

MECHANICS

S. L. Kakani

C. Hemrajani

Shubhra Kakani

Anshan

MECHANICS

S. L. Kakani

Director

Mewar Girls' Institute of Management and Technology
Mewar University Campus College, Chittorgarh (Raj.)

C. Hemrajani

Head, Department of Physics (Retd.)
MLV Govt. P.G. Collage, Bhilwara (Raj.)

Shubhra Kakani

Ph. D. Scholar, Department of Physics
College of Science, M. L. Sukhadia University, Udaipur (Raj.)

Anshan


Copyright © S.L. Kakani, C. Hemrajani, Shubhra Kakani

First Published in India by Viva Books Private Limited, 2005.

This edition published in 2005 by

Anshan Limited

6 Newlands Road

Tunbridge Wells

Kent TN4 9AT

UK.

Tel/Fax: +44 (0) 1892 557767

E-mail: info@anshan.co.uk

www.anshan.co.uk

Published by arrangement with

Viva Books Private Limited, 4262/3, Ansari Road, New Delhi - 110 002. India

All rights reserved. No part of this book may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recorded or otherwise, without the written permission of the publisher.

All care has been taken to bring this publication to incorporate the correct information. Notwithstanding, errors, omissions and / or discrepancies may not be completely ruled out. The publication is sold/ exhibited subject to the condition that the author, editor, publisher, typesetter and printers jointly or severally undertake no liability in respect of any damage, injury or loss caused to the purchaser and / or any person whatsoever by reason of such sale or exhibition.

British Library Cataloguing in Publication Data

A catalogue record for this book is available from the British Library

ISBN 1-904798-46-2

Printed in India.

PREFACE

The book has been designed to serve as a textbook in Mechanics for B. Sc. (General and Hons.) students of physics of Indian and other Universities, keeping in mind the latest proposed syllabus by University Grants Commission.

The subject matter has been selected and developed in such a manner so as to provide a bridge between introductory and advanced level courses in physics. Care has been taken to keep the level of the text in accordance with understanding of wider group of undergraduate students of physics. The salient features of the book are:

- Basic concepts and fundamental principles are explained in simple and lucid language.
- A large number of solved typical problems of different types have been given in each chapter. These solved problems illustrate or supplement the text.
- A large number of self explanatory accurate diagrams and tables have been used to supplement the text.
- At the end of each chapter, good number of review questions and problems are given.
- A large number of *short question answers and objective type questions* with answers have been given at the end of each chapter to make the book useful for various competitive entrance examinations, e.g. UGC, CSIR, NET-SLET, etc.
- Two special chapters on Analytical Mechanics and Non-linear Oscillations, Chaos and Fractals are given so that students may follow advance courses in Mechanics easily.

It is hoped that with these unique features the book will fulfill the genuine requirement of the students and teachers.

Much originality cannot be claimed in a book of this kind. The authors take this opportunity to place on record their indebtedness to the large number of books and journals that they have freely consulted in the preparation of this book. The authors heartily thank the publishers, M/s Viva Books, for taking keen interest in getting the book printed well in time.

In spite of all precautions and care taken to avoid errors and misprints, there might have crept some due to oversight and the authors will feel highly obliged to those fellow teachers and students who will bring them to their notice. Any suggestions for improvement will be thankfully acknowledged.

Authors

CONTENTS

<i>Preface</i>	<i>xi</i>
1. PHYSICAL FUNDAMENTALS OF MECHANICS	1-36
1.1 Introduction	1
1.2 Frame of reference and coordinate systems	3
1.3 Newton's laws of motion	10
1.4 Space-time symmetry	13
1.5 Galileo's principle of relativity and Galilean transformations	17
1.6 Fundamental interactions in nature	20
1.7 Elastic forces	23
1.8 Friction forces	23
2. NON-INERTIAL FRAMES	37-76
2.1 Non-inertial frames	37
2.2 Reference frame with translational acceleration and fictitious or pseudo or inertial force	38
2.3 Coriolis and centrifugal forces	43
2.4 Motion relative to the earth	46
2.5 Equation of motion and deviation of freely falling bodies on earth	51
2.6 Effect of coriolis force on nuclear and molecular motion	53
2.7 Satellite motion	54
3. LAWS OF CONSERVATION	77-163
3.1 Introduction	77
3.2 Law of conservation of mass	78
3.3 Conservation of energy	79

3.4	Work-energy theorem	81
3.5	Conservative forces	83
3.6	Potential energy	84
3.7	Rectilinear motion under conservative force	89
3.8	Non-conservative forces	90
3.9	Principle of conservation of momentum	107
3.10	Centre of mass (or centre of inertia)	109
3.11	Collisions	120
3.12	Collision in centre of mass frame	125
3.13	Relations between scattering angles in Labs and CM frames	126
3.14	Kinetic energy in collision	128
3.15	Systems with variable mass	131
3.16	Angular momentum (or moment of momentum)	144
3.17	Angular momentum of a system of particles or an extended system	146
3.18	Angular momentum of two particles relative to their C.M. or C-Frame	147
3.19	Law of conservation of angular momentum	149
3.20	Angular momentum of the extended system about an arbitrary point	151
3.21	Rutherford Scattering: Scattering of charged particles by heavy nuclei	152
4.	DYNAMICS OF RIGID BODIES	164-234
4.1	Introduction	164
4.2	Moment of inertia	173
4.3	Theorem of moment of inertia	175
4.4	Calculation of moment of inertia	178
4.5	Energy of a rotating rigid body	186
4.6	Gyroscopes	189
4.7	Spin precession in constant magnetic field	195
4.8	Fly-wheel	197
4.9	Angular momentum in quantum mechanics	199
4.10	Molecular rotations	201

5. MOTION UNDER CENTRAL FORCES: THE UNIVERSAL GRAVITATION	235-307
5.1 Introduction	235
5.2 Nature of central forces	236
5.3 Motion under central force	238
5.4 Newton's law of universal gravitation	242
5.5 Inertial and gravitational mass	243
5.6 Motion in gravitational field	244
5.7 Kepler's laws	251
5.8 Rutherford scattering	253
5.9 Gravitational field and potential	255
5.10 Three particles system	269
5.11 Inter-planetary flights	272
5.12 Binary stars	274
5.13 Gravitation and intermolecular forces	276
 6. ELASTIC PROPERTIES OF MATTER	 308-365
6.1 Introduction	308
6.2 Some definitions	309
6.3 Different types of co-efficients of elasticity	311
6.4 Theorems on stress and strain	314
6.5 Relations between elastic constants	317
6.6 Bending of beams	321
6.7 The cantilever-depression of its loaded end	326
6.8 Transverse vibration of a cantilever	330
6.9 Beam supported at both ends and loaded in the middle	331
6.10 Torsion of a cylinder	334
6.11 Torsional oscillations	337
6.12 Determination of elastic constants	337
6.13 Origin of elastic forces	340
6.14 Determination of γ by bending	354
6.15 Determination of poisson's ratio of rubber	355
 7. FLUID MECHANICS	 366-405
7.1 Introduction	366
7.2 Properties of fluids	369
7.3 Archimedes' principle	374
7.4 Euler's equation of motion for a moving fluid	374

7.5	Torricelli's theorem: Speed of efflux of a fluid from a large vessel	376
7.6	Irrotational continuous flow of inviscid fluids	377
7.7	The continuity equation	379
7.8	The Bernoulli's equation: Steady flow of fluids	380
7.9	Venturimeter	384
7.10	Laminar and turbulent flows	386
7.11	Coefficient of viscosity	387
7.12	Limiting or terminal velocity	388
7.13	Reynold's number	388
7.14	Flow of a liquid in a round pipe: Poiseuille's formula	389
7.15	Lift	393
8.	OSCILLATORY MOTION	406-483
8.1	Introduction	406
8.2	Oscillations in a potential well	407
8.3	Simple harmonic motion (SHM)	410
8.4	Some examples of free vibrations	416
8.5	Superposition of two simple harmonic motions	419
8.6	Superposition of mutually perpendicular two SHMs	422
8.7	Coupled oscillators	425
8.8	Anharmonic oscillations	434
8.9	Damped oscillations	440
8.10	Power dissipation in damping oscillations	444
8.11	Auto-oscillations	446
8.12	Forced oscillations	446
8.13	Fourier analysis of periodic motion	460
9.	FUNDAMENTALS OF ANALYTICAL MECHANICS	484-508
9.1	Introduction	484
9.2	Generalized coordinates	485
9.3	Lagrangian-hamilton's variation principle	486
9.4	Hamiltonian formalism	496
10.	NON-LINEAR OSCILLATIONS AND CHAOS	509-534
10.1	Introduction	509
10.2	Singular points of trajectories	512
10.3	Non-linear oscillations	514

10.4	Chaos	518
10.5	Logistic map	522
10.6	Fractals	528
11.	RELATIVISTIC MECHANICS	535-593
11.1	Introduction	535
11.2	Inertial frames of reference	535
11.3	Galilean transformations	536
11.4	Velocity of light	537
11.5	The Search for the ether	537
11.6	Michelson-Morley experiment	538
11.7	Lorentz and Fitzgerald hypothesis	542
11.8	Einstein's special theory of relativity	544
11.9	Lorentz transformations	545
11.10	Proper frame, proper length and proper time	551
11.11	Experimental verification of time dilation	552
11.12	Relativistic velocity transformation equations	558
11.13	Relativity of mass	562
11.14	Mass-energy equivalence	567
11.15	Relation between momentum and energy	570
11.16	Particles with zero rest mass	571
11.17	Speed limit for material particles	572
11.18	Space and time in relativity	572
11.19	Four-vector notation	574
11.20	The velocity four vector	577
11.21	The momentum four-vector	580
11.22	The four-force vector	581
11.23	Electromagnetic interaction	582
11.24	The current density 'Four-Vector'	583
11.25	The relativistic doppler effect	583
	<i>Suggested Readings</i>	594
	<i>Subject Index</i>	596
	<i>Appendix: Tables</i>	601

PHYSICAL FUNDAMENTALS OF MECHANICS

1.1 INTRODUCTION

Physics is one of the fundamental natural sciences studying the laws of inanimate nature. Obviously, studying physics is an exciting and challenging adventure. The word *physics* comes from a Greek term meaning nature; and, therefore, physics should be a science dedicated to the study of all natural phenomena.

The nature or say universe consists of material bodies which are in permanent interaction and motion. All the observable natural phenomena are governed by certain laws. The study and the explanation of the laws governing the connections between various processes and phenomena are the fundamental aim of every branch of physics and in general of science. The analysis of the interaction of material bodies and of the laws of electromagnetic phenomena is the aim of physics.

The motion of the substance has various forms: mechanical, electromagnetic, thermal, etc. The mechanical form of motion of the matter is the simplest one. It consists in the movement of bodies or their parts relative to one another. We can see movements of bodies everywhere in our daily life. The laws of mechanical motion are studied in '*mechanics*'. The other branches of physics cannot be studied without mechanics since various displacements are observed almost in all physical phenomena.

Mechanics is generally subdivided into three parts: *Kinematics*, *Statics*, and *Dynamics*. In kinematics the motion of the bodies is considered irrelative

to the factors causing the motion or changing its characters. Statics deals with the laws of equilibrium of the bodies and dynamics studies the laws of motion and the causes producing the motion and changing it. On the otherhand, if the laws of motion are known it is possible to derive the laws of equilibrium as a special case of the former. Therefore in physics, usually the laws of statics are not considered separately and are studied in connection with the general laws of dynamics.

Bodies are macroscopic systems consisting of a very large number of molecules or atoms, so that the sizes of these systems are many times larger than the intermolecular distances. We may note that a *material point* or *particle* is a body whose size and shape are of no consequence in the problem under consideration. For example, in studying the motion of the planets around the sun, they can be regarded as particles since the distances of the planets from the sun are many times greater than their sizes. *Classical*, or *Newtonian*, mechanics deals with the motion of bodies travelling at velocities that are very much less than that of light in a vacuum. The investigation of the motion of bodies travelling at velocities commensurate with the velocity of light is taken up in *relativistic mechanics* which is based on the theory of relativity. Specific features of the motion of microparticles are dealt with in *quantum* (wave) *mechanics*. Microparticles are particles whose rest mass is commensurate with, or smaller than, the rest mass of atoms.

Problems of the internal structure of bodies, as well as the nature and laws of their interaction are beyond the scope of mechanics, and constitute the content of other branches of physics.

Motion of a body or a particle occurs both in space and in time (space and time are in alienable forms of existence of matter). According to Newton, time is absolute and flows on without relation to the presence of any physical object or event. In this absolute space and time concept of Newton, motion of an object is characterized by change of its position in space as time evolves. Today, we know that in fact there is no object (in the universe) at rest in this 'absolute space', with respect to which one can define the position of another object. Obviously, at any instant of time, the position of an object is always defined by the '*observer*' with respect to himself. An observer, always, first creates a local space, in which positions of other objects are then described. This local space, to which is attached the observer, is called a *reference frame* or *frame of reference*. A frame of reference is a real or conditionally rigid body with respect to which the motion of the body being studied is to be considered. Rigidly fixed in the frame of reference is some kind of coordinate system so that the position of each point of a moving body can be uniquely determined by the three co-ordinates of the point.

We may note that the concept of rest and motion are relative, i.e., these are in relation to a frame of reference or observer. Absolute motion is a meaningless conception.

1.2 FRAME OF REFERENCE AND COORDINATE SYSTEMS

We have seen that both rest and motion are relative concepts; i.e., they depend on the condition of the object relative to the body that serves as reference. A tree and a house are at rest relative to the earth, but in motion relative to the sun.

To describe motion, therefore, the observer must define a *frame of reference* relative to which the motion is analyzed. The best that can be done is to define a particular reference system at rest relative to a particular set of stars, called the *fixed stars*. All other frames of reference can be defined relative to this particular frame of reference.

A frame of reference is a real or conditionally *rigid body* with respect to which the motion of the body being studied is to be considered. Rigidly fixed in the frame of reference is some kind of coordinate system so that the position of each point of a moving body can be uniquely determined by the three coordinates of the point. The following systems of coordinates are most frequently employed in mechanics:

- (i) Rectangular or Cartesian coordinates: (x, y, z) ,
- (ii) Spherical-polar coordinates: (r, θ, ϕ) , and
- (iii) Cylindrical coordinates: (ρ, ϕ, z) .

(i) Rectangular Cartesian Coordinate System

In this coordinates system, the three dimensions are represented by three axes x , y and z perpendicular to each other. The coordinates of any point in space are taken as distances from the origin along the three axes x , y and z and written as (x, y, z) . This system is generally used where independent rectangular coordinate systems possible in space: (a) *the left handed* and (b) *the right handed* which cannot be made to coincide with each other by any means of translation or rotation. Cartesian coordinate system by convention is chosen to be right handed (Fig. 1.1). Position vector \mathbf{r} of a particle $P(x, y, z)$ relative to the origin is given by

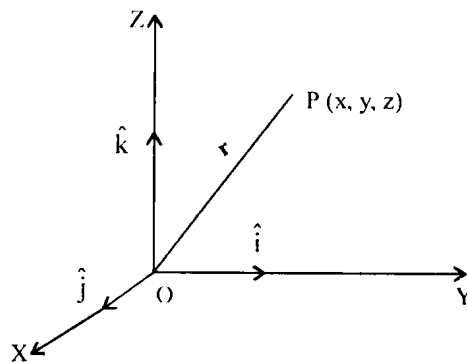


Fig. 1.1. Cartesian coordinate system.

\hat{i} , \hat{j} , and \hat{k} are unit vectors in the directions of x , y and z axes respectively.

$$\mathbf{r} = x\hat{i} + y\hat{j} + z\hat{k} \quad (1)$$

where \hat{i} , \hat{j} and \hat{k} denote constant unit vectors in the directions of X, Y, and Z respectively. The position of a particle P relative to the origin O is given by the position vector \mathbf{r} , characterized by its specific length and direction (Eq. 1).

The vectorial increment $d\mathbf{r}$ in \mathbf{r} can be written as

$$d\mathbf{r} = dx\hat{i} + dy\hat{j} + dz\hat{k} \quad (2)$$

The instantaneous velocity and acceleration of P, in Cartesian coordinate system will be

$$\mathbf{V} = \frac{d\mathbf{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\text{or} \quad \mathbf{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k} \quad (3)$$

The magnitude of \mathbf{V} is given by

$$|\mathbf{V}| = \sqrt{V_x^2 + V_y^2 + V_z^2} \quad (4)$$

Acceleration is the rate of change of velocity. Thus differentiating Eq. (3) with respect to time, one obtains,

$$\begin{aligned} \mathbf{a} &= \frac{d\mathbf{V}}{dt} = \frac{dV_x}{dt}\hat{i} + \frac{dV_y}{dt}\hat{j} + \frac{dV_z}{dt}\hat{k} \\ &= a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \end{aligned} \quad (5)$$

The magnitude of \mathbf{a} is given by

$$|\mathbf{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \quad (6)$$

(ii) Spherical Polar Coordinates

The coordinates of a point—say P in this coordinate system are represented by radial vector \mathbf{r} , the zenith, colatitude or polar angle θ ; and azimuthal or longitudinal angle ϕ as shown in Fig 1.2 . These coordinates are related to the rectangular cartesian coordinates x , y , and z through

$$x = r \sin \theta \cos \phi, \quad r = \sqrt{x^2 + y^2 + z^2} \quad (7a)$$

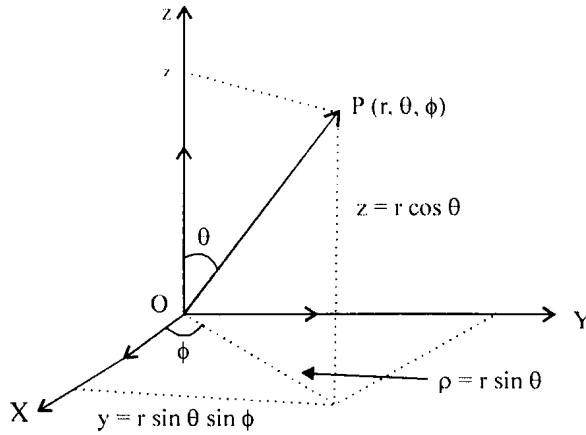


Fig. 1.2. Spherical Polar coordinates and their relationship with rectangular cartesian coordinates

$$y = r \sin \theta \sin \phi, \quad \tan \theta = \sqrt{\frac{x^2 + y^2}{z}} \quad (7b)$$

$$z = r \cos \theta \quad \tan \phi = \frac{y}{x} \quad (7c)$$

The coordinate surfaces are

- (i) Spheres with concentric with the origin ($r = \text{constant}$)
- (ii) Cones with the apex at the origin and along the Z-axis ($\theta = \text{constant}$) and
- (iii) Planes through the Z-axis ($\phi = \text{constant}$).

The unit vectors \hat{e}_r , \hat{e}_θ , \hat{e}_ϕ extend respectively in the directions of r increasing, θ increasing and ϕ increasing (Fig. 1.3)

One can easily write the general differential displacement of particle P in spherical polar coordinates as

$$d\vec{r} = dr \hat{e}_r + r d\theta \hat{e}_\theta + r \sin \theta d\phi \hat{e}_\phi \quad (8)$$

The unit vectors \hat{e}_r , \hat{e}_θ and \hat{e}_ϕ can be expressed in terms of unit vectors \hat{i} , \hat{j} , \hat{k} as follows:

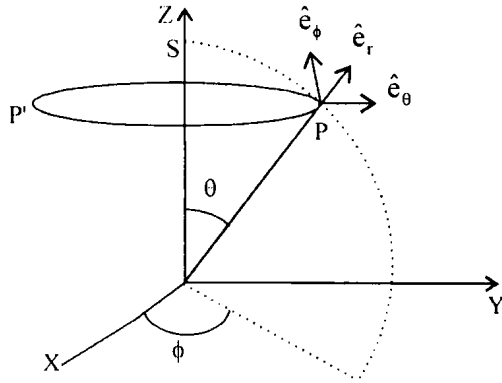


Fig. 1.3.

$$\hat{e}_r = \sin \theta \cos \phi \hat{i} + \sin \theta \sin \phi \hat{j} + \cos \theta \hat{k} \quad (9a)$$

$$\hat{e}_\theta = \cos \theta \cos \phi \hat{i} + \cos \theta \sin \phi \hat{j} - \sin \theta \hat{k} \quad (9b)$$

$$\hat{e}_\phi = -\sin \phi \hat{i} + \cos \phi \hat{j} \quad (9c)$$

The unit vectors (\hat{e}_r , \hat{e}_θ , \hat{e}_ϕ) in spherical coordinate system, unlike (\hat{i} , \hat{j} , \hat{k}) in cartesian coordinate system are not constant vectors but change in direction as coordinates θ and ϕ change. However, we can see that at each point they constitute an orthogonal right-handed coordinate system, i.e., we have

$$\begin{aligned} \hat{e}_r \cdot \hat{e}_\theta &= \hat{e}_r \cdot \hat{e}_\phi = \hat{e}_\theta \cdot \hat{e}_\phi = 0 \\ \hat{e}_r \times \hat{e}_\theta &= \hat{e}_\phi, \quad \hat{e}_\theta \times \hat{e}_\phi = \hat{e}_r \quad \text{and} \quad \hat{e}_\phi \times \hat{e}_r = \hat{e}_\theta \end{aligned} \quad (10)$$

The spherical polar coordinates are very convenient in those problems of physics where there is no preferred direction and the force in physical problem is spherically symmetrical, e.g., (i) Coulomb force due to a point charge, and (ii) gravitational force due to a point mass. We may note that these are also examples of *central forces*.

(iii) Motion in Two Dimensions

Usually, we find a particle constrained to move in a plane, rather than in three-dimensional space in general. Let us denote the two dimensional plane as XY-plane (Fig. 1.4). The position vector of the particle P is given by

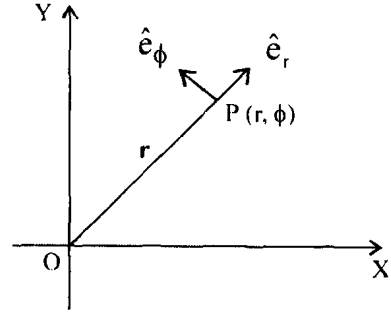


Fig. 1.4. Motion of a particle in two dimensional plane

$$\mathbf{r} = x\hat{i} + y\hat{j} \quad (11)$$

as $Z = 0$ always for the particle in motion. Since $\theta = 90^\circ$, for the particle P and hence the spherical polar coordinates (r , θ , ϕ) reduces to circular polar coordinates (r , ϕ). Now, the position of the particle P in terms of radial distance r from origin O and ϕ is given by

$$r = \sqrt{x^2 + y^2} \quad x = r \cos \phi \quad (11a)$$

$$\tan \phi = \frac{y}{x} \quad y = r \sin \phi \quad (11b)$$

The displacement vector $d\mathbf{r}$ in circular polar coordinates is obtained as (put $\theta = 90^\circ$ and $d\theta = 0$ in Eq. 8)

$$d\mathbf{r} = dr \hat{\mathbf{e}}_r + r d\phi \hat{\mathbf{e}}_\phi \quad (12)$$

where unit vectors $\hat{\mathbf{e}}_r$ and $\hat{\mathbf{e}}_\phi$ are given by

$$\hat{\mathbf{e}}_r = \cos \phi \hat{\mathbf{i}} + \sin \phi \hat{\mathbf{j}} \quad (13a)$$

$$\text{and} \quad \hat{\mathbf{e}}_\phi = -\sin \phi \hat{\mathbf{i}} + \cos \phi \hat{\mathbf{j}} \quad (13b)$$

(iv) Velocity and Acceleration (In Circular Polar Coordinates System)

Let us find the velocity and acceleration of a point object, moving in a plane, in terms of circular polar coordinates (r, ϕ) . Differentiating position vector

$\mathbf{r} = r \hat{\mathbf{e}}_r$ with respect to time.

We obtain

$$\mathbf{V} = \frac{d\mathbf{r}}{dt} = \frac{dr}{dt} \hat{\mathbf{e}}_r + r \frac{d\hat{\mathbf{e}}_r}{dt} \quad (14)$$

We may note that $\hat{\mathbf{e}}_r$ is not a constant vector. One can obtain its time rate by using Eq. (13a).

$$\text{One obtains,} \quad \frac{d\hat{\mathbf{e}}_r}{dt} = -\sin \phi \frac{d\phi}{dt} \hat{\mathbf{i}} + \cos \phi \frac{d\phi}{dt} \hat{\mathbf{j}} = \frac{d\phi}{dt} \hat{\mathbf{e}}_\phi \quad (15)$$

Similarly one obtains

$$\frac{d\hat{\mathbf{e}}_\phi}{dt} = -\frac{d\phi}{dt} \hat{\mathbf{e}}_r \quad (16)$$

Using (15) and (16), Eq. (14) takes the form

$$\mathbf{V} = V_r \hat{\mathbf{e}}_r + V_\phi \hat{\mathbf{e}}_\phi \quad (17)$$

where $V_r = \frac{dr}{dt} = \dot{r}$, and $V_\phi = r \frac{d\phi}{dt} = r\dot{\phi}$ are the radial and tangential (or azimuthal) components of velocity vector \mathbf{V} respectively.

In order to find the expression for acceleration, we have

$$\begin{aligned} \mathbf{a} = \frac{d\mathbf{v}}{dt} &= \left(\frac{dV_r}{dt} \hat{\mathbf{e}}_r + V_r \frac{d\hat{\mathbf{e}}_r}{dt} \right) + \left(\frac{dV_\phi}{dt} \hat{\mathbf{e}}_\phi + V_\phi \frac{d\hat{\mathbf{e}}_\phi}{dt} \right) \\ &= \left(\frac{dV_r}{dt} - V_\phi \frac{d\phi}{dt} \right) \hat{\mathbf{e}}_r + \left(V_r \frac{d\phi}{dt} + \frac{dV_\phi}{dt} \right) \hat{\mathbf{e}}_\phi \end{aligned}$$