COLLEGE ALGEBRA

H. T. DAVIS

COLLEGE ALGEBRA

by

HAROLD T. DAVIS
NORTHWESTERN UNIVERSITY

WITH REVISIONS AND ADDITIONAL PROBLEMS

NEW YORK
PRENTICE-HALL, INC.
1946

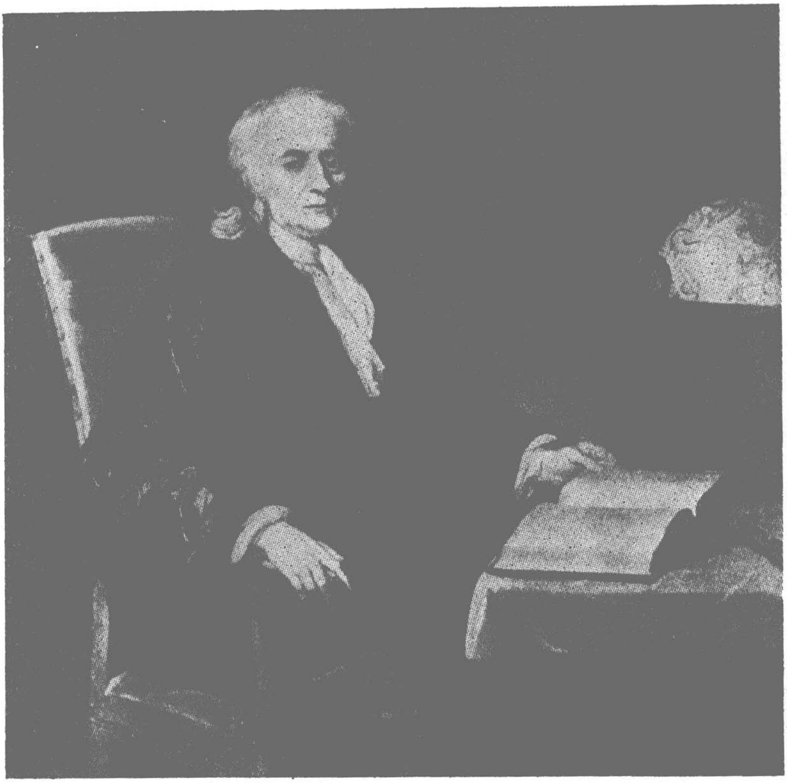
PRENTICE-HALL MATHEMATICS SERIES

ALBERT A. BENNETT, EDITOR

First Printing ....November, 1940
Second Printing.....................July, 1941
Third Printing................September, 1941
Fourth Printing...............November, 1941
Fifth Printing.................September, 1942
Sixth Printing....................January, 1943
Seventh Printing...............October, 1945
Eighth Printing..................March, 1946
Ninth Printing..................August, 1946
Tenth Printing...............September, 1946

PRINTED IN THE UNITED STATES OF AMERICA

COLLEGE ALGEBRA



SIR ISAAC NEWTON (1642–1727)

PREFACE

This text on college algebra presents in the first sixteen chapters the material usually treated in a first course in American colleges and universities. Since most students welcome a review of their earlier studies, the first three chapters develop the subject from the beginning. The concept of number is explained and deepened. Operations with polynomials, the use of simple identities in factoring, the combination of fractions, the application of simple equations, and other elementary topics are introduced and explained without the assumption of previous knowledge on the part of the student. In this way, preparation is made for an understanding of the more advanced topics which constitute the basis of a college course in algebra.

In recent years, the fact that mathematical teaching gains much from an intimate association with historical origins has been realized. Thus we find in the final report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics the following statement:

"The Commission believes that it should stress one subject, namely the history of mathematics. If the study of secondary mathematics is to reveal mathematics as one of the fundamental enterprises of man, which, though rooted in daily need, is an expression of deep, irrepressible, and idealistic impulses, then the teaching of it should constantly be associated with its history. One recalls the statement of Glaisher, 'I am sure that no subject loses more than mathematics by any attempt to dissociate it from its history.' "*

With this conclusion the author of the present text is in full accord. Consequently unusual attention has been given

* *Fifteenth Yearbook of the National Council of Teachers of Mathematics: The Place of Mathematics in Secondary Education*, p. 197. Edited by W. D. Reeve Columbia University, New York City, 1940.

to the historical incidents which marked the development of algebra. Numerous references to the history of the subject have been interpolated in the text, and the final chapter is devoted to a discussion of the meaning of mathematics. A brief account is given of a few of the contributions of mathematics to the development of knowledge. Biographical sketches of some of the greatest mathematicians are included, and pictures of a few of them are given in the text.

Recognition of the growing interest of the social sciences in algebra is made by the inclusion of a chapter on statistics. This material is related to the graphing of data, to the binomial theorem, and to other topics treated earlier in the book.

A course in algebra may be illuminated by reference to some of the problems included under the general title of "Mathematical Recreations." A chapter on this subject provides ample material for use in connection with other topics in the book.

An elementary course must provide among other things a systematic drill in operational techniques. Numerous examples have been included to provide the necessary instruction in the processes described, and extensive lists of problems supply the student with material for testing his manipulative skill.

PREFACE TO FIFTH PRINTING

In response to requests for more problems for classroom drill in certain parts of the text, this new edition has been provided. Some thousand new problems have been added and the supplementary lists for review have been incorporated in the body of the text and extended in their scope. In recognition of the need for problems illustrating the application of the algebraic processes to various domains of science, a number of new exercises of this character have been added.

The opportunity has also been used to enlarge several sections of the text and to include certain topics, which, although usually omitted in a course of three or four semester-hours, are desired in longer courses.

SIGNS AND SYMBOLS USED IN ALGEBRA

$+$, sign of addition, to be read *plus;* $-$, sign of subtraction, to be read *minus.*

$\times$, $\cdot$, signs of multiplication, to be read *times;* $\div$, sign of division, to be read *divided by.* The solidus, /, is also used to indicate division.

$=$, denotes equality, and is read *is equal to;* $\equiv$ is the identity symbol, and is read, *is identical with.*

$\neq$, the sign of inequality; to be read, *is not equal to.*

$>$, to be read, *is greater than;* $<$, to be read, *is less than.*

$\geqq$, to be read, *is greater than or equal to;* $\leqq$, to be read, *is less than or equal to.*

( ), parentheses; [ ], brackets; { }, braces.

$|a|$, denotes the absolute value of a.

$\sqrt{a}$, means the square root of a; $\sqrt[n]{a}$, denotes the nth root of a.

$n!$, means *factorial* n. This symbol is sometimes written $\underline{|n}$.

${}_nC_r$, denotes both the rth *binomial coefficient* and the *number of combinations of n things taken r at a time.*

${}_nP_r$, denotes the *number of permutations of n things taken r at a time.*

s_n, denotes the *amount of unit principal at compound interest,* and v^n the *discounted amount of unit principal at compound interest.*

$s_{\overline{n|}}$, denotes the *amount of an annuity of 1,* and $a_{\overline{n|}}$ the *present value of an annuity of 1.*

$\rightarrow$, to be read *approaches.* For example, $x \rightarrow a$, means x approaches a.

$\lim\limits_{x = a}$, means the limit as x approaches a.

$f(x)$, denotes a *function of x;* to be read, *f is a function of x.*

(x, y) designates *a point whose coordinates are x and y.*

GREEK ALPHABET

Letters	Names	Letters	Names	Letters	Names
Α α	Alpha	Ι ι	Iota	Ρ ρ	Rho
Β β	Beta	Κ κ	Kappa	Σ σ ς	Sigma
Γ γ	Gamma	Λ λ	Lambda	Τ τ	Tau
Δ δ	Delta	Μ μ	Mu	Υ υ	Upsilon
Ε ε	Epsilon	Ν ν	Nu	Φ φ	Phi
Ζ ζ	Zeta	Ξ ξ	Xi	Χ χ	Chi
Η η	Eta	Ο ο	Omicron	Ψ ψ	Psi
Θ θ	Theta	Π π	Pi	Ω ω	Omega

CONTENTS

TABLES

CHAPTER I

PRELIMINARY TOPICS

1. Algebra—its purposes and historical development. Algebra is the branch of mathematics that reasons about quantities by means of letters and other symbols. It is essentially a generalized form of common arithmetic in which symbols are used to represent numbers, and specific processes of arithmetic are stated more comprehensively. The word **algebra** is derived from the Arabic word *al-jebr*, which means *restoration*—that is to say, the restoration of an equation by transposing negative terms from one side to the other. The first book in which the word *al-jebr* was known to have been used was written by Mohammed ibn Mûsâ al-Khowârizmî, who lived during the reign of the Caliph Ma'mūn (813–833). The name of this author was in later years contracted to Algoritmi, from which we derive the word **algorithm**, which means the art of computing by some special method or process.

The subject of algebra originated in the very distant past. We find that simple problems appeared in an Egyptian papyrus written by Ahmes some time before 1700 B.C., and this work is believed to have been founded on an earlier work that may date as far back as 3400 B.C. Both arithmetical and geometrical progressions, which we shall study later in this book, are found in the Ahmes' papyrus. For example, the author requires that 100 loaves be so divided among five people that the number received will decrease from person to person by a constant amount and that the last two will receive one seventh of what the first three get. The student may verify the answer: $38\frac{1}{3}$, $29\frac{1}{6}$, 20, $10\frac{5}{6}$, $1\frac{2}{3}$. In this ancient manuscript also appears a problem which historians have interpreted as equivalent to the following: 7 people each have 7 cats; each cat eats 7 mice; each mouse eats 7 ears of barley; from each ear, 7 measures of barley may grow. What is the total number of people, cats, mice, ears of barley, and measures of barley? The student may verify Ahmes' answer of 19,607.

Among the Greeks the only algebraist of significance was Diophantus. His death occurred about 300 A.D. We know that he lived to be 84 years old from the following epitaph: Diophantus passed one sixth of his life in childhood, one twelfth in youth, and one seventh more as a bachelor; five years after his marriage, a son was born; the son died four years before his father at half his father's age. Diophantus wrote a treatise, entitled *Arithmetica*, in 13 books, of which seven are extant. This work is largely confined to problems in algebra and the theory of integers. He solved a special case of quadratic equations, but rejected both negative and irrational numbers.

Although the roots of some of the problems in algebra are to be found in Arabian and Hindu mathematics, and although some sporadic attempts to develop algebra were made in the Middle Ages (notably by Leonardo of Pisa, who flourished around 1200), the beginnings of algebra in any modern sense are found in the sixteenth century. The *Ars Magna* of G. Cardano (1501–1576), which was published in 1545, laid the foundations for the general theory of algebraic equations; the germs of the theory of exponents and of logarithms are to be found in the *Arithmetica Integra* of Michael Stifel (1486?–1567), which appeared in Germany in 1544.

It is a significant fact that the development of algebra and the development of science were simultaneous. Thus François Vieta (1540–1603), in his numerous writings on algebra, was introducing the use of letters for algebraical quantities, while Tycho Brahe (1546–1601), his contemporary, was collecting the first modern set of observations of the motions of the planets. John Napier (1550–1617), Baron of Merchiston in Scotland, published his theory of logarithms in 1614, and Henry Briggs (1561–1630) produced the first table of common logarithms in 1624—works contemporary with the discoveries of Johann Kepler (1571–1630), the astronomer. The *Géométrie* of René Descartes (1596–1650), which connected the appar-