 1.1). Position vector **r** of a particle P(x, y, z) relative to the origin is given by

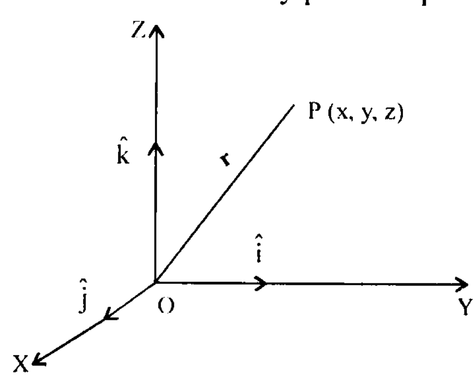

Fig. 1.1. Cartesian coordinate system.

$\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$ are unit vectors in the directions of x, y and z axes respectively.

$$\mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k} \tag{1}$$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ denote constant unit vectors in the directions of X, Y, and Z respectively. The position of a particle P relative to the origin O is given by the position vector **r**, characterized by its specific length and direction (Eq. 1).

The vectorial increment **dr** in **r** can be written as

$$\mathbf{dr} = dx\hat{i} + dy\hat{j} + dz\hat{k} \tag{2}$$

The instantaneous velocity and acceleration of P, in Cartesian coordinate system will be

$$\mathbf{V} = \frac{d\mathbf{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

or

$$\mathbf{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k} \tag{3}$$

The magnitude of **V** is given by

$$|\mathbf{V}| = \sqrt{V_x^2 + V_y^2 + V_z^2} \tag{4}$$

Acceleration is the rate of change of velocity. Thus differentiating Eq. (3) with respect to time, one obtains,

$$\mathbf{a} = \frac{d\mathbf{V}}{dt} = \frac{dV_x}{dt}\hat{i} + \frac{dV_y}{dt}\hat{j} + \frac{dV_z}{dt}\hat{k}$$

$$= a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \tag{5}$$

The magnitude of **a** is given by

$$|\mathbf{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \tag{6}$$

(ii) Spherical Polar Coordinates

The coordinates of a point—say P in this coordinate system are represented by radial vector **r**, the zenith, colatitude or polar angle θ; and azimuthal or longitudinal angle ϕ as shown in Fig 1.2 . These coordinates are related to the rectangular cartesian coordinates x, y, and z through

$$x = r \sin\theta \cos\phi, \qquad r = \sqrt{x^2 + y^2 + z^2} \tag{7a}$$

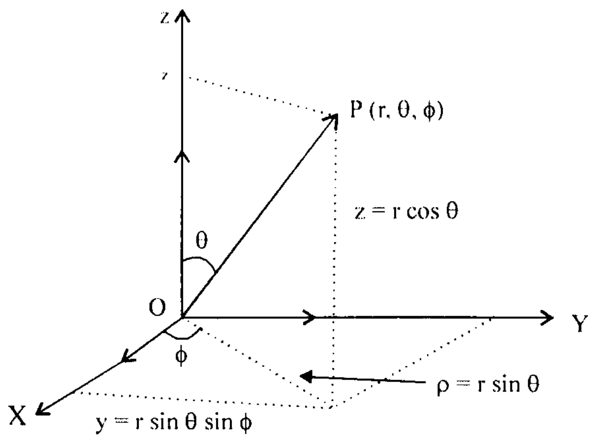

Fig. 1.2. Spherical Polar coordinates and their relationship with rectangular cartesian coordinates

$$y = r \sin\theta \sin\phi, \qquad \tan\theta = \sqrt{\frac{x^2 + y^2}{z}} \tag{7b}$$

$$z = r\cos\theta \qquad \tan\phi = \frac{y}{z} \tag{7c}$$

The coordinate surfaces are

(*i*) Spheres with concentric with the origin (r = constant)

(*ii*) Cones with the apex at the origin and along the Z-axis (θ = constant) and

(*iii*) Planes through the Z-axis (ϕ = constant).

The unit vectors $\hat{e}_r$, $\hat{e}_\theta$, $\hat{e}_\phi$ extend respectively in the directions of r increasing, θ increasing and ϕ increasing (Fig. 1.3)

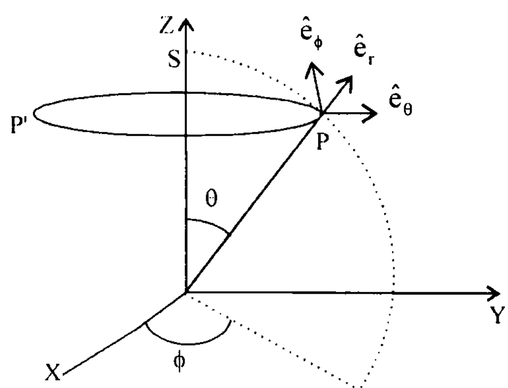

Fig. 1.3.

One can easily write the general differential displacement of particle P in spherical polar coordinates as

$$d\vec{r} = dr\,\hat{e}_r + rd\theta\,\hat{e}_\theta + r\sin\theta\,d\phi\,\hat{e}_\theta \tag{8}$$

The unit vectors $\hat{e}_r$, $\hat{e}_\theta$ and $\hat{e}_\phi$ can be expressed in terms of unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ as follows:

$$\hat{e}_r = \sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k} \quad (9a)$$

$$\hat{e}_\theta = \cos\theta\cos\phi\,\hat{i} + \cos\theta\sin\phi\,\hat{j} - \sin\theta\,\hat{k} \quad (9b)$$

$$\hat{e}_\phi = -\sin\phi\,\hat{i} + \cos\theta\,\hat{j} \quad (9c)$$

The unit vectors ($\hat{e}_r$, $\hat{e}_\theta$, $\hat{e}_\phi$) in spherical coordinate system, unlike ($\hat{i}$, $\hat{j}$, $\hat{k}$) in cartesian coordinate system are not constant vectors but change in direction as coordinates θ and φ change. However, we can see that at each point they constitute an orthogonal right-handed coordinate system, i.e., we have

$$\hat{e}_r \cdot \hat{e}_\theta = \hat{e}_r \cdot \hat{e}_\phi = \hat{e}_\theta \cdot \hat{e}_\phi = 0$$

$$\hat{e}_r \times \hat{e}_\theta = \hat{e}_\phi,\ \hat{e}_\theta \times \hat{e}_\phi = \hat{e}_r \text{ and } \hat{e}_\phi \times \hat{e}_r = \hat{e}_\theta \quad (10)$$

The spherical polar coordinates are very convenient in those problems of physics where there is no preferred direction and the force in physical problem is spherically symmetrical, e.g., (*i*) Coulomb force due to a point charge, and (*ii*) gravitational force due to a point mass. We may note that these are also examples of *central forces.*

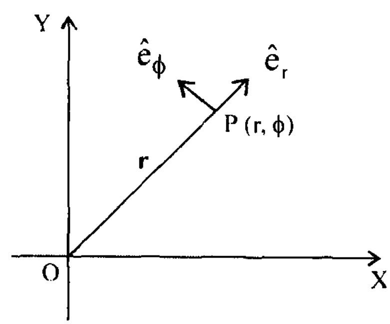

Fig. 1.4. Motion of a particle in two dimensional plane

(iii) Motion in Two Dimensions

Usually, we find a particle constrained to move in a plane, rather than in three-dimensional space in general. Let us denote the two dimensional plane as XY-plane (Fig. 1.4). The position vector of the particle P is given by

$$\mathbf{r} = x\hat{i} + y\hat{j} \quad (11)$$

as Z = 0 always for the particle in motion. Since θ = 90°, for the particle P and hence the spherical polar coordinates (r, θ, φ) reduces to circular polar coordinates (r, φ). Now, the position of the particle P in terms of radial distance r from origin O and φ is given by

$$r = \sqrt{x^2 + y^2} \qquad x = r\cos\phi \quad (11a)$$

$$\tan\phi = \frac{y}{x} \qquad y = r\sin\phi \quad (11b)$$

The displacement vector **dr** in circular polar coordinates is obtaind as (put θ = 90° and dθ = 0 in Eq. 8)

$$\mathbf{dr} = dr\,\hat{e}_r + rd\phi\,\hat{e}_\phi \tag{12}$$

where unit vectors $\hat{e}_r$ and $\hat{e}_\phi$ are given by

$$\hat{e}_r = \cos\phi\,\hat{i} + \sin\phi\,\hat{j} \tag{13a}$$

and

$$\hat{e}_\phi = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j} \tag{13b}$$

(iv) Velocity and Acceleration (In Circular Polar Coordinates System)

Let us find the velocity and acceleration of a point object, moving in a plane, in terms of circular polar coordinates (r, ϕ). Differentiating position vector $\mathbf{r} = r\,\hat{e}_r$ with respect to time.

We obtain

$$\mathbf{V} = \frac{d\mathbf{r}}{dt} = \frac{dr}{dt}\hat{e}_r + r\frac{d\hat{e}_r}{dt} \tag{14}$$

We may note that $\hat{e}_r$ is not a constant vector. One can obtain its time rate by using Eq. (13a).

One obtains,

$$\frac{d\hat{e}_r}{dt} = -\sin\phi\frac{d\phi}{dt}\hat{i} + \cos\phi\frac{d\phi}{dt}\hat{j} = \frac{d\phi}{dt}\hat{e}_\phi \tag{15}$$

Similarly one obtains

$$\frac{d\hat{e}_\phi}{dt} = -\frac{d\phi}{dt}\hat{e}_r \tag{16}$$

Using (15) and (16), Eq. (14) takes the form

$$\mathbf{V} = V_r\,\hat{e}_r + V_\phi\,\hat{e}_\phi \tag{17}$$

where $V_r = \frac{dr}{dt} = \dot{r}$, and $V_\phi = r\frac{d\phi}{dt} = r\dot{\phi}$ are the radial and tangential (or azimuthal) components of velocity vector **V** respecitvely.

In order to find the expression for acceleration, we have

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \left(\frac{dV_r}{dt}\hat{e}_r + V_r\frac{d\hat{e}_r}{dt}\right) + \left(\frac{dV_\phi}{dt}\hat{e}_\phi + V_\phi\frac{d\hat{e}_\phi}{dt}\right)$$

$$= \left(\frac{dV_r}{dt} - V_\phi\frac{d\phi}{dt}\right)\hat{e}_r + \left(V_r\frac{d\phi}{dt} + \frac{dV_\phi}{dt}\right)\hat{e}_\phi$$