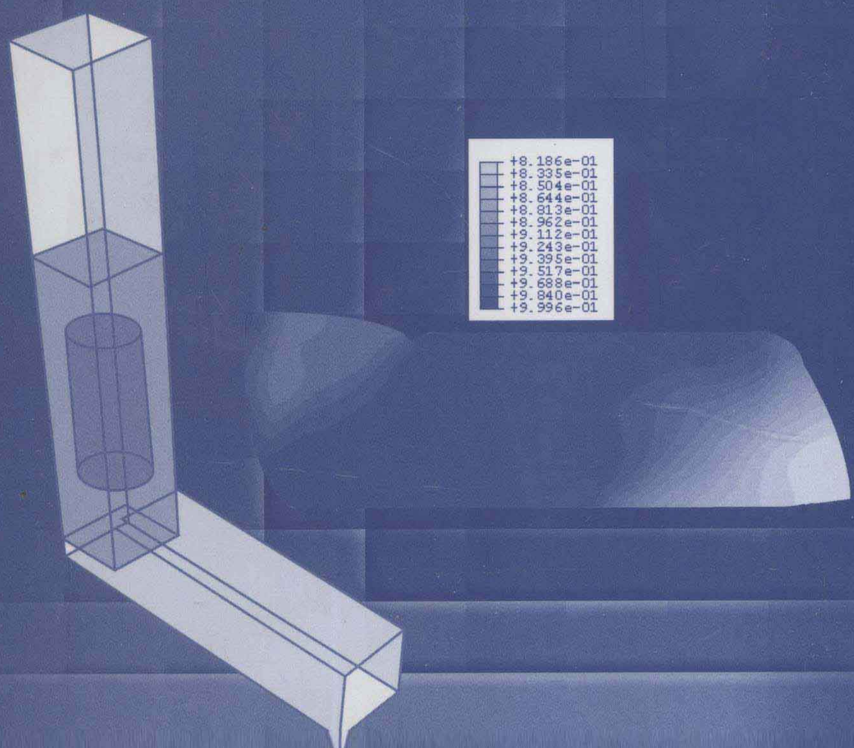
Series on Advances in Mathematics for Applied Sciences – Vol. 80

INELASTICITY OF MATERIALS

An Engineering Approach and a Practical Guide

Arun R Srinivasa
Sivakumar M Srinivasan



Series on Advances in Mathematics for Applied Sciences – Vol. 80

INELASTICITY OF MATERIALS

An Engineering Approach and a Practical Guide

Arun R Srinivasa
Texas A&M University, USA

Sivakumar M Srinivasan
Indian Institute of Technology, Madras, India

iitmadras
CELEBRATING TECHNOLOGY
Golden Jubilee 2008-09

World Scientific

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.
5 Toh Tuck Link, Singapore 596224
USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601
UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

Library of Congress Cataloging-in-Publication Data
Srinivasa, Arun R. (Arun Ramaswamy)
Inelasticity of materials : an engineering approach and a practical guide / by Arun R Srinivasa & Srinivasan M. Sivakumar.
p. cm. -- (Series on advances in mathematics for applied sciences ; v. 80)
Includes bibliographical references and index.
ISBN-13: 978-981-283-749-3 (hardcover : alk. paper)
ISBN-10: 981-283-749-3 (hardcover : alk. paper)
1. Hardness--Mathematical models. 2. Brittleness--Mathematical models. 3. Plasticity--Mathematical models. 4. Materials--Testing--Computer simulation. 5. Strains and stresses.
I. Sivakumar, Srinivasan M. II. Title.
TA418.42.S75 2009
620.1'126--dc22

2009008937

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library.

Printed in Singapore by World Scientific Printers

INELASTICITY OF MATERIALS

An Engineering Approach and a Practical Guide

Sri Ramajayam.

This book is dedicated:

to my father Prathivadibhayankaram Srinivasa Ramaswamy Iyengar, my mother Saroja, my wife Prabha, my children Vishnu and Divya, and to my Ph.D. advisor Prof. P.M. Naghdi .

–Arun Srinivasa

to my dad, Panchanathan Srinivasan as he turns 80 and, my mom, Pattammal to my family, Leela, Satchit and Aniketh, and to my friend Dr. Dilip Veeraraghavan.

–Sivakumar Srinivasan

We also dedicate this book to our friend and mentor Prof. K.R. Rajagopal.

Foreword

It is a pleasure, as well as a responsibility, to write a foreword[1] to a book co-authored by two colleagues, when one has worked very closely with one of them for over a decade and interacted with the other for over two decades, lest one loses objectivity while penning it. As good luck would have it, the authors have made my task all the easier by writing a book that approaches the subject matter in an inimitable manner. The book is written as though one would deliver the material in class; the informality with which the subject matter is introduced is a welcome change to the sterile and formal pedagogy that one finds in most texts. Also, the emphasis of the book is towards educating the student by educing that which is inherent rather than that of teaching. The idea of presenting the material in a question and answer format makes the material very readable and easy to understand, the motivation behind many of the concepts and procedures are then well motivated and rendered transparent. Ernest Rutherford is supposed to have said that if one truly understands the material, one should be able to explain it to a barmaid. While the authors are not the sorts that hold conversations with bar maids, they seem to have taken Rutherford's comments to heart. The book is accessible to anyone that has taken an undergraduate course in mechanics of materials.

A very interesting feature of the book is its intended audience. One could cull material to offer a regular senior level undergraduate course, honors level undergraduate course, a graduate course in plasticity theory, and it also offers insights into the foundations of the field that will be both new and appealing to the experts in the field. The healthy balance that has been achieved between the theoretical and numerical and computational material, between worked out examples and home work problems, is commendable. The student is led through the complex maze of plasticity is small steps so that he can gain a proper understanding of the underpinnings of the subject. Whether it is a lumped parameter approach to one dimensional problems in inelasticity, small stain inelasticity or inelasticity associated with finite deformation, the authors are able to provide a clear and rationale thermodynamic basis. The authors appeal to the maximization of the rate of entropy production,

[1] According to Fowler [Fowler (1926)], the word foreword is to be preferred to preface as the former is of Anglo-Saxon origin while the latter is of Latin origin.

an approach that has proved very fruitful in describing the response of a plethora of bodies that produce entropy in disparate manner. The book is peppered with exercises with hints as to how to solve them. The exercises have been carefully selected so that the student will obtain useful, interesting and illuminating insights into subtle and interesting issues concerning inelasticity.

The initial material concerning one-dimensional response appealing to springs and dashpots provides the basis for the more abstract mathematical setting for the three dimensional response of inelastic bodies within the context of a sound thermo-mechanical framework. The initial lumped parameter model of a spring accounts for how the material stores energy and the dashpot how dissipates energy. The lumped parameter modeling places the main ideas concerning inelasticity within the grasp of a senior undergraduate student or a beginning graduate student. This early treatment of inelasticity is followed by a detailed and careful discussion of small strain plasticity. The reader is provided a reasonably thorough account of the historical development of small deformation plasticity. The authors start with a discussion of rigid-perfectly plastic bodies and then provide a reasonably exhaustive treatment of the behavior of elastic-plastic bodies. Plane strain and plane stress problems are discussed in great detail and the student is made aware of how the classical Airy's stress function can be put to use gainfully to study plane stress and plane strain problems for inelastic bodies. Numerical solutions are discussed which can be used to solve problems in more complex geometries. The final three chapters in the book are devoted to advanced topics in plasticity; a general introduction to inelasticity involving large deformations, a succinct treatment of crystal plasticity and three case studies devoted to shot peening, equal channel angle extrusion, and aging tissues. The final chapter devoted to case studies takes the reader to the very boundaries of the subject, showing them how to tackle complicated problems that have important technological ramifications. A person that has mastered the contents of the book will have no difficulty in tackling important open problem in inelasticity.

I found the book to be most instructive and unhesitatingly recommend it to my colleagues to use it as a text book for a senior undergraduate level or beginning graduate level course by restricting the course to the first two parts of the book, or to an advanced graduate course by mainly covering the final part of the book, or as a source of reference to one carrying out research in the field of inelasticity.

-K. R. Rajagopal
Distinguished Professor, Regents Professor,
Forsyth Chair in Mechanical Engineering,
Professor of Mathematics,
Professor of Biomedical Engineering,
Professor of Civil Engineering,
Professor of Chemical Engineering,
Texas A&M University
College Station, Texas, 77843, USA.

About the book...

The two of us have known each other since our days as undergraduate students at IIT, Madras and staying in the same dorm or Hostel—Godavari. We have collaborated with each other over many years and have shared a passion for teaching. This book is an outgrowth of teaching courses on plasticity at Texas A&M University, at the University of Pittsburgh and at the Indian Institute of Technology, Madras with both of us being quite dissatisfied with what we have taught. Both of us found plasticity to be fascinating in so many ways and have used it as a doorway to other theories, but we found that students were curiously unmoved by the subject. Not surprisingly we blamed it on the students and the material—how could anything be wrong with the way we teach.

But we did notice a curious thing—students were always fascinated by in class demonstrations and the application of plasticity to engineering tasks. Then, one summer, we decided to do a class lecture on thermodynamics and its applications—the students' interest was intense and they peppered us with questions. They had taken thermodynamics as undergraduates but had never really understood its place in the world—but they had had enough glimpses of its wide reach to really pique their curiosity; and here was a speaker talking to them about the thermodynamics of hurricanes and thunderstorms!

In talking things over, it dawned on us that our presentation style was at fault—we spent way too much time lecturing and not enough time challenging students and helping them discover the beauty of the subject. The difference is like that between a museum lecture on Egyptian artifacts where each item is brought before us and its relevance explained and actually visiting Egypt and seeing the artifacts in their original setting.

This book is a result of our intense discussions on how to present this matter to the students. After many false starts, we have finally managed to arrive at a consensus on how this book was to be written. We have written it in an informal style (like a conversation) with jokes thrown in[2]. We also agreed that it has to be a textbook and not a monograph and that it had to be engaging. We do not want it

[2] so don't be shocked by an occasional smiley face ":-))" in the text

to meet the fate of the book which was "so interesting that once you put it down, it was hard to pick up again"—a fate that is all too common to monographs.

Thus it is written more like a workbook. The text is frequently interrupted by exercises, and homework, just like what you would do in a class. We frequently make use of web research to help students discover some of the remarkable applications of plasticity. YouTube^{TM} is a great resource to help students see videos and animations and have served as an invaluable complement to the material in the book. Most of the chapters start with an engineering scenario where you would have to make a decision, and you have to use your knowledge of inelasticity to help you make the decision.

One of the major new features in the book is the extensive use of programming exercises to help students model realistic material behavior. We make extensive use of MATLAB'S ODE suite to solve many homogeneous deformation problems as well as other numerical techniques. In our opinion, this has been one of the features that has to be sorely lacking in many plasticity texts (since analytical solutions are all but impossible other than for a few special and simple cases).

Thus, the book has homework exercises and lots of programming exercises. In our classes, we have found that these programming exercises help students anchor the concepts firmly in their mind since it provides us a way to give instant feedback to the students.

We have also provided a number of case studies that can be used by students and instructors to see actual applications of the theory in real situations. Also, based on student feedback we have taken pains to ensure that there is not a heavy dependence of chapters on each other. The notable exceptions are Chapter 2 and Chapter 6—they form the foundations of the book.

Organization of the book

This book is written in a modular fashion, which provides adequate flexibility for adaptation in classes that cater to different audiences such as senior-level students, graduate students, research scholars, and practicing engineers. A graphical look at the set of chapters and their organization is given in Fig. 0.1.

Acquiring the capacity to model inelastic behavior and to choose the right model in a commercial analysis software has become a pressing need for practicing engineers with the advent of a host of new materials ranging from shape memory alloys to bio-materials to multiphase alloys. Even with the traditional materials, there is a continued emphasis on optimizing and extending their full range of capability in the applications. This book builds upon the existing knowledge of elasticity and thermodynamics, and allows the reader to gain confidence in extending ones skills in understanding and analyzing problems in inelasticity. By reading this book and working through the assigned exercises, you, the reader will gain a level of comfort and competence in developing and using inelasticity models.

This book is written in three parts:

Part I, the first part, is primarily focused on lumped parameter models and simple structural elements such as trusses and beams. In *Chapter 1*, the subject is introduced through discussing the appropriate sample scenarios in which inelastic modeling and analysis plays an important role. Further, in this chapter, formulation of the elastoplastic equations, together with the different spring and dashpot elements are introduced from a purely mechanistic approach directly exploiting an analogy. These analogies are meant to replicate, in a purely mechanical manner, the stress-strain response of many types of inelastic materials.

Chapter 2 on lumped parameter systems forms the most important chapter of this part which sets the tone of the book. The lumped parameter models considered as "zero" dimensional models use primarily springs, dashpots and masses, and the appropriate set of variables to define the state of the material including temperature to derive response equations. Both mechanistic and energetic approaches are used to demonstrate how governing equations for the inelastic behavior can be obtained. The maximum rate of dissipation hypothesis is introduced here.

The reader is seamlessly taken to a scenario of solving one-dimensional problems in the next chapter (*Chapter 3*) where a one-dimensional structure is solved for quantities appropriate to inelasticity. This chapter discusses how constitutive equations can be formulated for materials with hardening and how one can use the existing knowledge of the "accumulated" plastic strain calculated from the model for predicting the fatigue life of a one-dimensional component. Examples have been introduced along with MATLAB codes to provide the reader with a chance to try out examples.

More often than not, the typical structural element that an engineer encounters or considers in bearing mechanical loads are the beam elements. *Chapter 4* is dedicated to this structural action (beam action) for this reason. The reader is taken through a complete exercise of either modeling an existing beam or designing a beam for dimensions for inelastic deformations. An algorithmic approach to solving these one dimensional boundary value problems is given along with the appropriate MATLAB codes.

Two more solved real life examples have been introduced in *Chapter 5*. One is that of a crane girder made of a truss system. The other is that of a seismic damper frame that uses shape memory alloy cables for damping. The idealization of the systems, the analysis and modeling exercises are carried out and solved for appropriate quantities needed for design and understanding.

These essentially form the chapters in Part I dealing mainly with lumped systems and simple structural elements.

Part II of this book focuses on small deformation multi-dimensional inelasticity. Sufficient material is included on how to numerically implement an inelastic model and solve either using a simple stress function type of approach or using commercial software. Simple examples are included. There is also an extensive discussion of thermodynamics in the context of small deformations.

The simplest and most commonly used among the plasticity models are the J2 plasticity models. With a view on dealing with this model more thoroughly, *Chapter 6* provides a complete idea of using J2 plasticity model. In this chapter, the reader is taken through the history of the development of plasticity models that have led to the present prominence and popularity of the J2 plasticity models. The chapter explains, to start with, how to arrive at the governing set of equations for a rigid plastic model in this class of J2 plasticity models. Then, the model is relaxed to introduce elasticity into the model. This makes it suitable for structural applications. To make it more realistic hardening should be added to the model. A nice definition of hardening for the three dimensions is introduced subsequently before presenting the process of modeling for the same as extension from a one-dimensional model. The chapter ends with examples of different loading histories after explaining the procedure obtaining the tangent moduli of the material.

Now that the reader is ready for solving boundary value problems involving multi-dimensional inelasticity, *Chapter 7* deals with formulating and solving simple boundary value problems, especially, the two-dimensional problems. It is shown in the chapter how to derive an analogical set of equations of inelasticity that look similar to the biharmonic operators that are popular in isotropic small deformation elasticity. Airy stress functions are introduced, shown how they can be obtained for a set of cases before attempting to solve the equations.

J2 plasticity is not application in many applications and one has to resort to other yield surfaces and conditions. *Chapter 8* explains how the concepts learnt so far can be extended to other yield conditions and behaviors such as pressure dependency, anisotropic moduli and anisotropic yield.

Thus far, there remain many questions unanswered in a curious reader's mind in relation to the thermodynamic principles adopted and their basis. An entire chapter (*Chapter 9*) is devoted to explaining the thermodynamic principles involved in modeling these materials. This chapter is fundamentally different from the others in the book since it is more philosophical in nature and presents a point of view as to how one could unify the constitutive relations for different dissipative materials using a common thermodynamical framework. The approach presented in this chapter is more explanatory than procedural.

In general, in spite of such simplifications, the resulting boundary value problem may be complex for a simple analytical approach to work in solving it. Over simplification may lead to unreliable results. The direct numerical simulation comes to the rescue. Simulation of a plastic deformation process in a realistic engineering application such as components undergoing collapse and fatigue or a metal forming process, gives insight into the mechanics that occurs in the appropriate process and helps make design decisions. In *Chapter 10*, the authors walk you through a process of numerically solving the governing equations of a IBVP for the simulation of a general small deformation application. Simple examples are solved to elucidate the effectiveness of this solution. A meshless modeling exercise is introduced to give wider possibilities for simulation.

Part III: More advanced situations such as finite deformation inelasticity, thermodynamical ideas and crystal plasticity are dealt with in Part III. Some advanced case studies are also included.

In order to model finite elastoplastic deformation, either one has to be very familiar with the notation and terminology. *Chapter 11* provides a quick refresher on the concepts in continuum mechanics apart from an overview of the notations and terminologies used. The chapter is introduced essentially for completeness and is not a substitute of a continuum mechanics course. Specifically, the treatment is from the perspective of understanding the concepts introduced in later chapters on finite deformations focusing only on basic foundations for such discussions.

Chapter 12 discusses on finite deformation inelasticity for isotropic materials. The central idea is to find a way to adapt an existing finite strain elasticity to formulate an inelastic model. A novel idea is introduced in which one need not break his or her head on what objective rates but directly use the finite deformation elastic model and the thermodynamic restrictions in order come up a constitutive law. Several questions and issues that may arise are answered with clarity. The relationships with other models are also discussed to make it clear to the reader the connections that exist between the proposed model and the models existing in the literature.

We have addressed modeling an isotropic material for inelastic finite deformations but, in general, especially in metal forming applications, one may need to deal with anisotropic materials. *Chapter 13* deals with polycrystal plasticity where the orientations of the individual grains and their slip mechanisms play a role in the anisotropy of the material. An anisotropic plasticity model is introduced here borrowing concepts of crystal plasticity such as resolved shear stress on slip systems and of power law dashpots. Polycrystalline aggregates can be approximately simulated as a collection of single crystals (all with the same velocity gradient (the so-called Taylor assumption) or all with the same stress (the so-called Sach's assumption). There have been attempts to improve on these two assumptions for various special conditions. Finally, we end the discussions in *Chapter 14* with simple case studies of shot peening process, equal channel angular extrusion process, and ageing of skin tissues. The use of commercial packages in solving these problems are explained.

How to use this book

"You cannot build muscles by watching an exercise video"— these words[3] are the guiding motto of this book! We strongly believe that you get the most out of this book by actually doing all the exercises and not by simply looking at the answers. Don't take our word for anything in the book. Do it yourself and you will understand it in your own way. We can only give you an explanation, no one but yourself can give you an understanding. So the way the book is written, it is essential for you to get a notebook and interrupt your reading and doing the exercises that appear in the middle of the narrative.

If you are trying to learn plasticity by yourself by following this book, then you have to first assess your capabilities. Are you familiar with elasticity? or only strength of materials? If it is the former then the material up to Chapter 10 does not require additional mechanics knowledge. On the other hand if your knowledge is strength of materials then you can do the first six chapters and then attempt the remaining after taking a class on elasticity. If you want to do the numerical examples, learn MATLAB! you will thank us for it[4].

If you are more familiar with continuum mechanics, then after Chapter 2, skip to Chapter 10 and above.

Suggested course outlines

As mentioned in the previous section, the book is intended for different audiences ranging from the senior-level students, the graduate students, the research scholars, to practicing engineers.

Depending on which part of the book is covered, different pre-requisites may be needed. For example, for the first part of the book (part I) that deals with introductory concepts and structural elements, an elementary knowledge of structural / strength of materials and introductory thermodynamics may be necessary. Part II that deals the small deformation inelasticity of continua, requires that the student has undergone an advanced structural mechanics or an elasticity course apart from introductory thermodynamics. Part III, as is clear, required the student to have undergone a continuum mechanics course and is familiar with earlier parts of the book.

We suggest the following that would allow the instructor to pick the topics according to the focus areas of the students.

[3]The quote is to be credited to Dr. Bil Schneider, Designer of the TransHab inflatable space module and friend and colleague of one of the authors.

[4]While we absolutely love MATLAB, we have no affiliation with it–nor do we have stock in MATHWORKS (what a pity!) since it is a privately held company. But trust us when we tell you that for prototyping some numerical methods, MATLAB is hard to beat. Both of us have written programs in FORTRAN, C, C++, LabView, etc., but MATLAB is great for the tasks that we throw at it and its learning curve is really flat (if you are comfortable with Matrices).

Part	Chapters	No. of weeks	Level	Pre-requisites
I	1-5	8	Senior	(a) Strength of materials (b) Intro. to Thermodynamics
II	6-10	10	Grads Res. scholars	(a) Elasticity / Adv. Strength of Mat. (b) Intro. to Thermodynamics
III	11-14	8	Senior Grads Res. Scholars	(a) Elasticity (b) Continuum thermodynamics

For example, if it is a semester course (14 weeks) for just the senior level and first year graduate students, one can omit Part III of the book and only deal with small deformations (Parts I and II). Some portions of the book need not be covered. For example, one can omit Chapters 8 and 9, without affecting the flow of the book i.e. the 14 weeks' portion could be Chapters 1-7 and 10.

For a two-semester course on inelasticity, or if it is an advanced one-semester course for students with a continuum background, we suggest after a refresher on parts of Chapter 2 (sections 2.5-2.8) and Chapter 6 (J2 plasticity), Chapters 8 and 9 could be introduced in the first part of the course. The second part of the course could be devoted to the finite deformations part of the book (Part III).

Our addresses and websites are given below: Our websites will connect you to the book website which has MATLAB codes, solutions to exercises and additional material that could be used for teaching and learning purposes.

Arun R. Srinivasa,
Department of Mechanical Engineering,
Texas A & M University,
College Station, TX 77843, USA.
http://www2.mengr.tamu.edu/FacultyProfiles/facultyinformation.asp?LastName=asrinivasa

and

Sivakumar M. Srinivasan,
Department of Applied Mechanics,
Indian Institute of Technology Madras
Chennai 600036, INDIA.
http://apm.iitm.ac.in/smlab/mss

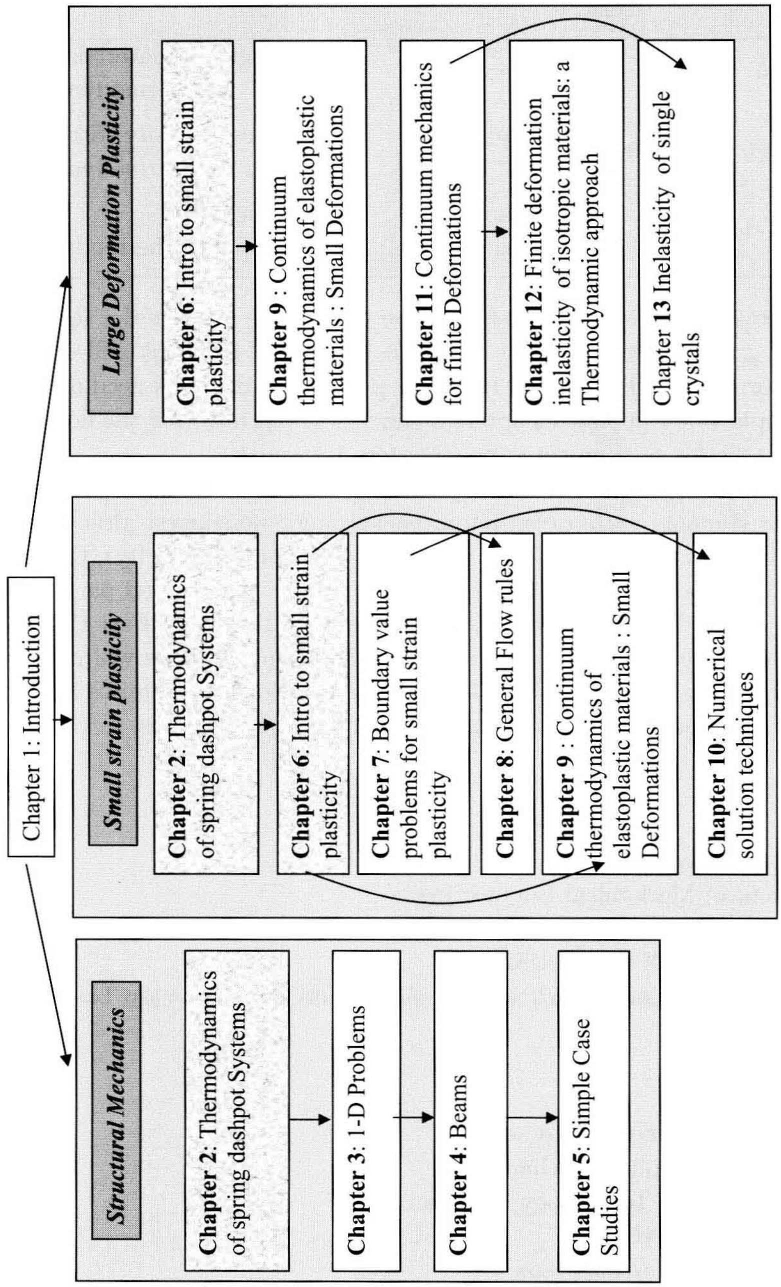


Fig. 0.1 A flow chart illustrating the grouping of the chapters. The dependencies of the chapters are shown by the arrows.

Acknowledgments

I thank my parents who have sacrificed so much, for my sake, and my loving wife who enabled me to take a four-month sabbatical to write this book while she looked after our children all by herself. I also thank my children Vishnu and Divya who showed me how to be joyful and restored my sense of perspective.

I would like to thank my advisor Prof. P. M. Naghdi, who believed in me and introduced me to the fascinating world of thermodynamics and inelasticity in his own inimitable way and my mentor Prof. K. R. Rajagopal who vastly broadened my vision well beyond mechanics. I cherish my interactions with them, and their words and actions continue to guide me.

I would like to acknowledge the insightful conversations that I had with my good friend Dr. Jeff Froyd from the center for teaching excellence at Texas A&M University who educated me on so many aspects of effective teaching. They were invaluable to me in figuring out how to present the material. I would also like to acknowledge Prof. M. S. Ananth, Director, IIT Madras, for allowing me to stay in the sylvan surroundings of IIT while we wrote this book. The environs of IIT inspire me like no other place I know. My undergraduate days there and my sabbatical stay there are definitely some of the best days of my life.

–Arun R. Srinivasa.

First of all, I should thank Arun for inviting me to coauthor this book. It was a great learning experience all the way and I relished every moment of our conversations which many times went beyond just the technical matter.

I started learning the ABC of plasticity when I was with Prof.G.Z.Voyiadjis, my Ph.D. guide. I thank him for providing that crucial start of this continuing journey.

I wish to thank Prof. K. Ramamurthy, chairman, CCE, my department chair, Prof. K. Ramesh and our Director Prof. M.S. Ananth for facilitating this through the Golden Jubilee Book Writing Scheme. Without this, it would have been a painful struggle to complete this task. They were also proactive in effecting Arun's visit to IITM that saw a step on the gas in this effort.

I remember the ready support my dad gave when I wanted to pursue higher studies in a situation that demanded my going for a job. It was my brother, Raman who provided the foundation stone to the technical me today that participated in writing this book. Watching my wife, Leela do the juggling between the work place and home deftly was more than just a performance to watch. The time I had with my children during bedtime gave me the much needed relaxation and joy. I thank my family for all this.

Special thanks to Baskaran Bhuvaraghan and Arun Chandran who helped in solving some of the problems and case studies inserted in this book and Mr.Sreekumar for help with svnhost.

The constant poking from my B.Tech classmates Aditya Gurajada and Shekar Ramakrishnan kept my desire burning, culminating in the completion of the book.

Thanks to them. I would like to thank my friend and mentor Prof. K. R. Rajagopal who taught me so many things well beyond just mechanics. The guidance and motivation from him kept me going.

–Sivakumar M. Srinivasan.

We would also like to acknowledge our conversations with Dr. Chandrasekaran, who was a consultant at IIT Madras. His enthusiasm for work, humility and deep knowledge served as an example to all of us who were in the Active materials lab at IIT Madras.

We acknowledge the tireless work of Pritha Gosh, Shriram Srinivasan, Satish Karra, Shreyas Balachandran, Guruprasad, Srikrishna Doraiswamy, H.S. Srivatsan, A. Jayavel, K. Jayabal, K.Srivatsan, S.Balaji, Arun Chandran, R.S. Priyadarsini, and R. Suresh Kumar—the students who proofread the material, checked the equations, and pointed out places where our explanations were murky and needed clarification. Without their eagerness to learn and their help, this book would not have been possible. Thanks to Dr. Mangala Sunder and Mr. Britto Jones for help with diagrams. The help and support from France Prakash and Dr. A. Arockiarajan deserve a special mention.

Lastly, we might have inadvertently missed out, in the above, some of those who have directly or indirectly helped us with this effort. Our sincere thanks to them all.

–Arun R. Srinivasa
& Sivakumar M. Srinivasan