

The background of the cover is a light gray grid. Overlaid on this grid are several horizontal and vertical lines in various colors: red, blue, green, and black. Some of these lines are solid, while others are broken or segmented. A prominent diagonal line runs from the top right towards the bottom left, creating a sense of depth and perspective. The overall design is minimalist and geometric.

**Lial/
Miller**

**MATHEMATICS WITH
APPLICATIONS**

Fourth Edition



Fourth Edition

MATHEMATICS WITH APPLICATIONS

**IN THE MANAGEMENT,
NATURAL, AND
SOCIAL SCIENCES**

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SCOTT, FORESMAN AND COMPANY
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London, England

Alternative Books

Mathematics and Calculus with Applications, Second Edition, is a longer book, featuring more topics, especially in calculus.

Finite Mathematics, Third Edition, features an introduction to the broad areas of linear mathematics and probability for students of business and life science. *Finite Mathematics* is written at a higher level than this text.

Calculus with Applications, Third Edition, gives an extensive treatment of calculus for those students needing a separate course in calculus.

Computer Applications for Finite Mathematics and Calculus by Donald R. Coscia (with accompanying diskettes for Apple II or IBM PC) is designed for courses in finite mathematics; calculus for the life, management, and social sciences; statistics; or discrete mathematics.

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Preface

Mathematics with Applications, Fourth Edition, introduces the mathematical topics needed by students of management, social science, and natural science for a combined finite mathematics and calculus course. It also can be used for a separate course in either finite mathematics or applied calculus as well as for a course in applied college algebra. Previous editions have been used in courses such as Mathematics with Applications, Mathematics for Management and Social Science, Mathematics for Business and Economics, Finite Mathematics, Introduction to Analysis, Algebra with Applications, and College Mathematics.

The only prerequisite assumed is a course in algebra. A review of algebra is given in Chapter 1, and a diagnostic pretest is included in the *Instructor's Guide* to help determine which students need review.

Features new in the fourth edition include the following.

The ***algebra review*** (Chapter 1) is presented in greater depth and includes more challenging exercises on rational expressions, exponents, and absolute value equations and inequalities.

The chapters on ***matrices and systems of linear equations*** have been combined into one chapter (Chapter 6) to provide a clearer presentation which uses systems to motivate matrices.

The chapter on ***linear programming*** (Chapter 7) has been thoroughly and carefully revised.

The discussion of ***permutations and combinations*** (Chapter 8) now provides a stronger distinction between permutations and combinations.

The ***derivatives of exponential and logarithmic functions*** are now included in the same chapter (Chapter 11) as all the other derivatives.

The treatment of ***multivariate functions*** (Chapter 12) has been expanded.

The ***fundamental theorem of calculus*** is now presented in a separate section (Chapter 13).

The following popular features are retained from previous editions.

Problems at the side help test student understanding. By working the more than 400 marginal problems as topics are encountered, students can quickly locate their source of difficulty.

Examples and exercises are extensive. This book continues to have substantially more examples (more than 400) and exercises (3900 drill and 1300 applications) than other leading books.

Realistic applications help students grasp new concepts and see the many interesting ways in which mathematics is used. The applications in this book are as realistic as possible.

One or more case studies follow most of the chapters of the book. These cases present the topics of the chapters as they apply to real-life situations. We have tried to keep the cases short and simple. The concepts of the course come alive when students can see how mathematics is used at Upjohn or Booz, Allen and Hamilton, for example.

The book can be used for a variety of courses, including the following.

Finite Mathematics and Calculus (one year or less) Use the entire book; cover topics from Chapters 1–4 as needed before proceeding to further topics.

Finite Mathematics (one semester or one or two quarters) Use as much of Chapters 1–4 as needed, and then go into the topics of Chapters 5–10 as time and local needs permit.

Calculus (one semester or quarter) Use Chapters 1–4 as necessary, and then use Chapters 11–13.

College Algebra with Applications (one semester or quarter) Use Chapters 1–8 with the topics of Chapters 7 and 8 being optional.

Chapter interdependence is as follows.

Chapter	Prerequisite
1 Fundamentals of Algebra	None
2 Linear Models	None
3 Polynomial and Rational Models	Chapter 2
4 Exponential and Logarithmic Models	Chapter 2
5 Mathematics of Finance	None
6 Systems of Linear Equations and Matrices	None
7 Linear Programming	Chapters 2 and 6
8 Sets, Counting, and Probability	None
9 Further Topics in Probability	Chapter 8
10 Statistics	Chapter 8
11 Differential Calculus	Chapters 2–4
12 Applications of the Derivative	Chapter 11
13 Integral Calculus	Chapters 11–12

Additional materials for this book include the following.

The *Instructor's Guide* contains a complete testing program, including an algebra pretest (with answers), a bank of test items for each chapter (with answers), and answers to even-numbered text exercises. In addition, the manual offers background material for instructors and presents computer programs in BASIC, for Leontief models and for Markov chains, and the program LINPRO, for the simplex method of linear programming.

The *Study Guide and Student's Solutions Manual* contains the solutions to the odd-numbered exercises. Detailed solutions are provided for each major concept presented within each section, with helpful hints, cautions, and suggestions. More condensed solutions are then presented for the remaining odd-numbered problems.




Many instructors helped us prepare this revision. In particular, we would like to thank Garret Etgen, University of Houston; George Evanovich, Iona College; Patricia Hirschy, Delaware Technical and Community College; Alec Ingraham, New Hampshire College; Donald Mason, Elmhurst College; Carol Nessmith, Georgia Southern College; and Daniel Symancyk, Anne Arundel Community College.

The text answers were checked by James Hodge, College of Lake County; Louis F. Hoelzle, Bucks County Community College; and Wing M. Park, College of Lake County.

At Scott Foresman we were very fortunate to be able to work with an extremely talented group of people: Bill Poole, Pam Carlson, and Marge Prullage helped establish the overall framework of the revision, and Adam Bryer helped with the detailed changes.

Margaret L. Lial
Charles D. Miller

TO THE STUDENT

Using the text Side problems, which help reinforce skills and pinpoint sources of difficulty, are referred to in the text by numbers within colored squares:  at the ends of examples and  elsewhere. Squares without numbers () mark the ends of examples that have no side problems.

Computer exercises Section, chapter review, and case exercises requiring the use of a computer are highlighted by colored exercise numbers.

Additional help If you would like more help with mathematics, you may want to get a copy of the *Study Guide and Student's Solutions Manual*, containing solutions to the odd-numbered exercises. Detailed hints and cautions are provided for the major concepts of each section, and more condensed solutions are presented for the remaining problems. Your local college bookstore either has this book or can order it for you.

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1

Fundamentals of Algebra

This book is about the application of mathematics to various subjects, mainly business, social science, and biology. Almost all of these applications begin with some real-world problem that is solved by creating a mathematical model. Mathematical models are discussed in more detail later. Basically, a **mathematical model** is an equation (or other mathematical relationship) that represents a given problem.

For example, suppose the real-world problem is to find the area of a floor, 12 feet by 18 feet. Here, a mathematical model would be the formula for the area of a rectangle, $\text{Area} = \text{Length} \times \text{Width}$, or $A = LW$. Substituting the numbers, 18 for L and 12 for W into this formula gives the area as $A = 18 \times 12 = 216$ square feet.

Most mathematical models involve algebra. Algebra is used first to set up a model and then to simplify the resulting equations. Since algebra is so vital to a study of the applications of mathematics, this book begins with a review of some of the fundamental ideas of algebra.

1.1 The Real Numbers

The various types of numbers used in this book can be explained with a diagram called a **number line**. Draw a number line by choosing any point on a horizontal line and labeling it 0. Then choose any point to the right of 0 and label it 1. The distance between 0 and 1 gives a unit of measure that can be used repeatedly to locate points to the right of 1, labeled 2, 3, 4, and so on, and points to the left of 0, labeled -1 , -2 , -3 , -4 , and so on. A number line with several sample numbers located (or **graphed**) on it is shown in Figure 1.1. [1]

- [1] Draw a number line and graph the numbers -4 , -1 , 0 , 1 , 2.5 , $13/4$ on it.

Answer:

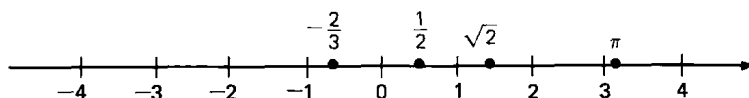
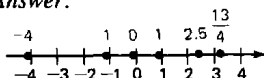


Figure 1.1

Any number that can be associated with a point on the number line is called a **real number**.* All the numbers used in this book are real numbers. The names of the most common types of real numbers are as follows.

*Not all numbers are real numbers. An example of a number that is not a real number is $\sqrt{-1}$.

The Real Numbers

Natural (counting) numbers	1, 2, 3, 4, . . .
Whole numbers	0, 1, 2, 3, 4, . . .
Integers	. . . , -3, -2, -1, 0, 1, 2, 3, . . .
Rational numbers	All numbers of the form p/q , where p and q are integers, with $q \neq 0$
Irrational numbers	Real numbers that are not rational

The three dots in this box show that the numbers continue indefinitely in the same way. The relationships among these types of numbers are shown in Figure 1.2. Notice, for example, that the integers are also rational numbers and real numbers, but the integers are not irrational numbers.

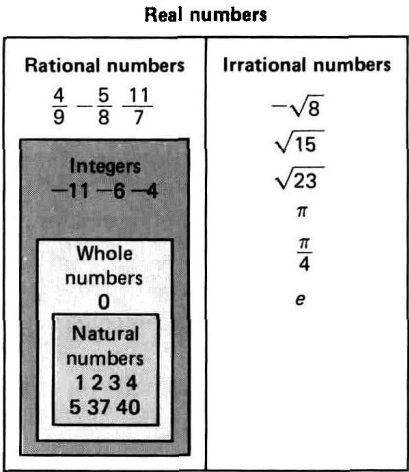


Figure 1.2

One example of an irrational number is π , the ratio of the circumference of a circle to its diameter. The number π can be approximated by writing $\pi \approx 3.14159$ or $\pi \approx 22/7$ (\approx means “is approximately equal to”), but there is no rational number that is exactly equal to π . Another irrational number can be found by constructing a triangle having a 90° angle, with the two shortest sides each 1 unit long, as shown in Figure 1.3. The third side can be shown to have a length which is irrational (the length is $\sqrt{2}$ units).

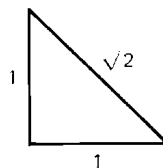


Figure 1.3

Many whole numbers have square roots which are irrational numbers; in fact, if a whole number is not the square of an integer, then its square root is irrational.

Example 1 List all the names of sets of numbers that apply to the following.

(a) 6

Consult Figure 1.2 to see that 6 is a counting number, whole number, integer, rational number, and real number.

(b) $3/4$

This number is rational and real.

(c) $\sqrt{8}$

Since 8 is not the square of an integer, $\sqrt{8}$ is irrational and real. 2

2 Name all the types of numbers that apply to the following.

- (a) -2
- (b) $-5/8$
- (c) π

Answer:

- (a) Integer, rational, real
- (b) Rational, real
- (c) Irrational, real

Some basic properties of the real numbers are given below.

Properties of the Real Numbers For all real numbers a , b , and c :

Commutative properties $a + b = b + a$
 $ab = ba$

Associative properties $(a + b) + c = a + (b + c)$
 $(ab)c = a(bc)$

Identity properties There exists a unique real number 0 such that
 $a + 0 = a$ and $0 + a = a$.

There exists a unique real number 1 such that
 $a \cdot 1 = a$ and $1 \cdot a = a$.

Inverse properties There exists a unique real number $-a$ such that
 $a + (-a) = 0$ and $(-a) + a = 0$.

If $a \neq 0$, there exists a unique real number $1/a$ such that

$$a \cdot \frac{1}{a} = 1 \quad \text{and} \quad \frac{1}{a} \cdot a = 1.$$

Distributive property $a(b + c) = ab + ac$

3 Name the property illustrated in each of the following examples.

- (a) $(2 + 3) + 9 = (3 + 2) + 9$
 (b) $(2 + 3) + 9 = 2 + (3 + 9)$
 (c) $(2 + 3) + 9 = 9 + (2 + 3)$
 (d) $(4 \cdot 6)p = (6 \cdot 4)p$
 (e) $4(6p) = (4 \cdot 6)p$

Answer:

- (a) Commutative property
 (b) Associative property
 (c) Commutative property
 (d) Commutative property
 (e) Associative property

4 Name the property illustrated in each of the following examples.

- (a) $2 + 0 = 2$
 (b) $-\frac{1}{4} \cdot (-4) = 1$
 (c) $-\frac{1}{4} + \frac{1}{4} = 0$
 (d) $1 \cdot \frac{2}{3} = \frac{2}{3}$

Answer:

- (a) identity property
 (b) inverse property
 (c) inverse property
 (d) identity property

Example 2 The following statements are examples of the commutative property. Notice that the order of the numbers changes from one side of the equals sign to the other.

- (a) $6 + x = x + 6$
 (b) $(6 + x) + 9 = (x + 6) + 9$
 (c) $(6 + x) + 9 = 9 + (6 + x)$
 (d) $5 \cdot (9 \cdot 8) = (9 \cdot 8) \cdot 5$
 (e) $5 \cdot (9 \cdot 8) = 5 \cdot (8 \cdot 9)$ ■

Example 3 The following statements are examples of the associative properties. Here the order of the numbers does not change, but the placement of the parentheses does change.

- (a) $4 + (9 + 8) = (4 + 9) + 8$
 (b) $3(9x) = (3 \cdot 9)x$
 (c) $(\sqrt{3} + \sqrt{7}) + 2\sqrt{6} = \sqrt{3} + (\sqrt{7} + 2\sqrt{6})$ ■

The identity properties give some special properties of the numbers 0 and 1. Since 0 preserves the identity of a real number under addition, 0 is the **identity element for addition**. In the same way, 1 preserves the identity of a real number under multiplication and is the **identity element for multiplication**.

Example 4 By the identity properties.

- (a) $-8 + 0 = -8$,
 (b) $(-9)1 = -9$. ■

The number $-a$ is the **additive inverse** of a , and $1/a$ is the **multiplicative inverse** of the nonzero real number a .

Example 5 By the inverse properties,

- (a) $9 + (-9) = 0$,
 (b) $-15 + 15 = 0$,
 (c) $6 \cdot \frac{1}{6} = 1$,
 (d) $-8 \cdot \left(\frac{1}{-8}\right) = 1$,
 (e) $\frac{1}{\sqrt{5}} \cdot \sqrt{5} = 1$.

(f) There is no real number x such that $0 \cdot x = 1$, so 0 has no inverse for multiplication. ■

One of the most important properties of the real numbers, and the only one that involves both addition and multiplication, is the distributive property. The next example shows how this property is applied.