quantum collision theory

charles j. joachain

ISBN: 0 444 86773 2

First edition 1975
Second edition 1979
Third edition 1983

Published by:

North-Holland Physics Publishing

a division of

Elsevier Science Publishers B.V.

P.O. Box 103
1000 AC Amsterdam
The Netherlands

Sole distributors for the U.S.A. and Canada:

Elsevier Science Publishing Company Inc.
52 Vanderbilt Avenue
New York, NY 10017
U.S.A.

Library of Congress Cataloging in Publication Data

Joachain, Charles J.
Quantum collision theory.

Outgrowth of lectures which the author has given at the Universities of Berkeley, Calif., Brussels and Louvain.
Includes bibliographical references and indexes.
1. Collisions (Nuclear physics) 2. Potential scattering. 3. Scattering (Physics) 4. Quantum theory. I. Title.
QC794.6.C6J6 1979 539.7'54 78-24542
ISBN 0-444-86773-2 (Elsevier)

Printed in The Netherlands

Preface

The purpose of this book is to give a self-contained and unified presentation of the methods of quantum collision theory, with applications to atomic, nuclear and high-energy processes. It is primarily aimed at graduate students in theoretical physics, although it is hoped that any physicist whose main interest lies in microphysics will also find it useful.

The book is divided into four parts. The first one is devoted to the presentation of the *basic definitions* and to the study of collision *kinematics*. The former is done in Chapter 1, where the various types of collisions and the concepts of channels and cross sections are defined. Chapter 2 entirely deals with kinematical questions, first in the non-relativistic case and then for relativistic collisions.

The second part (Chapters 3–12) contains a detailed discussion of the simplest collision problem, namely the non-relativistic scattering of two particles interacting through a potential which depends only on their relative coordinate. Because of its simplicity, this problem provides a good introduction to the methods of collision theory. Moreover, since "exact" solutions can often be readily obtained in this case, potential scattering is a convenient "laboratory" in which one can test approximation methods that become unavoidable in more complicated situations.

The *general features of potential scattering* are first discussed in Chapter 3. The following chapter is devoted to the *method of partial waves*, while the *Lippmann–Schwinger equation* for potential scattering is studied in Chapter 5. The particular case of the *Coulomb potential* is treated in Chapter 6. It is followed in Chapter 7 by an analysis of the potential scattering of *identical particles*. A number of important approximation methods are studied in Chapters 8–10: the *Born series* is discussed in Chapter 8, *semi-classical* *approximations* are taken up in Chapter 9 and *variational methods* are considered in Chapter 10. The next chapter, which is of a more advanced nature, is entirely devoted to the *analytic properties of scattering amplitudes*. It provides an introduction to some of the most fruitful techniques recently used in elementary particles physics. Finally, the *time-dependent approach to potential scattering* presented in Chapter 12 paves the way for the general treatment of quantum collisions developed in Part III.

With the exception of Chapter 11 and of Section 2.2 on relativistic kinematics (which may both be studied at a later stage), the material contained in Parts I and II constitutes the basic subject matter of quantum scattering theory. Graduate students in physics may be expected to have already encountered parts of this material in various courses, but I believe that the unified approach given in this book will help them to gain a deeper understanding of the subject. I have also stressed in the first two parts the pedagogical approach to scattering theory, the beginning sections being purposely designed to have a minimum of notational complication. In the same spirit I have also included a series of problems at the end of both Parts I and II. Having mastered the basic material contained in these first two parts, the student is then prepared to go on to the more advanced topics contained in the remaining parts of the book.

The *general treatment of quantum collisions* is the subject of Part III. This part begins in Chapter 13 with general notions of *quantum dynamics*. This formalism is then applied in Chapter 14 to analyze quantum collisions from a time-dependent point of view. It leads to the central concepts of *S- and T-matrices*. The calculation of the *transition probabilities and cross sections* is carried out in Chapter 15. This chapter also contains a discussion of the Lorentz invariance of the cross sections, together with an investigation of some important consequences of the unitarity of the *S*-matrix.

Chapter 16 is devoted to the *determination of the S- and T-matrices*. It contains the derivation of the general Lippmann–Schwinger equations, together with a discussion of Born expansions, variational principles and the Low equations. The reaction matrix is then introduced and the Heitler equations are proved. The modifications necessary to treat general collisions involving identical particles are then given, and the chapter ends with a study of the role played by invariance principles in the determination of the collision matrix. Chapter 17, which concludes Part III, deals with an important application of formal scattering theory, namely the problem of *two-potential scattering*.

Part IV of the book (Chapters 18–21) is devoted to the application of the general theory developed in Part III to selected problems in atomic, nuclear and high-energy physics. *Two-body* collisions are first considered in Chapter 18, first for central forces and then for non-central interactions. Generalized partial wave expansions are obtained for the collision matrix, and a detailed discussion is given for the case of spin zero–spin one-half scattering. The Regge pole concept introduced in Chapter 11 is also used to describe some features of two-body collisions of hadrons at high energies.

Chapter 19 deals entirely with the *three-body problem*. In particular, an introduction to the Faddeev theory is given, together with a discussion of various multiple scattering expansions. Two important problems are studied in detail: electron collisions with atomic hydrogen and high-energy hadron–deuteron scattering. In the next chapter the *optical potential* method is presented and applied to high-energy hadron–nucleus scattering and to the elastic scattering of charged particles by atoms. Finally, in Chapter 21, a few typical *multiparticle scattering processes* are considered, such as electron–helium collisions and nuclear stripping or pick-up reactions. Also discussed in the last chapter is the theory of final state interactions.

This book is the outgrowth of lectures which I have given at the Universities of California (Berkeley), Brussels and Louvain over the past nine years. I wish to thank my colleagues and students at these institutions for numerous fruitful discussions and remarks. I am particularly indebted to Professor F. W. Byron, Jr. for a careful reading of the manuscript and for many helpful suggestions and comments. The proofreading of the manuscript by Dr. E. H. Mund and Dr. K. H. Winters has been of great help. It is also a pleasure to thank Mr. C. Depraetere, who drew most of the figures of this book, and Mmes G. Janssens, T. Köke, G. Mobers and E. Péan for their expert and careful typing of the final manuscript.

CHARLES J. JOACHAIN

Contents

APPENDICES

PART I

DESCRIPTION OF COLLISION PROCESSES

1

Basic Definitions

This chapter contains a simple, descriptive introduction to quantum collision processes. We first classify the various types of collisions in Section 1.1. The concept of channel is then introduced in Section 1.2 and the cross sections are defined in Section 1.3.

1.1. Types of collisions

Let us consider a typical collision experiment which is illustrated by the schematic drawing of Fig. 1.1. A beam of particles A, well collimated and nearly monoenergetic, is directed towards a target. The incident beam should be neither too intense – so that the interaction between the incident particles may be neglected – nor too weak, because one wants to observe a reasonable number of "events" during the experiment.

The target usually consists of a macroscopic sample containing a large number of scatterers B. The distances between these scatterers are in general

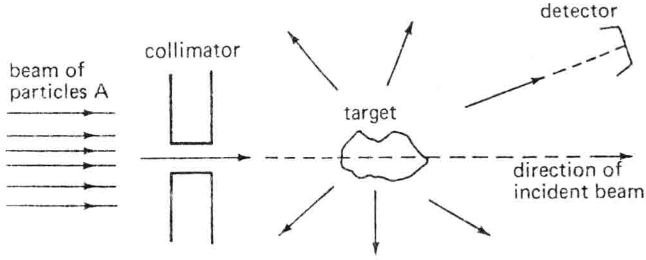

Fig. 1.1. Schematic drawing of a collision process.

References and notes on page 11

quite large with respect to the de Broglie wavelength of the incident particle, in which case one can neglect coherence effects between the waves scattered by each of the scattering centers [1]. In addition, if the target is sufficiently thin, multiple scattering by several scatterers can be neglected. One may then consider that each scatterer B acts as if it were alone, and focus one's attention on the study of a typical collision between a particle A of the incident beam and a scatterer B of the target [2].

After the collision, some or all outgoing particles are registered by detectors [3], located at a macroscopic distance from the target. Several processes can occur:

1) *Elastic scattering:* the two particles A and B are simply scattered without any change in their internal structure,

$$A + B \to A + B. \tag{1.1}$$

2) *Inelastic scattering:* the two particles A and (or) B undergo a change of their internal quantum state during the collision process [4]. Denoting by A′ and B′ these new internal states, we may have

$$A + B \to A' + B$$

or

$$A + B \to A + B' \tag{1.2}$$

or

$$A + B \to A' + B'.$$

3) *Reactions:* the composite system (A + B) splits into two particles different from A and B, or into $n > 2$ particles. That is,

$$A + B \to C + D \tag{1.3}$$

or

$$A + B \to C_1 + C_2 + \ldots C_n. \tag{1.4}$$

It is convenient at this point to define the terms "elementary" and "composite" particles. Since the rigorous definition of an "elementary particle" is presently rather arbitrary we shall use a practical definition and decide that *a particle is elementary if, in the particular phenomenon under study we may suppose that it is not a bound system of other particles.* For example, in low energy atomic collisions the atomic nuclei may be considered as elementary, while they appear as composite structures in nuclear reactions.

Let us now return to the reaction (1.3). If the particles A, B, C and D are all "elementary", we shall call this process a *two-body reaction.* If on the contrary the particles A and (or) B are composite so that the reactions (1.3) and (1.4) occur because of the *exchange* of one or several elementary constituents, we shall call these processes *rearrangement collisions.* Chemical reactions are rearrangement collisions between atoms, ions and molecules; nuclear reactions are rearrangement collisions between atomic nuclei. In particular, we shall call *binary* rearrangement collisions those of the type (1.3), where two particles emerge in the final state.

As an illustration of these various types of collision processes, consider a positron e^+ incident on an hydrogen atom in its ground state, for energies above the ionization threshold. Among the processes which can occur are

$$\begin{aligned} e^+ + H &\to e^+ + H && \text{: elastic scattering} \\ &\to e^+ + H^* && \text{: inelastic scattering} \\ &\to e^+ + p + e^- && \text{: reaction : ionization} \\ &\to p + 2\gamma && \text{: reaction : annihilation} \\ &\to p + (e^+e^-) && \text{: binary rearrangement collision} \\ & && \text{(formation of positronium)} \end{aligned} \tag{1.5}$$

where H^* denotes an hydrogen atom in an excited state, p is a proton, e^- an electron, γ a photon and (e^+e^-) represents positronium, a bound state of the electron–positron system.

In many experimental situations it is difficult to keep track of all the outgoing particles. For example, in high energy collisions of hadrons [5], neutral particles are difficult to detect. Therefore, following Feynman [6], it is convenient to define *exclusive experiments*, where the nature and momenta of *all* outgoing particles are measured, and *inclusive* experiments where only some of the outgoing particles are detected. For example the two-body reaction (1.3) is an exclusive experiment, whereas the reaction

$$A + B \to C + \cdots \tag{1.6}$$

– where the particle C is produced together with some other outgoing particles – is an inclusive reaction.

Before we conclude this section, it is worth pointing out that many experimental settings may differ appreciably from the simple one shown in Fig. 1.1. For example, in experiments with colliding beams (as in the case of "intersecting storage rings"), crossed beams [e.g. 7] or merged beams [e.g. 8], the target itself consists of a beam of particles B. We shall return to this question in Chapter 2.

1.2. Channels

A channel is a possible mode of fragmentation of the composite system (A + B) during the collision. It is characterized by the number and the nature of the fragments into which the system (A + B) can be decomposed. For example, we have listed above in (1.5) five possible final channels into which the composite system $(e^+ + H)$ may dissociate. The definition of the channels contains some arbitrariness, related to our way of specifying the precise "nature" of the fragments. For example, in the inelastic process

$$e^+ + H \to e^+ + H^* \tag{1.7}$$

we may consider each of the excited states H^* as corresponding to a different channel, or group them into one "inelastic" channel. Ambiguities of this type

References and notes on page 11

also arise in particle physics, where two particles may often be equally well considered as "different", or "different states" of a same particle.

One of the possible modes of fragmentation of the system gives back the two original particles A and B, i.e. the initial channel. In an elastic collision, the two colliding particles remain in the initial channel. A channel is *open* if the corresponding collision is allowed by known conservation laws (energy conservation, charge conservation, etc.). Otherwise it is *closed.*

As an example, consider proton–proton scattering at energies below the threshold for production of two pions. Four possible open final channels are then

$$\begin{aligned} \mathrm{p} + \mathrm{p} &\to \mathrm{p} + \mathrm{p} \\ &\to \mathrm{p} + \mathrm{p} + \pi^{\circ} \\ &\to \mathrm{p} + \mathrm{n} + \pi^{+} \\ &\to \mathrm{d} + \pi^{+}. \end{aligned} \tag{1.8}$$

1.3. Cross sections. Laboratory and center of mass systems

The results of collision experiments are usually expressed in terms of characteristic quantities called *cross sections.* They are defined as follows: *The cross section of a certain type of event in a given collision is the ratio of the number of events of this type per unit time and per unit scatterer, to the relative flux of the incident particles with respect to the target.*

To illustrate this definition, let us first consider the case of *total cross sections.* We assume that the initial channel consists of two particles A and B which are both in well defined quantum states [2]. We also suppose that the incident beam of particles A satisfies the physical requirements stated in Section 1.1. Thus a sufficiently large (but not too large) number N_{A} of particles A reach per unit time the target (made of particles B) with nearly parallel directions and a kinetic energy distribution sharply peaked about a given value. Let us denote by $\mathcal{N}_{\mathrm{A}}$ the average number of particles A per unit volume in the incident beam and by v_{i} the average magnitude of their velocity with respect to the target. The flux Φ_{A} of incident particles relative to the target (i.e. the number of particles A crossing per unit time a unit area perpendicular to the direction of the incident beam and at rest with respect to the target) is then given by

$$\Phi_{\mathrm{A}} = \mathcal{N}_{\mathrm{A}} v_{\mathrm{i}} = N_{\mathrm{A}}/S \tag{1.9}$$

where S is the cross-sectional area of the incident beam, as shown in Fig. 1.2.

Next, let us assume that the target is sufficiently thin and denote by n_{B} the number of particles B within the "effective" target volume interacting with the incident beam. For example, if we imagine the target to be a very thin layer of thickness l perpendicular to the incident beam (see Fig. 1.2) we have

$$n_{\mathrm{B}} = S \cdot l \cdot \mathcal{N}_{\mathrm{B}} = S\hat{\mathcal{N}}_{\mathrm{B}} \tag{1.10}$$

where $\mathcal{N}_B$ is the number of particles B per unit volume in the target and $\hat{\mathcal{N}}_B = \mathcal{N}_B\, l$ is the (average) *surface density* of the target particles.

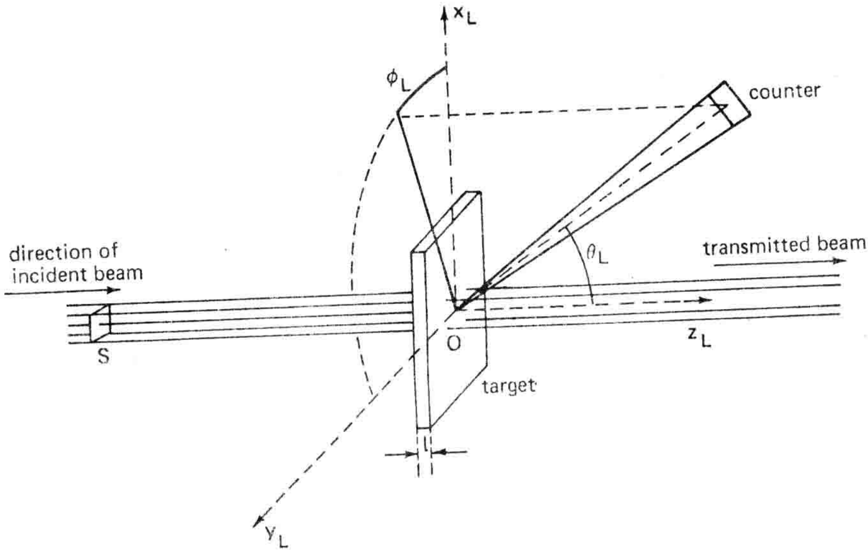

Fig. 1.2. Illustration of various quantities used in the definition of cross sections.

Let us now denote by N_{tot} the total number of particles A which have interacted per unit time with target scatterers. Under the experimental conditions assumed here, the quantity N_{tot} is directly proportional to the relative incident flux Φ_A and the number n_B of target scatterers. We may then write

$$N_{tot} = \Phi_A n_B \sigma_{tot}. \tag{1.11}$$

According to the definition given above, the quantity σ_{tot} is called the *total cross section* for scattering of the particle A by the particle B. We note from eq. (1.11) that σ_{tot} has the dimension of an area. We also remark that if we define

$$P_{tot} = N_{tot}/N_A \tag{1.12}$$

as the total probability that an incident particle has interacted with a target scatterer and has therefore been removed from the incident beam, then by using eqs. (1.9)–(1.11) we have

$$P_{tot} = \hat{\mathcal{N}}_B \sigma_{tot}. \tag{1.13}$$

We emphasize that the definition (1.11) of σ_{tot} only holds for the case of a thin target (such that $P_{tot} \ll 1$). Corrections must be applied when this condition is not satisfied [9].

It is important to note that in contrast with P_{tot} which depends upon various experimental parameters (such as the values of N_A, $\mathcal{N}_B$, $\hat{\mathcal{N}}_B$, l), the

References and notes on page 11