A FIRST COURSE IN
Rational Continuum Mechanics

VOLUME 1

C. TRUESDELL

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General Concepts

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Preface

The mechanics of finite systems of points and rigid bodies was given a fairly definitive form by LAGRANGE's exposition in his *Méchanique Analitique*, 1788. While that book covers only certain aspects of the rational mechanics created by LAGRANGE's great predecessors, it presents them systematically and as a branch of mathematics: "Ceux qui aiment l'Analyse, verront avec plaisir la Méchanique en devenir une nouvelle branche, ..." The physics and the applications are omitted. He who will apply and interpret the theory, or dwell upon the intricacies and mysteries of its place among the relations between mind and external nature, is expected to learn it first. While the knowledge he thus acquires does not of itself put applications into his hands, it gives him the tools to fashion them efficiently, or at least to classify, describe, and teach the applications already known. By consistently leaving applications to the appliers, LAGRANGE set them on common ground with the theorists who sought to pursue the mathematics further: Both had been trained in the same workshop and spoke the same jargon. Even today this comradeship of infancy lingers on, provided discrete systems and rigid bodies exhaust the universe of mechanical discourse.

In 1788 the mechanics of deformable bodies, which is inherently not only subtler, more beautiful, and grander but also far closer to nature than is the rather arid special case called "analytical mechanics", had been explored only in terms of isolated examples, brilliant but untypical. Unfortunately most of these fitted into LAGRANGE's scheme; those that did not, he passed over in silence. Further brilliant examples, feigned mainly upon the framework of NEWTON's and EULER's concepts and not easily subsumed under LAGRANGE's, were created in the next century but were studied mainly for their own sakes, separately, and did not lead to a general doctrine, despite the deep and original syntheses of stress and strain forged by CAUCHY.

A hundred years after CAUCHY died began a renascence of "classical" mechanics as a whole, taking the deformable continuum as the typical body and describing it in terms of an equally specific concept of material, which had been left nebulous and physical or metaphysical before then. This new general doctrine is now fit to be learned and used by mathematicians, experimentists, and engineers and to join the old analytical mechanics as an element of common education. Physicists should be able to understand it if they wished to. Like geometry, it is a part of mathematics.

In writing a textbook of continuum mechanics at this time I imitate the example of LAGRANGE in several ways. My book offers merely a selection from the wondrous harvest of the last few decades; leaving much else unmentioned, it bases that selection on criteria of naturalness, ease, and subsumption to a general method and conceptual frame. Thus it is a short book, designed for readers who know already that applications to further cases are numberless and possibilities for further mathematical study infinite. As LAGRANGE wrote, "On ne trouvera point de Figures dans cet Ouvrage. Les méthodes que j'y expose, ne demandent ni constructions, ni raisonnemens géométriques où méchaniques, mais seulement des opérations algébriques, assujetties à une marche regulière & uniforme." This claim is as true—or as false—of the present book as of LAGRANGE's. Of course, many proofs are easier to grasp if a figure is drawn, and both teacher and student should illumine and enrich the "regular and uniform course" by sketches. Finally—and here, perhaps, lies the greatest difference between this book and others with similar titles—it follows LAGRANGE's example in presuming that the reader commands the elementary mathematics of his own day[1], making no attempt to offer a shadowy substitute for

[1] The reader is expected to know the elements of measure theory. For almost everything else needed in "pure" mathematics, more than sufficient background is given in the book by H. K. NICKERSON, D. C. SPENCER, & N. E. STEENROD, *Advanced Calculus*, Princeton, Van Nostrand, 1959, reprinted in 1968 by Affiliated East-West Press Pyt. Ltd., New Delhi. The same may be said of the recent textbook by R. M. BOWEN & C.-C. WANG, designed especially for students of continuum mechanics: *Introduction to Vectors and Tensors*, 2 volumes, New York and London, Plenum Press, 1976. While in the text below certain more specialized works are cited in reference to some particular theorems, most of those may be found also in the two undergraduate texts just cited.

decent modern training in algebra and calculus or to appease the notorious reluctance of old men to learn anything new. The student may well find this book easier than his teacher does.

In three respects, however, I depart from LAGRANGE's model. First, I leave important if small pieces of the arguments, and some illustrations of them, as exercises for the reader, since my experience in teaching the new mechanics as it sprouted and grew has assured me that he who does not for himself re-create and digest the mathematics step by step will never master this doctrine. Second, while LAGRANGE's presentation bestowed upon the subject a gloss of closure and completeness which by the passage of time has been abundantly proved specious, in this book I try to present the science of "classical" mechanics even to the beginner as what it is: a magnificent array of ordered concepts and proved theorems, some of them old, even very old, and some on the frontiers of research into great unsolved problems and not yet distilled experience of nature as human eyes see it and human hands feel it. Third, the frequent attributions of major ideas and results to others will make it clear that I claim little of the substance for my own. The citations of other works, however, are intended not as acknowledgments of sources but as aids to the student. Those at the ends of the chapters direct him to places where further matters closely related to the text are developed; those in the footnotes, to specific details passed over in the text such as counterexamples, direct generalizations, proofs of theorems cited from other parts of mathematics, and tangent domains of modern mechanics.

Finally, I wish to thank those who have helped me to understand mechanics and to complete and purify this book. Thus above all I thank WALTER NOLL, and after him J. L. ERICKSEN, R. A. TOUPIN, B. D. COLEMAN, M. E. GURTIN, C.-C. WANG, W. O. WILLIAMS, L. SOLOMON, T. TOKUOKA, W.-L. YIN, R. C. BATRA, and D. EUVRARD. I am indebted to Mr. BATRA also for a full set of solutions to the exercises.

"Il Palazzetto"
Baltimore
May 1, 1972

Addendum. Parts 1 through 4 of this work, expertly translated into French by D. EUVRARD from my text of 1972, were published in December, 1973, by Masson et Cie in a single volume with the title *Introduction à la Mécanique Rationnelle des Milieux Continus.* Parts 1 through 5 appeared in 1975 in Russian, Первоначальный Курс Рациональной Механики Сплошных Сред, Moscow, Мир, translated from my text of

1973 by R. V. GOLDSHTEIN & V. M. ENTOV under the guidance of P. A. ZHILIN & A. I. LUR'E. Since that time I have been able to add some material and also to work through the text again and make numerous improvements, partly in response to criticisms and suggestions offered by readers of the French book.

A question has been raised regarding the knowledge of mechanics the student is expected to have already. A good treatise on the theory of functions of a real variable does not strictly require of its readers any previous acquaintance with the subject, even in the most elementary aspects of infinitesimal calculus, yet a student armed with no more than a naked, virgin mind is unlikely to survive the first few pages. In the same way, although this book does not call upon any previous knowledge of continuum mechanics, or even of schoolboy mechanics, it is designed for students not altogether innocent of hydrodynamics and elasticity. Much as a crude and awkward first affair may furnish knowledge that, however elementary, is indispensable to him who aspires toward Venus' ultimate refinements, a bad course—something nowadays cheaply found—will serve well enough here, too.

Some comments on the preliminary editions in French and Russian suggest need for reminder that this is a mathematical textbook, not a treatise or a history. In attaching names to a proposition I follow the commonest usage in the mathematical literature, proclaiming respect for those to whom I think we owe that proposition, be it in entirety, be it for discovery and proof of a pilot case, be it for clearest statement or most elegant proof; a second name never indicates rediscovery but always some major improvement, and of course it would not be feasible in any discipline so broadly cultivated as rational mechanics now is to list all the persons who have done something valuable, even if I knew of them all.

Volume 1 contains Part 1 only. Volume 2, containing Part 2 on fluid mechanics and Part 3 on elasticity, is presently being polished for the press. I plan to complete the textbook by a third volume, to concern fading memory, thermodynamics, statics, and thermostatics.

I thank Mr. BATRA for further suggestions and for checking the manuscript of this volume. I am deeply grateful to him and to Messrs. DAFERMOS, ERICKSEN, GURTIN, MUNCASTER, NOLL, and WILLIAMS for their generous gift of time and care in correcting the proofsheets so as to remove errors and obscurities even at the last moment. For such faults as, alas, surely remain I bear an uncommon charge, for seldom has an author had the benefit of such abundant and expert aid.

I owe a double debt of gratitude to the U.S. National Science Foundation for its continued and generous support: first, for the work of some of the great savants whose discoveries are incorporated here; second, for my

own long effort to compose the essence of modern rational mechanics into an easy union with the magnificent tradition from which it sprang, so that beginners might learn both old and new together and in such a way as to see each illuminate and ennoble the other.

December 20, 1976 C.T.

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PART 6 STATICS AND THERMOSTATICS

Contents

PART 1

GENERAL CONCEPTS

In the following Chapters on Abstract Dynamics we confine ourselves mainly to the general principles, and the fundamental formulas and equations of the mathematics of this extensive subject; and neither seeking nor avoiding mathematical exercitations, we enter on special problems solely with a view to possible usefulness for physical science, whether in the way of the *material* of experimental investigation, or for illustrating physical principles, or for aiding in speculations of Natural Philosophy.

THOMSON & TAIT
Treatise on Natural Philosophy
(2^{nd} ed., 1883), §453

Chapter I

Bodies, Forces, Motions, and Energies

I have restored to the concepts of *mass* and *force* their old rights. Beyond all doubt we need these *things*, for without them, there is no mechanics. Force is more than mass times acceleration, as may be seen from the basic equation itself, which always asserts that mass times acceleration equals the *sum* of the forces. Therefore, why not use the good old *words*? The concepts themselves are not unclear; it is just that the books described them often in a very metaphysical and dark way. And what matter, if the concepts are remarkably useful—perhaps a bit riddling?—if the concepts of mechanics are deeper than many find convenient, and cannot be disposed of with a few elegant words like convention and economy of thought, abstraction and idealisation?

HAMEL
On the foundations of mechanics,
Mathematische Annalen **66**, 350–397
(1909).

Space, time, and force are *a priori* forms; they can be derived only from contemplation and from general principles of research. Their common relation to each other in mechanics must be regarded as something inspired indeed by experience but in its generality fixed by convention.

HAMEL
Elementare Mechanik (1912), ¶5