elementary theory of structural strength

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to my wife, Elsie,
and
to our daughters, Misty and Lisa—
here is what I have to show
for those countless hours of togetherness
we have done without

PREFACE

Writing yet another treatise on a subject well covered by a plethora of books is somewhat like putting out still another body style of automobile amid an already staggering array of choices. Undoubtedly, books are written for many different reasons; a common one is probably that of the author being dissatisfied with certain aspects of existing works—this, coupled with a conviction that he has something to contribute.

This book is in part an outgrowth of a nagging dissatisfaction with the treatment of the subject variously known as strength of materials, mechanics of materials, resistance of materials. This discontent has led to the development of an approach to the subject which is hereby offered for consideration as a small contribution to the teaching of strength theory.

I like the word "strength" for its terse cogency and have used it freely; indeed, it forms the central theme of the book. My dissatisfaction comes from the fact that strength has not been used as meaningfully as I feel its pithiness suggests it could be used. Worse than leaving the meaning implicit has been the actual misuse of the term. Example: When an author writes of "a beam of uniform strength," he means really a beam of variable strength which is everywhere stressed to the same maximum. Within the limitations of language, and of my own command of it, I have tried to define strength and have used it in that sense. In the hope of instilling in the student a proper appreciation of the strength concept, I have introduced the notion of utilization factors and have touched upon optimization.

I have deemphasized statical indeterminateness and stressed constraint redundancy instead. I believe that constraint redundancy is *very* germane to strength and have treated it accordingly. Statical indeterminateness (I have used it, but most sparingly) would certainly be appropriate for a chapter heading in a book entitled, say, "How to Solve Problems in Structural Mechanics." The usual treatment of this topic has been largely unsatisfactory to me for the one reason that the physical significance of redundancy has seldom been brought out, if at all. I have long had the feeling that statical indeterminateness has provided little more than a convenient excuse for demonstrating the niceties of certain mathematical techniques. It is really a simple matter to comment, even only as an afterthought, on why indeterminate structures are *purposely* made that way.

Equally as basic a reason as the preceding, for my soft-pedaling statical indeterminateness, is that of consistency. I have not classed multimaterial tension and compression members and torsion members as indeterminate, just as I have not multimaterial beams (they never are), because they are no more indeterminate than a simple tension member. As soon as we consider the distribution of internal forces, we have indeterminateness. Hence we should not assume, for instance, that one-half of a tension bar's cross section transmits one-half of the tension load, unless we *explicitly* point out that this *assumption* is needed to break the deadlock of indeterminateness.

At the risk of having them loom as a stumbling block, I have introduced in Chapter 2, as part of the background material, the fundamental differential equations (only in cartesian form) of equilibrium and compatibility. I see no other way out if the student is to be made aware of (1) what is the problem, when are solutions possible, and when are possible solutions correct, and (2) what are the bases of strength theory? This also explains why I have chosen to present the general statement of Hooke's law first, from which are then derived the special cases. Of the several topics in this chapter, I ask the reader to pay special attention to the discussion of signs. Modesty aside and ignorance confessed, I must say that I am not aware of any previous exposition of this dichotomy in the sign conventions and of why signs are so often a source of difficulty.

In Chapters 3 to 8, the one item that probably deserves special mention is the singularity function. Beam loading presents such a good opportunity for introducing the technique of handling stepwise continuous functions that it would be a shame not to use it. Furthermore, this treatment of the subject paves the way and furnishes as good a motivation as any, especially to nonelectrical engineers, for the later limit design is a natural. For the last topic, the riveted connection is used to illustrate limit analysis; I have long believed that this is the only logical way of presenting this subject.

It is a foregone conclusion that not everybody will be pleased. No doubt some will miss temperature effects; others, stress concentration, fatigue, and dynamic effects. I can only say that I have had to omit some topics in order that I might do justice to the exposition of an introductory viewpoint: the concept of optimum load-carrying capacity.

In these days of the feverish rush to get on the science bandwagon, when it seems the fashion to subserve engineering to science, some will look upon this work as a needless manicuring of an old viewpoint. Of course, I am prejudiced when I say that to me it represents a great deal more than this—in a small way I have tried here, as others have been trying elsewhere, again to draw attention to the engineer's sense of values and way of thinking, to the vitally necessary art of making meaningful assumptions and extrapolations.

This book, which presupposes a solid background in statics, would probably appear too lengthy for a three-hour course; but this is only because I have not spared words in explaining fundamental concepts. Paucity of words is not necessarily a virtue in a textbook, and I am asking the student to read many paragraphs uninterrupted by equations (he should do more of this, anyway). If desired, Chapters 10 to 13 may be skipped for a brief course.

Some of the problems are much more than mere exercises; they form an extension of the theory. The answers to most of the even-numbered ones are given at the end of the book. Experience has shown that nothing is more frustrating to a student, in trying to develop facility at this one of the many roles he is expected to play (that of being a computist), than not having any kind of guide or yardstick by which to gage whether or not he is in the "right ball park."

The influence of the many treatises on the subject is obvious—this book owes its very existence to them. I have learned much from my teachers, in particular, the late J. A. Van den Broek, of whose sense of values I hope I have retained a measure; for this I am truly grateful. My debt to many students in more than twenty years of teaching is a very real one: from their fresh viewpoints and unintentionally probing questions I have profited. Among several I cite John E. Edinger, Union, 1960, and Martin P. Einert, Union, 1961, for the many constructive comments they made on the class notes which were the rough draft of this book.

I acknowledge with appreciation the steadfast encouragement and moral support of my colleagues at Union College: Professors Joseph

study of the Laplace transformation, now fast becoming standard in required mathematics courses.

In regard to columns, two things have long impressed me. One is the artificial, therefore needless, separation of the centrally loaded from the eccentrically loaded member; the other is the near absolute silence on the most important one among the salient aspects of column behavior, namely, that a column is most sensitive to errors of one kind or another in the vicinity of the critical slenderness and becomes less so away from this region. It is inevitable therefore (1) that the centrally loaded column is presented as a special case of the eccentrically loaded member, and later it is remarked that the eccentrically loaded member may just as validly be considered a special case of a centrally loaded column; and (2) that empiricism in column design is discussed.

The variation of stress at a point comes late in the book. I am convinced that this variation should be presented only once, but fully and correctly. To confine the discussion to the one family of planes in two-dimensional problems could be misleading. Example: To tell the student that, when the two nonzero principal stresses have the same sign, the maximum shear stress is one-half of their difference, is a disservice to him because this is incorrect. Somehow, it is never easy for the student to reconcile the two ideas: that, in dealing with two-dimensional stress, one need not confine the discussion to two dimensions only.

Elegant as tensor notation is, I am not quite ready to say that it properly belongs here where the emphasis has been on strength. If, however, we were studying the mechanics of continua, then, certainly, the tensor should be used right from the start, beginning with the cartesian, to provide background, and building up to the general tensor. Nevertheless, some of the essential characteristics of cartesian tensors are pointed out; in fact, on the basis of the properties that are common to the strain and the stress tensors, the transformation equations for strain are derived from those previously obtained for stress.

As an alternative and powerful method of handling deflections, Castigliano's theorem is discussed. The sequence of presentation is not the usual one in that reciprocity of deflections is taken up first, in quite a general form, and then, as a concomitant, Castigliano's second theorem is brought out. The significance of what is commonly referred to as the Theorem of Least Work is explained from the supplementary view of differentiation relative to independent variables.

In keeping with the emphasis on strength, I have treated the role of ductility in one separate and final chapter. Once again, the significance of constraint redundancy is brought out, this time coupled with the added benefits of material ductility. And, of course, an introduction to

Modrey, Gardner M. Ketchum, and Raymond Eisenstadt, and Anthony Hoadley who read portions of the manuscript. The grant by the Trustees of Union College, through President Carter Davidson, of a sabbatical leave is sincerely appreciated; without that break this book would have been further delayed. I also want to express my gratitude to Carole Walck for typing the bulk of an earlier version of the manuscript for class use, to my former students Paul Snyder and Robert Marquez for preparing many of the sketches for reproduction, and to Dr. J. H. Smith of the Knolls Atomic Power Laboratory for a number of suggestions. Special thanks are due to Professors Lucien A. Schmit, Jr., of Case Institute of Technology and Samuel T. Carpenter of Swarthmore College for their critical reviews of the final manuscript, and to John Wiley and Sons, Inc., for special considerations during production of the book. Finally, I sincerely thank my wife, to whom this work is dedicated, for her monumental patience (many times sorely tried) at my being such a slow worker.

No effort has been spared to see to it that there are no errors as to both content and typography; for those that surely must have escaped hours of proofreading, I cheerfully assume sole responsibility. May I hope that they will be called to my attention by discerning readers.

FILADELFO PANLILIO

Schenectady, New York
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CHAPTER

1

INTRODUCTION

This book is concerned with the subject of structural strength, which in its widest sense means the strength of entire structures such as buildings, bridges, automobile frames, spacecraft frames. In the more limited scope which is both desirable and proper for an introductory text, it is confined to a theory of strength of the elements which constitute a structure. To gain some perspective and develop a proper appreciation of the nature of the subject, the student is invited to take a look back at some aspects of the background material.

1.1 Analysis and Design. In *statics*, the subject that is basic and preparatory to this one, there is a group of problems on frames and trusses that require determination of tension and compression forces in the component parts of these structures induced by specified loads. One important assumption tacitly made is that the dimensional changes caused by the loads are so slight as to be negligible. Furthermore, it is generally implied that the component parts possess the necessary load-carrying capacities which enable the entire structure to withstand the given loads. The process involved in the solution of such problems is one of *analysis*. If the problems were so changed as to necessitate calculation of the proper sizes for the component parts in order that the structure as a whole could safely carry the given loads, then the process involved in their solution would be one of *design*.

Analysis and design are inverse processes. To analyze a structure means to solve either one of two problems: (1) from given or assumed dimensions, forms, and material properties, to determine the internal forces and accompanying dimensional changes produced by given or assumed loads—a process that may be described as a *force-and-deformation analysis;* (2) from given or assumed dimensions, forms, material properties, and prescribed limitations on the induced internal forces and dimensional changes, to determine the load-carrying capacity of the structure—a process that may be called a *strength analysis.*

On the other hand, to design a structure is to determine, from given or assumed loads and major dimensional limitations, the appropriate sizes and forms of the component parts (sometimes also the proper material to use when there are two or more materials to choose from) in order that the structure will function effectively, safely, and economically.

Let us consider, for example, the horizontal bar supporting a load as represented in Fig. 1.1. If all the dimensions of the bar, the kind of material it is made of, and the magnitude of the load W are assumed known, the maximum deflection or sag can be found in a deformation analysis. If it were desired to calculate the maximum value that W may have without the deflection exceeding a given amount, then the answer would follow from a strength analysis. Finally, if, say, the cross-sectional dimensions of the bar are not fixed and we would answer such questions as what these dimensions should be and which is the most economical of several available cross sections so that, under the given load, the maximum deflection shall not exceed a prescribed amount, then we would have a problem in design.

One of the most important objectives of the engineer is to design structures and machines for the prospective benefit of his fellowmen. In order to achieve his goal, he must necessarily be conversant with methods of analysis. An analysis may precede a design as a preliminary step; another analysis may follow the design as a final step. The intermediate step, however, which is the solution of the design problem proper, requires something else in addition to analysis. This means that a knowledge of analysis alone is not adequate, because separating analysis and design is a gap that can be bridged effectively only by a strength theory.

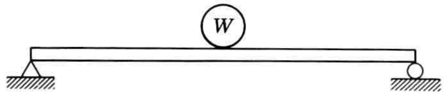

Fig. 1.1