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AN INTRODUCTION

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Centre de Recherches Nucléaires
Université Louis Pasteur, Strasbourg

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Preface

In the last fifteen years the attempts at a unified description of the fundamental physical interactions by gauge field theories have given rise to exciting developments in particle physics.

In this book, which grew out of lectures I gave in the last few years in several places, at Strasbourg University, at the 1980 XIV Curso Centro Americano de Fisica, held at the University of Panama, at the Federal University of Rio de Janeiro and the Centro Brasileiro de Pesquisas Fisicas, I try to explain in an elementary way the basic notions and principles of gauge theories. In particular, the Salam-Weinberg model of electro-weak interactions is developed in some detail including its verification in the study of neutrino-lepton scattering and the parton model. This model is at present the most successful attempt at a unified theory of physical interactions.

The aim of this book is to give a self-contained introduction to these theories.

The reader will be assumed to know basic quantum mechanics and special relativity together with the elements of group theory needed for these disciplines ; a knowledge of the qualitative description of elementary particles and their quantum numbers will also be required, as well as the elements of the Feynman diagrams technique.

The first Chapter contains the basic notions of classical field theory and the all important Noether's theorem. An introduction is also given to solitons and instantons and the topological quantum numbers, subjects which arose from the study of the non-linear field equations in gauge theories and which have been developed in the recent few years.

Besides the study of the electromagnetic and the Yang-Mills gauge fields, a chapter on the gravitational field is included. We think that this chapter is of interest for two reasons : firstly, it may be suggestive for the graduate students to learn that there are several common features between this and the Yang-Mills field-non-linear equations, similar covariance behaviour of certain quantities such as the gauge field and its source, under the corresponding groups. Secondly, it is precisely the unification of gravitational with strong, electromagnetic and weak interactions, which present the greatest challenge to theoreticians nowadays. It would be stimulating that the young readers acquire a basic knowledge of the situation for each gauge field, gravity included.

Perturbation calculations, renormalization and path-integral quantization are not studied in this book. Two excellent books on this subject which were recently published, one by C. Nash, the other by J.C. Taylor, are indicated in the bibliography ; they fully develop the basic ideas and techniques in this domain. The reader is invited to consult excellent reports and review articles mentioned in the bibliography.

A section in Chapter IX deals with very recent speculations on possible lepton and quark structures, for which there is so far no experimental evidence. An introduction to the SU(5) model of grand unification is presented in Chapter X. Problems are given for each chapter and solutions are collected at the end of the book.

I am most grateful to Abdus Salam, Director of the International Centre for Theoretical Physics, for sponsoring my lectures in Panama and to Mario Bunge for his support and encouragement ; my best thanks are also due to B. Fernandez and his colleagues of the University of Panama, to R. Lobo, E. Lerner and their colleagues of the Centro Brasileiro de Pesquisas Fisicas and of the University of Rio de Janeiro, respectively for the humanly warm and kind hospitality. The author greatly profited from conversations with J.J. Giambiagi, Ch. Ragiadakos, C.A. Savoy, J.A. Martins Simões and D. Spehler on topics of this book.

I am deeply grateful to the authors and to the Nobel Foundation for their kind permission to reprint the lectures given by the 1979 Nobel Laureates Sheldon L. Glashow, Abdus Salam and Steven Weinberg. I am equally grateful to the Physical Society of Japan and to the author for kindly permitting the reproduction of the Table IX from the article by C. Baltay in the Proceedings of the 19th International Conference on High Energy Physics, 885-903, Tokyo (1978).

Madame Erice North prepared the typescript with great ability and patience, my warmest thanks go to her.

J. LEITE LOPES

Strasbourg, January 1981

Contents

Introduction

1. - You all know that the philosophical dream of physicists has always been to reduce (and thus "explain") the enormous varieties of material bodies and events in nature to configurations of a small number of basic constituents and their interactions -the Greek atoms, the atoms and molecules of the chemistry and physics of the XVII th century, the elementary particles of the last fifty years, the quarks, leptons and fundamental bosons of today.

The ninety two elements of the Mendelejev table were explained in terms of three particles, the electron and the proton and neutron ; these, together with the photons, responsible for the electromagnetic interaction among electrons and nuclei, were the primordial objects of the physicists around 1934.

The later discovery of pions, postulated by Yukawa in 1935, to describe the nucleon interactions, and then of muons and neutrinos, of strange particles and ressonances, seemed to suggest that the underlying reality of fundamental particles was perhaps too rich to be possibly reduced to a small number of objects.The number of supposed elementary particles soon became at least as large as the number of elements in the Mendelejev table

2. - On the other hand the idea that physical forces propagate in space with a finite velocity through the action of a field was introduced by Maxwell and Lorentz in electrodynamics. This idea was further developed by Einstein and in his relativistic theory of gravitation -perhaps the most beautiful achievement in theoretical physics up to our days- the unifying power of the description by the field concept was greatly enhanced, the gravitational field being identified with the metric tensor in a Riemannian space-time.

With the development of quantum mechanics and of the principles of quantum field theory, physicists were led to associate a field to each particle. However, the large number of elementary particles which were discovered in the fifties discouraged many physicists in their belief of the unifying rôle of field theory. The efforts developed by Einstein to find a unitary theory of the gravitational and the electromagnetic field, seemed meaningless to quantum and particle physicists since many other fields would have to be taken into account in such a unifying theory. It was mainly in the domain of strong interaction physics that the notion of field seemed useless.

The developments in the last ten years which culminated with the Salam-Weinberg model of gauge fields which unify electromagnetic and weak interactions and, more recently, the discovery of quantum chromodynamics, restored the full theoretical value of field theory. It is believed that the unification which we must seek is rather that of the basic forces of nature, rather than of the bodies and their constituents. The elementary particles are now reduced to leptons and quarks but the number of these admitted basic objects seems to be increasing. Instead, the Salam-Weinberg model opened up a new style and a new aim, in the spirit of the great unification of physical fields as dreamed of by Einstein. Strong interactions are now assumed to be described by massless vector gauge fields associated with the colour degrees of freedom of quarks. And this theory is expected to reproduce the strong interactions between hadrons, although for the moment mathematical difficulties have been preventing an early completion of this program.

Current research actively develops efforts in the sense of a "grand" unification of all the basic interactions of nature, such as the Georgi-Glashow SU(5) model which attempts to unify the strong, weak and electromagnetic interactions.

3. - We know that the elementary particles are classified according to their spin into bosons -particles with integral spin, obeying therefore to the Bose-Einstein statistics- and fermions particles with half-integral spin and which obey the Fermi-Dirac statistics.

I will show you now a table (Table I) which indicates the basic physical interactions between particles. These are in small number : all forces in nature result from the interplay of : 1) gravitational interactions which are created by and act upon all forms of energy and matter ; 2) weak interactions, which act between leptons (electrons, muons, tauons and their neutrinos) and also hadrons ; 3) electromagnetic interactions, created by and acting upon all particles with a charge, a dipole moment ; 4) strong interactions, which act only on heavy particles called hadrons. As we said above, it is today believed that these interactions may be described in a unified way : massless vector fields -the gauge fields- are defined in association with the postulated invariance of the theory under gauge transformation and these fields give rise after a spontaneous break-down of the symmetry, to the fields of the weak and electromagnetic interactions. The strong interactions are assumed to be governed by another gauge theory with unbroken symmetry, the colour SU(3) gauge group, which introduces eight massless gauge fields, the gluons.

TABLE I - Basic interactions

Interactions	strength of coupling constant	Transmitted by	Gauge fields	
Gravitation	$\mathcal{G}\frac{m_e^2}{e^2} \sim 0.2\times10^{-42}$	spin 2 massless field quantum : graviton	general coordin. transform gauge field	
Weak	$G_F\frac{(m_p c)^2}{\hbar^3} \sim 1.01\times10^{-5}$	spin 1 massive fields quanta : W^+, W^-, Z^0	$SU(2) \otimes U(1)$ gauge fields	The SU(5) model defines 24 gauge fields
Electromagnetic	$\alpha = \frac{e^2}{4\pi\hbar c} \sim \frac{1}{137} \sim 10^{-2}$	spin 1 massless field quantum : photon		
Strong	$\frac{g^2}{4\pi\hbar c} \sim 10$ for hadronic. matter Momentum transfer dependent $\alpha_s(Q^2)$ for quark interaction	spin 1 massless fields gluons	colour SU(3) gauge fields	

Supergravity postulates a massless spin 3/2 quantum -the gravitino- in addition to the graviton

TABLE II - Observed fermions

	Display	observed
Leptons	weak and electromagnetic forces	$e^-, \nu_e; \mu^-, \nu_\mu; \tau^-, \nu_\tau$ and their antiparticles (spin 1/2)
Baryons	weak, electromagnetic and strong forces	nucleons;hyperons; baryonic resonances (spin 1/2, 3/2,...)

TABLE III - Observed bosons (1980)

Photons	
Hadronic Mesons	π, ρ, κ, ϕ, D, ψ, Υ, etc.

TABLE IV - Quark quantum numbers

Flavours	Q	I	I_3	B	Y	S	C	b	t(?)
u_a	2/3	1/2	1/2	1/3	1/3	0	0	0	0
d_a	- 1/3	1/2	- 1/2	1/3	1/3	0	0	0	0
s_a	- 1/3	0	0	1/3	- 2/3	- 1	0	0	0
c_a	2/3	0	0	1/3	- 2/3	0	1	0	0
t_a ?	2/3	0	0	1/3	- 2/3	0	0	0	1
b_a	- 1/3	0	0	1/3	- 2/3	0	0	- 1	0
⋮(?)									

Q = charge
I = isospin
I_3 = third component of isospin
B = baryon number
Y = hypercharge
S = strangeness
C = charm
b = beauty (bottom-ology ; bottomness ?)
t = top-ology (topness ?)

$Y = B + S - C + b - t$

$Q = I_3 + \frac{1}{2}(B + S + C + b + t)$

Each quark is assumed to exist in three states which differ among themselves only by a new quantum number, the colour a = 1,2,3

TABLE V - Lepton quantum numbers

	L_e	L_μ	L_τ
ν_μ	0	1	0
μ^-	0	1	
ν_e	1	0	
e	1	0	
$\bar{\nu}_\mu$	0	-1	
μ^+	0	-1	
$\bar{\nu}e$	-1	0	
e^+	-1	0	
ν_τ	0	0	1
τ	0	0	1
τ^+	0	0	-1
$\bar{\nu}_\tau$	0	0	-1

L_a : a-onic lepton quantum number