Theoretical Principles in Astrophysics and Relativity

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Preface

On May 27–29, 1975, a Symposium on Theoretical Principles in Astrophysics and Relativity was held at the University of Chicago in honor of S. Chandrasekhar, in his 65th year. This book, the proceedings of that symposium, is likewise dedicated to Chandra. We know that his many friends throughout the world join with us in this expression of our admiration and affection.

Chandra's research has ranged wide. It is not possible in a volume of this size to cover all the areas in which he has made important contributions. We have tried to select those areas that reflect the increasing influence that the directions of research in astrophysics and in relativity have on one another. We hope and believe that this book will be useful to those interested in research in either or both of these fields.

It is a pleasure to thank the speakers for participating in the symposium and for preparing their articles for publication. We also thank the National Science Foundation for its financial support of the symposium and the University of Chicago Press for its assistance with the publication of this volume.

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1 The Study of Stellar Structure

Martin Schwarzschild

Introduction

In a normal scientific colloquium it would be my assignment, under the general topic given me, to present to you a review of selected significant recent achievements in the field of stellar structure and evolution. The special character of this symposium, however, gives me, I believe—and I hope you agree—the permission and rare opportunity to review with you not the scientific topic of stellar structure but rather that human endeavor we call the study of stellar structure. Since I have no qualifications as a science historian, I would not want to concentrate on the actual history of this scientific endeavor, with its attendant difficult task of proper references and correct assignment of credit. Instead I would like to concentrate on the physical principles and processes as they have been introduced one by one into the study of stellar structure during the past 100 years, leading up to the present fascinatingly unfinished state.

I will restrict myself to those processes dominating the simplest stars in their various evolution phases. Accordingly I shall omit topics like instabilities (such as pulsations or thermal flashes), rotation and the related topic of close binaries, and magnetic fields. I am truly aware that these difficult topics are vital and in many cases decisive parts of the theory of the stellar interior. I feel justified in omitting them only because nearly all of them will be covered separately by subsequent speakers.

Hydrostatic Equilibrium

The apparent unchangeability of the majority of stars convinced researchers early that in general stars must be in strict hydrostatic equilibrium, i.e., that the gravitational force must be balanced by a pressure force everywhere within a star. The study of stellar models in hydrostatic equilibrium was well under way 100 years ago, and its classical period may be considered as essentially concluded when Emden wrote his summarizing book on this topic in 1907.

This first phase of stellar model making seems to me remarkable and instructive by the following circumstance. All investigators active in this development were fully aware that the application of hydrostatic equilibrium was by itself not sufficient to achieve unique stellar models. A pressure-density relation had

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to be more or less arbitrarily assumed. Much as is now the habit for formally representing an as yet unknown relation, a simple power law was used to represent the missing pressure-density relation, and the researchers of that epoch judged it worthwhile to study the thus defined polytropic star models in greatest detail. The value of such extensive theoretical work, based in part on pure assumption, is frequently questioned. The case we are here discussing is an example where such work has proved thoroughly valuable: the next phase of the study of stellar structure could surely not have proceeded as remarkably rapidly as it did if it had not been for the availability of extensive tabulations and full analysis of polytropic stellar models.

The story of the study of stellar models with hydrostatic equilibrium as the only equilibrium condition has two additional chapters, which occurred historically far later than the classical first chapter. In 1930 Chandrasekhar derived the pressure-density relation which holds in that density range in which electron degeneracy dominates the equation of state, and then he immediately proceeded to compute the corresponding stellar models. By this development he laid the foundation for the theory of white dwarfs, including the Chandrasekhar limit, i.e., the maximum mass possible for such stars. In contrast to the classical development of polytropic models, Chandrasekhar's development of white dwarf models contains no arbitrariness in the choice of the pressure-density relation—notwithstanding Eddington's violent and amazingly persistent objection to this relation, an objection which by now appears to have been based more on faith than on physics.

The third chapter in this study was opened by Oppenheimer and Volkoff in 1939 when they, for the first time, constructed models for neutron stars with a pressure-density relation dominated by ion degeneracy. This relation had to be modified for the effects of nuclear forces, effects even now not definitively known. Accordingly, this chapter of stellar structure, including the astronomically important determination of the upper limit for the masses of neutron stars, is still unfinished. This last chapter differs from the preceding ones in one fascinating aspect. The first neutron star models were derived as purely theoretical constructs without any observed astronomical objects known at that time to which they might apply. It took some 30 years after this initial theoretical construct for it to find its proper companion in the real world by the discovery of pulsars.

Energy Transport

Even though the fundamental importance of the role which energy transport mechanisms must play in the stellar interior was realized very early, I am not aware of any major efforts to determine such processes during the phase of polytrope building, though it appears that very general considerations of convective transport played a role in the choice of the pressure-density relations used during that phase.

In 1906, one year before Emden published his definitive book on polytropes, Schwarzschild showed that radiative energy transport was likely to be an important process for stars. But it was Eddington who, starting in 1916, solidly implanted radiative transport into the theory of the stellar interior. Even with this transport mechanism clearly defined, the problem of deriving a stellar model still did not permit a unique solution as long as the nuclear sources were unknown. Nevertheless, Eddington achieved the insight that the structure of a stellar model would not be greatly affected by the precise distribution of the nuclear sources within the star over a reasonable range of such distributions, a range we now know to be adequate for main-sequence stars but not for later evolution phases. The most striking result of this development by Eddington was the derivation of the theoretical mass-luminosity law which could be brought into fine agreement with the observed mass-luminosity relation for main-sequence stars when sufficient hydrogen was admitted as a major constituent of the stellar interior. Eddington curiously delayed the acceptance of sufficient hydrogen; when he finally took this step, it was with reluctance since it spoiled the balance between gas and radiation pressure for a star of average mass, a balance which appears to have had a high philosophical attraction for him. His book on the stellar interior, written in 1926, gives a lucid account of this phase of the study of stellar structure. We now know that Eddington's "standard model" is an amazingly good representation of most main-sequence stars, though we also know that it seriously fails to represent the main features of red giants.

After this period in which radiative energy transport ruled supreme, the role of convective transport in the stellar interior was reconsidered by Biermann and Cowling in the early 1930s. These investigations were based on Prandtl's mixing-length approach, the only approach for approximating turbulent convection then available. They showed convincingly that convective cores as well as deep convective envelopes might be expected in stellar models, depending on the specific character of the nuclear energy sources. These early investigations in which the superadiabatic gradient was generally neglected were followed up in due course by investigations, particularly on deep convective envelopes, in which the superadiabatic gradient was found in many cases to be far from ignorable and was computed by a detailed application of the mixing-length approach. This procedure was clearly dangerous but, I believe, unavoidable.

To complete the list of presently known energy transport mechanisms within stars, I should add the discovery that ordinary conduction is highly effective wherever the density is sufficiently high for electron degeneracy to affect the equation of state.

One more development properly belongs at this point of the story of the study of stellar structure, though actually it was not until 1962 that Hayashi accomplished it in a decisive manner. For a star of a given mass and composition which is in hydrostatic equilibrium and exploits the energy transport mechanisms just discussed, one can derive a limit, the Hayashi limit, which can be represented in the Hertzsprung-Russell diagram by a more or less vertical line. The significance of this line is that no models for the star can exist—under the conditions stated—to the right of the Hayashi limit, ie.., at lower effective temperatures, whatever the character of the energy sources of the star might be. The existence of the Hayashi limit tells us that the life of a star, from its hydrostatic pre-main-sequence contraction to its dying phase, is restricted in terms of the Hertzsprung-Russell diagram to run its course between the main sequence and the Hayashi limit.

Nuclear Sources

For the study of stellar structure, 1938 turned out to be a red-letter year. It was in this year that Bethe and von Weizsäcker independently detailed the nuclear processes by which hydrogen transmutation into helium occurs within stars. It was not the concept of such transmutation that was new, since the idea that the main sources of stellar energy must be "subatomic" had been accepted throughout the preceding period. What was new was the detailed enumeration of the specific nuclear processes through which hydrogen fusion proceeds within the stellar interior and, equally importantly, the daring tentative derivation of the rates with which these processes proceed, depending on the prevailing temperature.

This fundamental addition turned the study of stellar structure from one containing a substantial degree of arbitrariness to one in which definitive models could be derived for any given star in any given state of evolution. The basic character of this great change is not diminished by the realization that the physical concepts and processes have time and again undergone—and are sure to continue to undergo in the future—alterations, both qualitatively and quantitatively.

In the same eventful year Chandrasekhar published his first book, *An Introduction to the Study of Stellar Structure.* This book is of unique character, in part because of its timing. It contains a complete exposition of the preceding periods, including a most rigorous discussion of polytropic and fully degenerate models, as well as radiative transport and models dominated by it. At the same time it contains in its last chapter, added just before publication, a first report on the new developments regarding the nuclear sources for stars, and thus it points acutely in the direction of the next period of this research field.

The introduction of explicit nuclear sources was the opening step for the study of stellar evolution. The first endeavor in this direction was the construction of inhomogeneous stars, the inhomogeneity being a direct consequence of the nuclear transmutations. This endeavor had a most peculiar early history. The first substantive step in this endeavor was taken by Öpik who early in 1938, prior to the appearance of the famous papers by Bethe and von Weizsäcker, published two papers on inhomogeneous stellar models. His insight into the probable consequences of nuclear processes—though the details of them were still unknown—was so high that he was capable of constructing models representing quite advanced phases. In fact, the second of these papers contains one model that has all the essential characteristics of modern advanced evolution models containing two burning shells. For reasons that I now believe rest less on objective considerations than on weaknesses in the human character which time and again interfere with effective cooperation, none of us then working in the field studied these papers carefully; the consequence was an unnecessary delay in the development of our field.

The second effort in the endeavor of constructing inhomogeneous models was made by Hoyle and his group, but you might say for the wrong reason. They estimated at that time that accretion of hydrogen-rich interstellar matter should play a major role in the life of a star and thus cause internal inhomogeneities. In spite of this "wrong" reason this work was widely studied and contributed much to the first insight in the structure of inhomogeneous stellar models.

On this side of the Atlantic the study of inhomogeneous stars was taken up soon after 1938, particularly by Gamow and Chandrasekhar and their colleagues. This development led promptly to a temporary impasse. The discovery of the Schönberg-Chandrasekhar limit proved that nondegenerate stars in thermal equilibrium could exhaust their hydrogen fuel only in a quite limited central fraction of their entire mass. This scientifically fascinating impasse was not resolved as fast as it might have been in consequence of a short but spirited controversy about the possible influence of degeneracy on this limit.

Thus the study of stellar evolution was launched by a set of scientific undertakings, all brilliant but also violently disconnected. When the dust from this great launching settled, the study of stellar evolution was well under way.

Gravitational Sources

The idea that gravitational contraction can provide an energy source for stars is as old as the theory of the stellar interior itself. What is left of this classical idea now that we recognize nuclear processes as the main energy sources for most stars during most of their life?

The answer to this question appears to be that gravitational energy sources are the means by which a star can transit from one long-duration evolution phase to the next one. If such a transition is violent and accordingly dynamical, the effects of gravitational energy release can be spectacular. For the moment, however, let us restrict ourselves to more calm hydrostatic transitions.

The most classical of such evolutionary transitions is the one from an interstellar cloud to a main-sequence star. The theory of this transition has had a checkered history. First, this transition was thought to consist of a rather uneventful, basically homologous contraction throughout. Next, after the discovery of the Hayashi limit the transition was believed to consist of a short and harmless collapse to the Hayashi limit, followed by a long slide down along this limit and ending with the last portion of the classical contraction path. Most recently a penetrating analysis has shown that the dynamical phase represents a much larger portion of this transition phase than had previously been anticipated, with a corresponding shortening of the slide down the Hayashi limit.

This first transition phase starting from an interstellar cloud has a special character for those stars which have such low mass as never to reach hydrogen-burning temperatures prior to becoming degenerate. Since the onset of degeneracy precludes any further heating, these featherweight stars never have the opportunity to tap any of their major nuclear fuels. Thus for featherweight stars, in stark contrast to heavier stars, the first transition phase leads directly from an interstellar cloud to an ever cooling degenerate dead state. Even though the general outline of the short life of a featherweight star seems fairly well understood, I have the impression that not as much attention has been given to this topic recently (particularly as regards cooling rates) as the potential importance of featherweight stars to the overall makeup of a typical galaxy might warrant.

It would seem irrelevant for today's purposes to enter here into a detailed account of the role of gravitational energy sources during subsequent evolutionary transition phases. The further a star progresses in its evolution, the more frequently it has to call on its gravitational resources in different portions of its interior to manage the transitions from one nuclear source to the next. This circumstance is reflected by the fact that in modern stellar evolution calculations normally the gravitational sources are taken account of right along with the nuclear sources, i.e., that thermal equilibrium in the classical sense for the stellar interior is not used as a tolerable approximation for advanced phases.

I would, however, like to come back at this point to the impasse caused by the discovery of the Schönberg-Chandrasekhar limit referred to earlier. For low-mass stars this impasse was found to be resolved along the lines suggested by Gamow, namely, the occurrence of degeneracy in the stellar core, which lifts the Schönberg-Chandrasekhar limit. This way out, however, is not available to the more massive stars since their relatively lower densities do not lead to degeneracy. Here the resolution of the dilemma consists exactly in permitting the star to use its gravitational resources, however stingily, to heat up its core by contraction to the temperatures necessary to initiate helium-burning in the center and thus to start the star off on its next slow evolution phase.

Undramatic Mass Ejection

During the initial period of the study of stellar evolution there existed one school in Cambridge according to which on the average the star's mass increases during its evolution by the process of accretion, a process I have already referred to. At the same time there existed a school in Moscow who believed, on the basis of statistical observational data, that main-sequence stars, or at least upper main-sequence stars, steadily decrease in mass. Between these two diverging views it seems that we stellar model makers on this continent took the easy way out by assuming a star's mass to stay substantially constant during its life. It appears now that we have fared quite well with this prosaic assumption as far as the early evolution phases are concerned. On the other hand, observational evidence has been rapidly accumulating for substantial mass ejection from stars in advanced evolution phases and particularly in the final phases of their lives. Indeed, it seems now rather definite that we cannot follow any star to its death without properly taking account of mass loss—except presumably for the featherweight stars.

All the observational evidence for mass ejection from stars and the concurrent theoretical considerations may be divided roughly into three areas. The first area is centered around the solar wind which has recently been measured directly by space probes in fine detail. These measurements show that the rate of mass loss represented by the solar wind is so low as to be irrelevant for the Sun's evolution. The cause of the solar wind appears to be understood by a three-step mechanism. The first step consists of the emission of acoustical or hydromagnetic waves by subphotospheric turbulence. The second step consists of the deposition of the energy of these waves in the corona, thus keeping the corona hot, while the final step can be thought of as a simple evaporation of the hot corona causing a steady stream of matter away from the Sun. If we transplant this solar wind mechanism from the Sun to other stars with as proper scaling as we know how, the equivalent mass ejection rates appear to be always below the level which would have evolutionary consequences.

The second area of mass ejection refers to hot giants and supergiants. Early observational evidence in this area was found in the spectra of P Cygni stars. Much additional evidence has been obtained during the past decade by ultraviolet observations from rockets and satellites. Though the quantitative interpretation of these observations is still far from certain, present estimates based on these data strongly suggest that the indicated steady mass ejection rate is sufficiently high not to be ignorable in the theory of the late evolution phases of massive stars. Tentative theoretical investigations suggest that this substantial mass ejection is caused by radiation pressure acting in resonance lines. It is, however, still far from clear whether such radiation pressure can cause the observed mass loss unaided or requires the help of other mechanisms, such as the first two steps of the solar wind mechanism. Clearly in this second area, which is of distinct relevance to the theory of stellar evolution, we are not yet in a state to derive the expected mass loss rates directly from theoretical considerations.

The same statement can also be made for the third area, which involves mass ejection from red giants and supergiants. Early evidence of substantial mass ejection from such stars was derived from the spectra of double stars with one red supergiant component. But recent infrared investigations, in which the thermal emission from the grains embedded in the ejected matter has been measured, not only have shown the generality of substantial mass ejection from this class of stars but have also improved the estimates of the rate of ejection. Again the result is that mass ejection from red giants and supergiants, which presumably represent the late evolution phases of low- and medium-weight stars, is amply high enough to have evolutionary consequences. In this area present theoretical estimates suggest that the main driving mechanism for the ejection is radiation pressure on the grains contained in the ejected matter. But many of the detailed steps of this process—which in this case includes even the difficult problem of grain formation—seems still far from definite and at best in the very first phases of rough quantitative estimates.

Altogether then, as far as the unspectacular mass ejection is concerned we find ourselves in the following situation. No new evidence has appeared to suggest that our normal approximation of constant mass needs a revision as far as early evolution phases are concerned. In contrast, for late evolution phases the assumption of constant mass appears to be at least dangerous if not downright invalid in most cases. I feel sure that the causes as well as the consequences of substantial mass ejection from highly evolved stars will be a major subject in the study of stellar evolution for a good number of years.

Active Dynamics and Nucleosynthesis

All the developments in the study of stellar structure I have recounted this far are based on hydrostatic equilibrium. Even the processes causing quiet, steady mass ejection require only the mild modification of hydrostatic equilibrium represented by stationary flow. The overwhelming dominance of the concept of hydrostatic equilibrium over the past studies of stellar structure clearly reflects that the overwhelming majority of observed stars are in a very steady state. But observations have long told us that there also are some very significant exceptions, ranging from pulsating stars to supernovae. In spite of the rarity of these dynamical phases, they were early suspected of being of extraordinary importance, a suspicion which by now has developed into a well based conviction. It is accordingly not surprising that active dynamical evolution phases and the intimately connected fundamental topic of nucleosynthesis have become a leading part of the study of stellar structure in the most recent epoch.

Stellar pulsation has been the first of these dynamical topics to be studied in depth. It was one of Eddington's major achievements to lay the foundation for this field, largely on the basis of linear perturbation theory. But he also succeeded in delineating the possible physical processes which might energize pulsating stars and which should act dissipatively for all other stars. One of the processes discussed by Eddington was subsequently found to be indeed the driving mechanism for all major classes of pulsating stars. In 1963 Christie initiated the study of nonlinear finite-amplitude pulsations by a daring endeavor employing modern computers. This research field has blossomed in recent years into an extensive activity. The skeptics among you might well wonder: What is the justification for researchers spending effort and computer funds on trying to find out finicky details, such as the exact characteristics of a pulsating star which has a hump on its light curve one hour after its maximum, or the exact position of the demarcation line in the Hertzsprung-Russell diagram which separates stars pulsating in the fundamental mode from those pulsating in the first overtone? My answer to such skeptical questions is that today's studies of stellar pulsations are a clear example of exploiting tricky perturbed cases with the aim of finding the solutions to fundamental problems, solutions that appear not to be obtainable by the study of unperturbed cases. This procedure, after all, is a long established tool in all physical sciences. Thus modern pulsation research, though heavily involved in the theoretical reproduction of observed details, is aimed not at the understanding of these details for their own sake. Rather it is aimed at using the understanding of such details to test uncertain points in the fundamental assumptions underlying all stellar interior theory, as well as possibly deriving fundamental data such as, for example, the helium content of the oldest stellar population in our Galaxy.

Let us turn to wilder dynamic phenomena. While pulsational phases appear to leave stars essentially unaffected as far as their general evolution is concerned—with the possible exception that pulsations might enhance the quiet mass ejection rate—other dynamical phases now under study do represent strong transition phases in the evolution of stars. Starting at the birth of a star, we have earlier considered the second portion of the transition from an interstellar cloud to a main-sequence star. In contrast to this second portion which proceeds in hydrostatic equilibrium, the first portion of this transition proceeds truly dynamically because during this portion the dissociation of molecular hydrogen and the ionization of hydrogen and helium depress the ratio of specific heats below the classical critical value of 4/3. Hence a star's life starts with a hydrodynamic collapse—however slow this collapse may be in consequence of the low densities involved. Decisive computations of this collapse phase have been carried out and have given us much improved starting points for the subsequent hydrostatic contraction, as I have mentioned earlier. However, presently available investigations are quite naturally mainly concerned with the simplest case, that of spherical symmetry, though some computations already take account of the first complication, moderate rotation. But dense interstellar clouds are obviously not likely to be orderly at all. Again it cannot be our assignment to follow through all actually occurring classes of disorderliness. But surely it will be an aim in the future to follow sufficiently disorderly cases through their initial evolution to see how close double, or multiple, stars might be formed, or even how small bound clusters of stars are formed. It will be fascinating to see whether we will in fact need the brute force tool of three-dimensional dynamic calculations to gain reliable initial insight leading toward the eventual solutions of these problems. Finally, I feel duty bound to at least just state that the formation of a planetary system clearly is part of the general topic of the early phases of a star's formation. I feel hesitant to bring this topic up at all. Even if I were young now, I would—at least I hope I would—consider this particular problem not yet ready for a deductive attack. Shouldn't we at least wait until the planetologists have solved the ever so basic question as to the major chemical composition of the planets? Indeed, negatively, the immense complexity of the solar system gives me (on occasion) nightmares regarding the specific point of our standard assumption that stars, once they arrive on the main sequence, are homogeneous in chemical composition.

The other cases of dynamical evolution phases which are under active investigation at present refer to the very final phases of a star's life. The most moderate of these cases appears to be the one in which a lightweight star ejects a planetary nebula during the last throes of nuclear burning within the star and just prior to its cooling down to a white dwarf. The present working hypothesis for the cause of this dynamical ejection can be described as an extreme case of pulsational instability. This mechanism appears to be most effective for low-mass stars. In a number of calculations this instability has been followed to substantial amplitudes, under the assumption of spherical symmetry. These calculations each cover a sufficient time span to support strongly the working hypothesis that the process considered will lead in fact to mass ejection, but they do not cover long enough time spans to answer the question as to the total mass ejected by this process. This question is important, as we would surely like to know whether the ejection is limited to the envelope layers with a composition much like the original composition inherited by the star, or whether the ejection involves layers deep enough to be seriously affected by the nuclear transmutations during the past history of the star. If the latter were the case—which at present does not seem likely except for minor components—it could have a substantial influence on the progressive change of the chemical composition of the interstellar matter in the Galaxy. Even these one-dimensional dynamical computations have turned out to present more technical difficulties than we might have expected, largely owing to the complicated structure of the extremely extended envelopes of the red supergiants in question. In fact, it has recently been shown that in these cases the relevant differential equations combined with the standard local approximation of the mixing-length theory for convection are ill-posed, in the mathematical sense of this unfriendly adjective. Thus if one wants to represent the envelope with a sufficiently fine grid to represent securely the many physically different zones of the envelope, one finds oneself forced to represent the actual nonlocal character of convection in some approximation. In spite of these technical difficulties, I feel that it will be worthwhile in the near future to try to compute this entire ejection phase through to its termination, even if for no other reason than to establish whether estimates based on linear perturbation theory give us an adequate insight into the problem of the total amount of mass ejected and the character of the residual star.

For some more massive stars, life seems to end in a much more spectacular dynamical manner, namely, in the form of supernovae. The present main working hypothesis for the cause of such a sparkling explosion is not an envelope instability—like that presumed to be the cause of the ejection of planetary nebulae—but rather a core instability. Two candidates for such instabilities are presently in the running. Ignition of a nuclear fuel within a degenerate core might directly cause an explosion. Alternatively, an internal collapse caused by the core exceeding the Chandrasekhar limit might cause a sequence of dynamical events ending in the explosive ejection of the envelope. In both cases it appears plausible that, whatever happens in the core, the dynamics occurring in the envelope may, by and large, have the character of a single, very strong shock wave running outward with increasing peak temperatures. Thus while the collapsing core may—however hypothetically—provide us with neutron stars and black holes, the shock wave in the envelope may provide us with a perfect place for nucleosynthesis.

The idea of producing heavy elements within stars followed fast upon Baade's classification of stars in populations. However, the theory of stellar nucleosynthesis started in earnest only with the work which culminated in the spectacular paper of Burbidge, Burbidge, Fowler, and Hoyle in 1957. This paper contains the overall classification of the nuclear processes producing the heavy elements. This classification still seems to form the basis of the present-day theory of nucleosynthesis, though many modifications and additions have secured and enriched this topic vital to modern astrophysics. My subordinating nucleosynthesis to dynamical phases in stars in this presentation has the explicit intent of giving emphasis to one change in the overall picture of stellar nucleosynthesis that has been quite recently developed and that seems to me of particular importance. While in the early work in this field the physical conditions required for the production of the heaviest elements were more or less correctly identified, it was estimated that such conditions would be likely to be found only in the inner portions of collapsing stars and not in the envelopes. While the early estimates regarding the cores of the stars continue to seem basically correct, it now appears that the envelopes of highly evolved stars might also provide a seat for the production of heavy nuclei, albeit for the very short duration of an extremely strong shock wave. This turn in the theory would appear of particular significance since it provides, without any further hypotheses, for the ejection of the very heavy elements thus produced.

Flagrant Deficiencies

In the preceding two sections I have sketched two subject areas, quiet mass ejection and violent dynamical phases, which are relative newcomers to the study of stellar structure when viewed from the long-range view of the last 75 years, though both have strong old roots. From the tone of my discussion it should be clear that I feel very optimistic regarding substantive and exciting progress in these subjects in the near future. It might therefore be pleasant to stop my discussion here. However, this would have the danger of implying either that everything important in the study of stellar structure is done or that its progress is well under control. Clearly such assessment would in no way represent the actual situation. Flagrant deficiencies abound—even if we count only those that we are already aware of—and something needs saying about them.

Everybody in this field will have his personal list of the most outstanding deficiencies. May I be bold enough to describe my two top candidates for such a list of today's flagrant deficiencies in our field, namely, the missing theory for turbulent convection and the missing solar neutrinos? These two deficiencies, besides being flagrant indeed, also happen to be instructive examples, one of a very old unsolved problem—indeed, as old as the study of stellar structure itself—and the other of a basic discrepancy sprung on us suddenly and relatively recently.

The fundamental equations of hydrodynamics which presumably should form the basis of an exact statistical theory of turbulence have long been known. Very general methods have been developed with which the solution of these equations can be found for any particular case as long as such case falls into the laminar domain. Even the methods of deriving the limits of this domain, often expressible in terms of critical Reynolds or Rayleigh numbers, have been successfully developed. In contrast, for the turbulent domain at high Reynolds