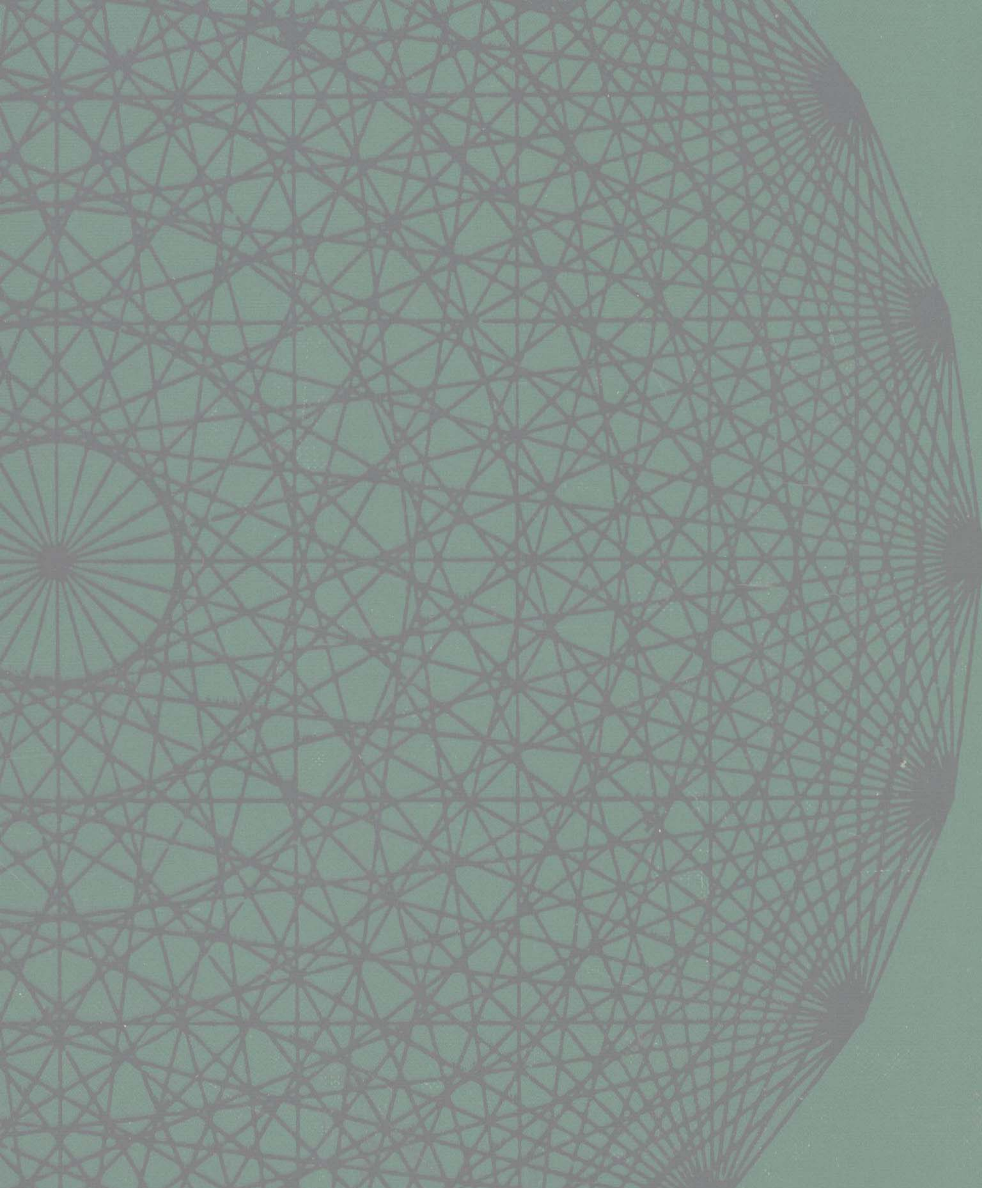

FUNDAMENTALS OF

College Mathematics

REVISED EDITION

JOHN C. BRIXEY

RICHARD V. ANDREE

The University of Oklahoma

FUNDAMENTALS OF *College Mathematics* REVISED EDITION

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This book is affectionately dedicated to

Dorothea B. Brixey and Josephine P. Andree

without whose understanding help
it could never have been written

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FUNDAMENTALS OF COLLEGE MATHEMATICS

Preface to the Instructor

This text presents a careful integration of college algebra, trigonometry, basic statistical reasoning, analytic geometry, and the elementary calculus needed to continue with current college courses in calculus or calculus with analytic geometry. It was originally intended for a one-year, college freshman course. That it has proved highly successful in courses for which it was intended is gratifying. Your authors were, however, surprised and delighted to discover that some of the better high schools are also using the text enthusiastically with their seniors, and that several colleges are using the same material in graduate courses for the retraining of teachers. This book was *designed to be read by the student*, and we find that students *do* read it.

Many students lack skill in algebraic manipulation. Hence, the delta process is introduced early (Chapter 5) and fundamental manipulative experience is gained while applying the delta process instead of by routine drill. Concepts from algebra are reviewed as needed rather than beginning with an indiscriminate review. Since problems are the meat of a mathematics text, this book provides a variety of well-motivated problems and applications from diverse fields. The number and variety of problems are large enough to accommodate individual differences among students, classes, and instructors. Topics for outside reports of historical and mathematical nature are indicated for teachers desiring them. The function concept is used throughout the text. Inequalities as well as equations are stressed. The derivative is introduced both as the rate of change of a function and as the slope of a line tangent to a curve.

The current practice in so-called "modern" texts is to sandwich in a chapter on set theory, often including truth tables and Boolean algebra, and then to forget all about set theory from there on. Your authors disagree with this practice. Even in the first (1954) edition, the concepts of set and of inequalities were used throughout the text.

This philosophy has been preserved. The phrase *solution set* is currently popular in place of the more cumbersome "set of all values that satisfy the equation or inequality," and has been used. Various set ideas are introduced and used where needed throughout the text. When the student is ready to study *Sample Spaces* in probability (Chapter 12), he already has the principal set concepts at his disposal.

The entire book has been reworked and many improvements have resulted from the suggestions of over 500 teachers who used the first edition. Many of these teachers wrote that when they examined the first edition they considered it too advanced. When they used it in class, however, they discovered that the students not only read but understood the presentation. More-over, the students were far better prepared for the more advanced work in calculus courses than ever before. Several colleagues have pointed out that our work with the Σ notation has prepared the student admirably to consider the integral as a limit of a sum. This is true. We have, therefore, developed the integral both as the limit of a sum (Sections 10–12 and 19–7) and also as a generalization of area (Sections 10–12, 19–5, and 19–7), as well as presenting the usual antiderivative approach. The instructor is, of course, free to pick the presentation that best fits his teaching practice. After some experience, we have concluded that it is highly worthwhile for the future development of the student to present all three views during the freshmen year. Similarly, it seems worthwhile to include brief experience with the ϵ, δ definition of limit (Section 5–4) in the first course to assist the student in his later work.

The notation of sequence unifies the chapters on progressions, permutations, combinations, and probability which climax in the mathematical statistics of the binomial distribution. The statistics does not consist of exercises in arithmetic, as beginning statistics often does, but orients the student in the general problems handled by modern statisticians and in the meaning and types of results to be expected.

The analytic portions of trigonometry (graphs, equations, inequalities) are stressed rather than the solution of triangles. The laws of sines and cosines and the expansions of $\cos(A \pm B)$ and $\sin(A \pm B)$ are obtained neatly, using methods of analytic geometry.

Throughout the text, examples, illustrations, and problems have been carefully selected to develop the understanding and the technique necessary for the student's applied work and for his later mathematical development. Many of the trigonometric identities, for example, are those that the student will encounter in further work in engineering, physics, and calculus. Intersections of polar coordinate loci are studied as problems involving trigonometric equations. Work on numerical analysis (Sections 10–9, 10–10, 10–11, 10–12, 19–8, and 19–9) is introduced with care and with an awareness of the importance of computers.

The authors feel that answers should provide additional instruction and be a vital teaching aid, rather than a mere check. With this in view, extensive skeleton solutions and over 400 sketches and graphs are included with the answers to the odd problems. Answers to the even problems and extensive comments on the text are included in a separate *Teacher's Guide* available from the publisher.

The two-semester course at The University of Oklahoma includes the unstarred sections in Chapters 1 to 22, with selected sections from Chapters 23, 24, and 25.

A one-semester course for better students has been taught using Chapters 1, 2, 5, 6, 7, 9, 10, 11, 12, 13, 17, 19, 20, 21, and part of Chapter 22.

A short course emphasizing modern trigonometry is contained in Chapters 1, 2, 8, 14, 15, and 16.

A course emphasizing college algebra, including statistics, is available in Chapters 1, 2, 3, 4, 8, 9, 10, 11, 12, and 13.

Analytic geometry may be emphasized by using Chapters 1, 2, 5, 6, 9, 17, 20, 21, 22, 23, with Chapter 24 giving the essentials of solid analytic geometry.

Several possible terminal courses, providing mathematical background desirable for a liberal (or general) education, may be selected from Chapters 1, 2, 4, 5, 6, 7, 9, 10, 11, 12, 13, and 25.

Chapter 25 is independent of the rest of the text, and provides the student with some interesting glimpses of advanced mathematics. The better students seem to relish an assignment from Chapter 25 during review periods.

The authors wish to acknowledge indebtedness to colleagues and students near and far, without whose encouragement and assistance this book could not have been written. We also thank Professor A. Gelbart, editor of *Scripta Mathematica*, for permission to use pictorial material appearing at the ends of chapters, and Keuffel & Esser Co. for illustrations of the slide rule and special graph papers.

The authors welcome your comments and suggestions concerning this text. We hope it meets your needs.

J. C. B.
R. V. A.

April, 1961
Norman, Oklahoma

Preface to the Student

The choice of text and problem material in this book was influenced by extensive interviews and correspondence with many engineers, physicists, chemists, social scientists, and mathematicians.

The material covered in this book represents more than one fourth of your first year of college study. It contains knowledge needed in more advanced courses. Your best reference book is usually the one with which you are most familiar. If you use the wide margins of this text to record points your instructor makes during his lectures, and your own related observations, you can build this text into a personal reference book that will serve you well in future years.

Throughout the text, sections and figures are numbered consecutively within each chapter. Thus, 12–3 refers to Chapter 12, Section 3. A decimal notation is used for reference equations: 5.1 indicates the first reference equation in Chapter 5.

Numerous problems in this book indicate that mathematics may be applied to other fields. Unfortunately they cannot indicate the most important role of a mathematician — that of discovery and invention. Before the planet Neptune was discovered, astronomers knew that it must exist since it had a disturbing influence on the orbit of Uranus. J. C. Adams and U. J. J. Leverrier determined the position of Neptune using purely mathematical methods. Only then were astronomers able to pinpoint their telescopes in the exact portion of the sky, and Neptune was observed. The planet Pluto was first observed in the year 1930 as a result of a similar purely mathematical discovery by Lowell and Pickering.

James Clerk Maxwell deduced the existence and nature of electromagnetic waves using differential equations. Later experiments carried out by H. Hertz confirmed Maxwell's discovery. Maxwell's equations are the basis of radio and television communication theory. Albert Einstein was a well-known mathematical physicist whose mathematical reasoning established, among other things, a connection between gravitation and the propagation of light in space.

Prior to World War II it was common engineering practice to construct test models when a new airplane or projectile was being designed. In the design of modern rockets and supersonic aircraft it is not feasible to make models and test each possible design and variation. Instead, mathematicians set up mathematical models (equations) that simulate possible significant effects, and computing machines are programmed to solve these equations. Twenty-five different rocket designs may be tested in two minutes using modern computers, once the computer has been programmed. Thus, many thousands of different designs may be completely tested in the mathematical laboratory in less time than is required to construct one model — and at much lower cost. Models are finally constructed and tested only for the few best designs.

A few remarks on **how to study** college mathematics may be appropriate. Is mathematics hard for you? If you read carefully and are willing to work, the battle is half won! In mathematics much difficulty is avoided by learning principles and not by memorizing rules. An understanding of the principles makes the rules easier to remember and apply. We, the authors, with the help of your instructor, will try to state these principles clearly, and provide you with numerous illustrative examples and with interesting problems.

For your part, read the material and study the examples before beginning to solve the problems assigned by your instructor. Please do not make the error of trying to work the assignment first, looking back at the examples only when trouble arises and finally reading the text as a last resort, if at all. That is *not* the way to study *this* course. **Read the text carefully** first. Test each statement. Study all illustrative examples, supplying any missing steps or reasons. Be sure you can solve each example without the text. Then begin to solve the problems assigned by your instructor. You will find an "Answers and Hints" section at the back of the book which should be consulted only after you have completed your solution. Self-tests have been included to help check your understanding of the material.

We sincerely hope you enjoy this book and that it will indeed meet your needs. Chapter 25 is included for *your* pleasure.

J. C. B.
R. V. A.

April, 1961
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Contents

1

The discovery of the immense power of modern mathematics can be a most rewarding and exciting experience. As a first step in this direction Chapter 1 develops certain familiar ideas in terms of what may prove to be a new technical vocabulary. It also places new emphasis on certain important concepts. The reader has been using the "basic laws of algebra" for several years, but even so may not realize that they are all derived from a simple set of postulates. He already knows how to solve equations such as $\sqrt{13x - x^2} = -6$ mechanically by squaring each member, but a more critical examination is necessary in order to determine why this equation has no solution (both $x = 4$ and $x = 9$ are extraneous values that *do not* satisfy the equation) whereas the equation $\sqrt{10x - x^2} = 4$ has two valid solutions: $x = 2$ and $x = 8$.

In short, both the new vocabulary and an understanding of the basic theory behind certain algebraic manipulations are essential to an appreciation of mathematics.

1–1. The "set." † In mathematics the word **set** is actually an undefined term, just as "line" and "point" are undefined terms in geometry. The word **set** describes any collection of objects. For example, one speaks of the set of rational numbers less than 237, the set of points inside of and on a circle, or the set of distinct letters on this page. Actually the elements of a set need have nothing in common except that they form a collection. All the sets mentioned above might be combined into a single set. The reader of this book will find a number of specific, important sets discussed — for example, the set of all solutions of a given equation or inequality (solution set), the set of continuous functions, the set of problems one can work correctly, or the set of all ways in which a seven can be thrown on a pair of dice. The word **subset** is used to describe a collection of part (or even all, or none) of the elements of a given set. Every set is a subset of itself. Another unusual, but important, concept is that of the **empty set** or **null set** which consists of a set containing no elements at all. It is reasonably easy to describe such sets; for example, the set of college freshmen who are less than 4 years old is probably an empty set. The set of pages in this text that are numbered above 900 is an empty set; and the set of negative numbers that are also perfect squares of integers (whole numbers) is an empty set. The symbol $\emptyset$ is sometimes used to represent the empty set.

1–2. Basic rules of algebra. The reader is familiar with the basic concepts of high-school algebra — for example, $(a + b) \cdot x = a \cdot x + b \cdot x$. He

† A more elaborate treatment of sets appears in Chapter 12.

has worked regularly with these rules, but may never have seen them collected together. Many students know that geometry has a set of axioms or postulates from which theorems are derived, but do not realize that algebra has a similar structure. Since advanced **mathematics** involves **the study of structure,** it seems desirable to state specifically "the rules of ordinary algebra," or, in modern mathematical language, the **postulates for a field.**

Field postulates (rules of ordinary algebra). A field is defined to be a set of elements (or numbers) having two operations "+" and " · ," and having an equals † relation "=" such that the following postulates are satisfied:

1. *Closure:* For each pair x, y of elements in the set, the sum $x + y$ and the product $x \cdot y$ are in the set.

2. *Commutative:* For each pair x, y of elements in the set, $x + y = y + x$ and $x \cdot y = y \cdot x$.

3. *Associative:* For each triple x, y, z of elements in the set, $x + (y + z) = (x + y) + z$ and $x \cdot (y \cdot z) = (x \cdot y) \cdot z$.

4. *Additive Identity (zero):* There exists an element, 0, in the set such that for every x in the set $0 + x = x + 0 = x$ and $0 \cdot x = x \cdot 0 = 0$.

5. *Multiplicative Identity (one):* There exists an element, 1, in the set such that for every x in the set, $1 \cdot x = x \cdot 1 = x$.

6. *Inverses:* For each element x in the set, there is an element, $(-x)$, in the set such that $(x) + (-x) = 0$. If x is not zero, there is an element $\left(\frac{1}{x}\right)$ in the set such that $x \cdot \left(\frac{1}{x}\right) = 1$.‡ The element $(-x)$ is called the additive inverse§ of x. The element $\left(\frac{1}{x}\right)$ is called the multiplicative inverse‖ of x.

7. *Distributive:* For each triple $a, b, x,$ of elements in the set, $(a + b) \cdot x = a \cdot x + b \cdot x$ and $x \cdot (a + b) = x \cdot a + x \cdot b$.

Some typical examples of fields with which the reader is already familiar include the field of complex numbers, the field of real numbers, and the field of rational numbers.# The set of all integers is *not* a field, however, since 1 is the only nonzero integer having a multiplicative inverse that is an integer. [What does this statement mean?]

† Actually, "an equals relation" must satisfy a set of three postulates as given in Problem 14, Set 1–16, and should be "well defined," which means that $a = b$ must imply that $a + k = b + k$ and that $a \cdot k = b \cdot k$.

‡ The symbols $(-x)$ and $\left(\frac{1}{x}\right)$ represent single elements, *not* the results of a "subtraction" or of a "division."

§ Or negative of.

‖ Or reciprocal of.

A more complete discussion of real, rational, and complex numbers is given in Section 1–7.

The reader may wonder why the postulates fail to mention either subtraction or division. The omission is deliberate and not an oversight. This omission simplifies later theoretical work and, surprisingly, is no restriction. Postulate 6 guarantees the existence of an additive inverse, $(-x)$, which has the property that $x + (-x) = 0$. Subtraction really means "add the additive inverse." In a similar fashion Postulate 6 guarantees that if $x \neq 0$, then there is an element known as the multiplicative inverse $\left(\frac{1}{x}\right)$, which has the property that $\left(\frac{1}{x}\right) \cdot x = 1$. Division really means "multiplication by the multiplicative inverse." In brief, the expression $(-b)$ is the solution z, of the equation $b + z = 0$; and the expression $\left(\frac{1}{b}\right)$ is the solution w of the equation $b \cdot w = 1$. Clearly, $0 \cdot w = 1$ has no solution.

- ***Example 1.*** In the set of all decimals, the multiplicative inverse of the element 2.5 is the decimal 0.4, since $(2.5)(0.4) = 1$.

- ***Example 2.*** An integer (whole number) is *even* if it is an integral multiple of 2. Thus, 74, 14936, 0, -4, 2, and -768 are even integers. However, 17, 2π, and $4\sqrt{34}$ are *not* even integers.

The set of all even integers is closed under both addition and multiplication. (This follows since both the sum and the product of two even integers are even integers.) The set of even integers contains the element zero, and for each even integer x the number $(-x)$ is an even integer. Thus Postulates 1, 4, part of 6, and certain others are satisfied. There are, however, two field properties that the set of even integers does not possess. The reader should be able to determine them.

no. 5
Part. of 6

This discussion may appear as obvious to the reader. What may be new, however, is the fact that from this simple set of postulates it is possible to derive the important theorems of ordinary algebra. For example, using only the postulates for a field it is possible to derive the following theorem, which is the basic theorem used in solving equations.

Theorem. In a field, if $A \cdot B = 0$, then $A = 0$, or $B = 0$, or both $A = 0$ and $B = 0$.

PROOF: Given: $A, B,$ elements of a field such that $A \cdot B = 0$.
To Prove: Either $A = 0$, or $B = 0$, (or both)

If $A = 0$, the conclusion of the theorem holds.

If $A \neq 0$,† then by Postulate 6 there exists an element $\left(\frac{1}{A}\right)$ in the field such that $\left(\frac{1}{A}\right) \cdot A = 1$. We use this property in the proof.

† The symbol $\neq$ means the two members are *not* equal.

$A \cdot B = 0$	Given
$\left(\frac{1}{A}\right) \cdot [A \cdot B] = \left(\frac{1}{A}\right) \cdot 0$	Equals relation is well defined
$\left[\left(\frac{1}{A}\right) \cdot A\right] \cdot B = \left(\frac{1}{A}\right) \cdot 0$	Postulate 3 (Associative Postulate of Multiplication)
$[1] \cdot B = \left(\frac{1}{A}\right) \cdot 0$	Postulate 6 (Multiplicative inverse)
$B = \left(\frac{1}{A}\right) \cdot 0$	Postulate 5 (Multiplicative identity)
$B = 0$	Postulate 4 (Multiplicative property of additive inverse)

Hence the theorem is proved.

- ***Illustration.*** Find the (rational) solutions of $(x+4)(2x-3)(x-2)=0$. The theorem states that if the product $(x+4)(2x-3)(x-2)$ is to be zero, then a factor must be zero. On the other hand, if a factor is zero, then the product is zero. All solutions are given by
  $x+4=0,\ 2x-3=0,\ x-2=0$.
  Using the distributive law $(ax+ay)=a(x+y)$ with $a=2$, one obtains $2x-3=2(x-\frac{3}{2})=0$. Hence $x-\frac{3}{2}=0$. (Why?) The set of solutions is $\{-4, \frac{3}{2}, 2\}$.

The set of postulates given is *not* the most concise set possible. For example, in Postulate 4, the statement $0 \cdot x = x \cdot 0 = 0$ can actually be proved from the remaining postulates. The purpose here is not to give an extremely brief set of postulates, but rather to give a set that conveys the idea of a field to the reader.

Section 1–16 of this text describes a simple system, which does *not* obey the theorem that "a product is zero if and only if at least one of the factors is zero." Such a system cannot be a field. (Why not?)

PROBLEM SET 1–2

1. Let S be the set of distinct letters appearing in this sentence. Which of the following are subsets of S?

(a) $\{a, b, e, t, w\}$
(b) $\{w, g, a, p, f, t\}$
(c) $\{s, t, n, c, d, p, g\}$