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Interpreting Mathematical Economics and Econometrics

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Preface

This book is written for those who fear mathematics. If you often say, 'I was never very good at maths', but now have to confront aspects of the subject, this book is for you. The title has been chosen carefully. The operative word is **interpreting.** The complex mathematical and statistical techniques which are used to arrive at sophisticated results are ignored. Only the results are important. But even these are usually presented in an esoteric, jargon-laden fashion. Fortunately, the interpretation of these results is straightforward. Unfortunately, the textbooks which are the usual route to such an interpretative understanding assume a substantial amount of mathematical training. When confronted with page upon page of matrix algebra, most of the lower case and half of the upper case of the Greek alphabet, all but the most dedicated students decide that they 'didn't really need that course anyway'. The lay person who wishes to be informed about the details of the results of some recent research will be quickly turned away by the existence of 'econometric estimations', and discussions of 'regression coefficients', 'standard errors', and so on. But such is the stuff of research – mathematical/statistical jargon is pervasive. The ideas represented by the jargon are simple. There is no good reason for the use of Greek letters, except 'by convention'. If someone were to simply interpret and explain 'in plain English' the meaning of the jargon and of the mathematical and statistical (Greek) symbols, the difficulty would disappear. This book provides such an interpretation.

The encroachment of mathematics into many social sciences has been accelerated by the computer. Computers have enabled more and more sophisticated analyses to be done by less and less sophisticated computer users. The advent of the 'personal' or 'home' computer is further amplifying this effect. An increasing number of people are undertaking complicated analyses using the 'packaged' computer programs available on all ranges of computers without the ability to fully interpret the results of their work. These 'packages', or, as they are called in the jargon of the computer industry, 'software', are themselves getting more sophisticated enabling the most naive of computer users to undertake the most complicated of statistical analyses. The missing link is the ability to interpret these results. Part II of this book provides that link.

This book came to be written because of an expressed need by many generations of students for a non-technical, non-mathematical, non-statistical introduction to the meaning of mathematical symbolism and econometric analyses.

My motivation has been to enable those with no training in calculus or statistics to understand the results of the applications of these powerful tools. This is the book I searched for in vain when first confronted with the terrifying requirement that I must understand the meaning of 'partial differentials' and 'partial regression coefficients'. There is no pretence of rigour in the presentation of the mathematics. Stress is placed on the understanding of the results of research from an intuitive viewpoint.

The book is divided into two major parts. The first, consisting of Chapters 1 to 4, deals with the tools of the differential and integral calculus. Chapter 1 introduces mathematical notation for the most extreme form of novice and builds on that foundation in Chapter 2 as the use of equations is explained. Chapters 3 and 4 discuss the meaning of the principal results of differentiation and integration.

Part II addresses econometric analyses. Chapter 5 bridges the gap from the theoretical modelling to the statistical testing of those models. Included in Chapter 6 is a discussion of the most important statistics associated with econometric analyses. These form the actual 'testing' of a theory. Chapter 7 discusses the most serious problems which can arise in regression analyses.

The context of the book is economics but the applications are general. Anywhere that mathematics is used or statistical analyses are required the same interpretations are valid. Parts of the book have been used to assist students undertaking research in such varied fields as economics, psychology, sociology, political science, history, chemistry, biology, physics, business studies and physical education.

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To Rachel, Alexander and Timothy for helping me to interpret what's important

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PART I

MATHEMATICAL ECONOMICS

1

Mathematical Symbolism

Introduction

The first thing we must do is introduce some basic mathematical concepts. I am going to assume you have no mathematical background. The first thing a mathematical economist does is replace words with symbols so that price may become p, quantity may become Q, and so on. Of course, the symbol used is completely arbitrary: price could be represented by Y and quantity by X, for example. Most writing, however, prefers to relate the symbols in some way to the words they represent. The words represented and the symbols used are referred to as **parameters** and **variables.**

Variables

Mathematical symbolism may refer to constants or variables. A constant is a quantity which takes a fixed value in a specific problem. The constant may be presented as a number or a symbol denoting a number in which case it is called a **numerical** or **absolute** constant. Alternatively, the constant may be presented as a symbol and is called an **arbitrary** or **parametric** constant, usually abbreviated simply to parameter. Absolute constants take on the same value in all situations whereas parameters, while assuming only one value in a specific problem, may take different values in other problems. Parameters are most frequently represented by letters at the beginning of the alphabet, although there are sufficient exceptions not to make this a definitional rule. Some writers prefer the Greek alphabet with the parameters represented by its first letters, α, β, γ, δ, etc.

Table 1.1 presents the Greek alphabet. Familiarity with it should remove the anxiety resulting from the esoteric quality surrounding its use.

Variables can be thought of as quantities which assume a variety of values in a specific problem. (The set of possible values is called the range of the variable.) Commonly, pure mathematics uses the letters at the end of the alphabet, for example, X, Y, Z, to represent variables but in applied mathematics this convention is sometimes broken; the symbol used is often simply the first letter of the variable name, for example, p represents the variable price, q represents quantity, and so on.

Variables may be classified in many ways. One classification scheme dichotimises variables as either continuous or discrete. **Continuous variables** can take values within a specified interval of real numbers. Because **any** value in the interval can be taken, and because these values can differ by infinitely small amounts, it is not possible to count all the values in, say, the interval between 1 and 10. Put another way, there are no gaps in a continuous variable over the range between 1 and 10. **Discrete** (discontinuous) **variables**, on the other hand, **are** countable and are often defined as being able to take only

Table 1.1 *The Greek alphabet and English couterparts*

Greek			*English*		*Common usage*
Capital	*Small*				
Α	α	alpha	A	a	constant
Β	β	beta	B	b	constant
Γ	γ	gamma	C	c	constant
Δ	δ	delta	D	d	special operator: increments
Ε	ϵ	epsilon	E	e	variable
Ζ	ζ	zeta	Z	z	variable
Η	η	eta	Y	y	variable
Θ	θ	theta	–	–	functional operator
Ι	ι	iota	I	i	parameter
Κ	κ	kappa	K	k	parameter
Λ	λ	lambda	L	l	parameter
Μ	μ	mu	M	m	parameter
Ν	ν	nu	N	n	parameter
Ξ	ξ	xi	X	x	variable
Ο	o	omicron	O	o	–
Π	π	pi	P	p	Π: special constant
Ρ	ρ	rho	R	r	variable
Σ	σ	sigma	S	s	summation operator
Τ	τ	tau	T	t	variable
Υ	υ	upsilon	U	u	variable
Φ	ϕ	phi	F	f	functional operator
Χ	χ	chi	–	–	statistic
Ψ	ψ	psi	G	g	functional operator
Ω	ω	omega	W	w	variable

values which are specified in a **countable** range. In other words, discrete variables do have gaps. An example of a discrete variable is the price of a commodity if the price is given only in pennies. The price, say p, is then a discrete variable which assumes values which are a set of integers – a range which is discontinuous.

Another popular classification scheme divides variables into four categories: independent, dependent, exogenous and endogenous. **Independent** variables are those that do not depend on other variables. Now, it is important to distinguish between the mathematical and economic concepts of dependence. Dependence in the purely mathematical sense does not require **causality**. The mathematics only establishes a rule for associating the two variables; a systematic relationship is suggested but there is no information about **why** the variables move together. We do not know whether X influences Y, or Y influences X, whether some external factor makes X and Y move together, or whether it is by pure chance that X and Y move together. All that can be said is that if we know the value of one variable we can find the value of another.

Dependence in the economic context is not arbitrary. Economists usually try to write what they think is the **dependent** variable as the one that is being changed by the **independent** variable – the independent variables cause the change. For example, if we say that an increase in the price of a pizza causes fewer pizzas to be demanded each week then we are saying that the quantity of pizza demanded **depends** on the price. Quantity is therefore a dependent variable and price is an independent variable. Price is an independent variable because we have not said anything **affects it**. Notice that if we said that price depends on something else, we would make pizza price a dependent variable and something else would be independent. It is all a matter of what affects what. If our hypothesis is simply that price affects quantity demanded **and we stop there** then quantity demanded is a dependent variable and price is independent.

Exogenous and **endogenous** variable classifications refer to the broader context of a complete model but are related in some ways to the dependent–independent dichotomy. An exogenous variable is the easiest one to define because it is placed so as to depend on nothing in the model. Nothing affects it as far as we are concerned. It is something which is taken as given in our economic model – values for it come from outside the model. Some writers use the term **autonomous** or **predetermined** rather than exogenous. If the quantity of pizza demanded depends not only on price but also on, say, the population of a city, and we have no interest in explaining what determines the population of a city, then the population variable is exogenous. It is **outside** ('ex') the scope of what we want to explain and we treat it merely as given in our model of what determines the quantity of pizza demanded.

An endogenous variable is, like a dependent variable, one which is dependent on others. In the pizza example, quantity demanded is an endogenous variable because it depends on such things as the price of the pizza and the population of the city. Hence, **if a variable is dependent then it is also endogenous**. But if a variable is endogenous, it is not necessarily dependent. To illustrate, we will expand our discussion a little about how much pizza is sold by adding one more variable, the price of wheat. We suggested that the quantity of pizza demanded is determined by the price of the pizza and population. Now, let us add that the price of the pizza is affected by the price of wheat. We now have pizza price **affecting** quantity demanded and being **affected by** wheat price. Pizza price is therefore an independent variable when we say it **affects** quantity and a dependent variable when we say it is **affected by** wheat price. Whether we consider pizza price dependent or independent depends entirely on which part of the model is being referenced.

But with the endogenous–exogenous split, the answer is more clear cut. The first hypothesis has quantity demanded depending on pizza price and population. The second hypothesis has pizza price depending on wheat price. The total model therefore has two exogenous variables (they depend on nothing within our model) – population and wheat price are exogenous because our hypotheses do not try to explain how they are determined.

The dependent–independent split only comes into play when we consider each hypothesis **separately**. Hypothesis one (that quantity of pizza demanded is dependent on pizza price and population) has quantity demanded as dependent and pizza price and population as independent **in that hypothesis**. Hypothesis two (that pizza price depends on wheat price) has pizza price as dependent and wheat price as independent **in that hypothesis**.

The dependent–independent split relates to each hypothesis **separately**; the endogenous–exogenous split relates to all the hypotheses together, i.e. the 'model'.

We see then that variables can be dependent, independent, endogenous or exogenous and often more than one of these at the same time. But how are variables related mathematically? That is, how are the verbal hypotheses put into mathematical hypotheses? The answer is through equations.