

Quantitative Methods
for
business decisions

Second Edition

JON CURWIN and ROGER SLATER



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Preface to the Second Edition

This second edition of *QM for Business Decisions* has been updated to reflect the changes to some of the statistics produced in the UK and the development of the subject. It has also benefited considerably from the comments we have received from readers of the first edition; comments for which we are very grateful.

We have found that the book has become recommended reading on a wide range of courses where students are being introduced to Quantitative Methods for the first time; from BTEC courses where it may take two years to cover all of the material, to MBA courses where the material is covered in a term or a year, and in courses for the professions, for example accountancy. This wide usage reflects the fact that whenever students come upon QM for the first time they are starting from similar positions, although their rate of progress from this starting point may be very different.

From our contacts with other polytechnics and universities we have found that the content of first stage QM courses has not changed radically since the first edition was published, although the technology used for the solution of problems has now moved on to include the use of computer packages. Many of the examples and exercises in this edition can be solved using computer packages, but we find that end of year examinations still require students to perform calculations by hand, and thus this edition still includes full details for the solution of problems by hand. We have not selected a particular statistical package since different institutions have different packages and we feel that it is essential for students to become familiar with the packages in their own environment.

We have included a final section of typical examination questions, Part VIII, which you may wish to use as a form of revision. This section also includes some questions from past papers from the professional accountancy bodies.

How to use this book

The book has been arranged in eight parts, the first seven of which include two, three or four chapters. These groups of chapters all develop a theme within quantitative methods.

<i>Part</i>	<i>Theme</i>
I	Quantitative information
II	Descriptive statistics
III	Mathematical methods
IV	Measuring uncertainty
V	Statistical inference
VI	Relating two or more variables
VII	Mathematical models

There is a general introduction to each part which should provide a business-related context for what follows. These introductions include ideas for supportive work which you may choose to undertake as you read the relevant chapters. This work may, for example, take the form of collecting recent data on a topic of interest and applying the described methods of analysis. Also, at the end of each part an exercise has been included for practice, revision and discussion.

As with all books of this kind, some chapters may be familiar and easily understood, and other chapters new and apparently difficult. You may, for example, understand descriptive statistics from your lecture notes but not statistical inference. **What you need to do is establish a way to use this book to your own maximum advantage.** For some, it may be a matter of using the book for supportive reading only. You will find plenty of worked examples to complement those given in lectures and seminars. For others, the book may need to provide a self-contained course in quantitative methods. You will find a description of all the methods used, exercises included at the end of chapters (except chapter 18) and selected answers (given in Appendix 7).

In working through the book, you will need to observe the following interdependencies:

- (1) Chapter 4 requires Chapter 3;
- (2) Chapter 10 requires Chapter 9;
- (3) Chapter 11 requires Chapter 9;
- (4) Chapter 12 requires Chapters 3, 4 and 11;
- (5) Chapter 13 requires Chapter 12;
- (6) Chapter 14 requires Chapters 11, 12 and 13;

- (7) Chapter 17 requires Chapter 16;
- (8) Chapter 18 requires Chapters 16 and 17.

You will see that basic descriptive statistics (Chapters 3 and 4) and an understanding of probability (Chapter 9) are fundamental. We recommend that you become fully acquainted with these chapters as you proceed through the book.

The use of computers

The availability of computer technology to a wide range of users of quantitative methods has continued to improve. We would now expect first-year students on degree courses in Accountancy, Business Studies and Economics and BTEC courses in Business Studies and Public Administration to have access to at least one computer package able to generate basic descriptive statistics.

Given current trends, it is quite likely that this computer experience is now on a microcomputer using specialist business software such as SUPERCALC or a more integrated package combining word-processing, spreadsheets and databases such as FRAMEWORK. In many first-year units on quantitative methods the analysis of collected survey data or economic trends over time is a particularly important element which is often assessed.

We have resisted the idea of directing any of the text or exercises in this second edition to a particular type of computer package. It would be impossible to retain the strength of the book while attempting to address the diverse computing needs of an Accounting student using a spreadsheet or an Economics student using the regression facilities of a survey analysis package.

However, we expect students to be making more use of computing software and confronting more information in a quantitative form. The book should continue to support the understanding of data from computer screens or print-outs and facilitate the formulation of problems.

We would recommend to students who have acquired skills with a particular computer package to develop these skills further by attempting appropriate exercises from the book on their computers. (Most of the exercises, except for those on algebra and calculus, can be solved by using a package or a spreadsheet.)

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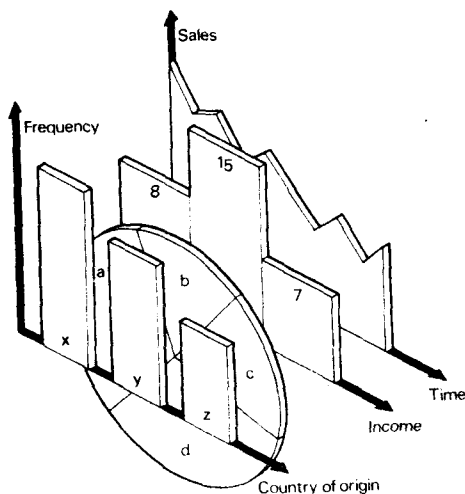
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Part I

QUANTITATIVE INFORMATION



In this part we are concerned with the ways data can be collected and how such data can be effectively presented.

Mastery of the magnitude and the complexity of the world around us demands more than a verbal description. We need to understand the numbers; how many, how large and how soon. Collected together, these numbers provide our data.

Some information is available from government publications:

Annual Abstract of Statistics;
Economic Trends;
Monthly Digest of Statistics;
Regional Statistics;
Social Trends; and others.

Exercise

We suggest you visit your local library and make a list of the range of information available from these publications.

We may also require information not available from published sources.

Exercise

Make a list of the information not available to you from these publications.

Your first list is limited to information of a more general form that is of interest to government and other organizations. There will be figures on the number of people by age, by sex and by income or region. What will often be lacking is particular detail. If we want information on the attitude towards frozen foods or the preferred colour of cars we will need to collect it.

Consider, for example, employment. You will need to define what is meant by this term and who should be included: the **qualitative factors**. You may choose to include the numbers: the **quantitative factors**. A single figure, such as the total number employed in the previous year in the UK, is precise, needs little in the way of presentation, and will be easily available from published statistics. It will, however, communicate little about the nature of employment. If you investigate further you will find the numbers employed by region, by industry and a range of other factors. As you include more detail, the information will become more difficult to communicate and more difficult to present. Quantitative methods aim at the communication, presentation and analysis of numerical information. If you investigate further still you will discover that there are a number of items relevant to employment not found in the published statistics. You will find that for some issues no data have been collected in a formal way. If you want information about a particular product or particular group of individuals and it is not available, you will have to collect it. If you want to know how workers perceive their training opportunities, chances of promotion or payment of overtime working you will need to organize and implement a survey.

Exercises

1. Find out what information you can on the number of working days lost through strikes in the UK. Discuss the information not available on the number of working days lost through strikes in the UK and suggest how this might be obtained.
2. Explain how you would obtain information on the selling price of second-hand cars.

1 Data collection

Many business, social and economic questions can be assessed and evaluated more closely if some data are available upon which to base discussions and decisions. These discussions and decisions are, however, basically pointless if the data used are biased or misleading; some people would even argue that it might be better to do without such data. Managers are often faced with other people's data, so need to be able to assess if this information is likely to be biased, or if it really will be useful for their decision-making process. This is often a case of knowing which questions to ask! Even if you can ask the right questions, you also need to be able to understand the answers that you are given by the 'experts'.

Questions about the data fall into broad groups, and this chapter will consider each group, looking at the issues that may be raised. These groups are:

- (a) What is the relevant population?
- (b) How many people were asked and how were they selected?
- (c) How was the information collected from these respondents?
- (d) Who did not respond?

1.1 POPULATION

Identification of the relevant population is essential since data collection can be a costly exercise and contacting large numbers of people who could have nothing to do with the survey will only waste these valuable resources. For example, if you were concerned with the acceptability to women of a new contraceptive pill it would be pointless contacting a group of people, half of whom were men. A similar problem can arise if the group you have identified as the relevant population does not include everyone for whom the survey is relevant, since a range of views or information will be totally missed. If you were interested in why people bought foreign-built cars, but failed to contact purchasers of certain Ford models, for example, then you may fail to identify the fact that some buyers do not realize that their car is foreign-built. The term 'population' can also be used to describe all the items or organizations of interest. An audit, for example, is concerned with the correctness of financial statements. The population of interest to the auditor could be the accounting records, invoices or wage sheets. If we were concerned with job opportunities, the population could be all the local businesses or organizations employing one or more persons.

Exercises

1. What is the relevant population to contact regarding a new nappy?
2. Who would be interested in a new metal paint to be sold in large quantities?

Having considered the type of people who would fall into the relevant population, the next problem is to try to identify who these people are; and perhaps even to get a list of their names and addresses. If this list can be obtained, it is called a **sampling frame**. Many surveys, particularly in market research, need a general population of adults, and make use of the Electoral Register. This contains a list of most people in the UK over 18 years old and is updated annually; of course as about 10% of the population of the UK move each year, others emigrate, some die and there is some immigration, the list cannot be 100% accurate, but it is easily available and very widely used. A study paper from the Central Statistical Office suggests that the Electoral Register is even more accurate in identifying addresses rather than people. (Houses move less frequently than people.) Other groups in the general population may appear on separate lists; for example, all those who are members of the RAC, or all those who are on a credit blacklist. These lists may not be generally available.

When a list does not exist or is not generally available, then those collecting the information may either try to compile a list, or use a method of collection which does not require a sampling frame (see next section). A method used by the Central Statistical Office to identify the population of disabled people, when a new Act of Parliament came into force, was to contact a large number of people in the general population, asking a few simple questions on whether they themselves were disabled or if they knew anyone who was, and if so who they were. This gave a fairly comprehensive list of the disabled, who could then be contacted and asked for more detailed information.

1.2 NUMBERS AND SELECTION

1.2.1 A census

One type of data collection does not require a selection procedure, and this is a census, or complete enumeration of the identified population. The best example of this type of survey is the Population Census which has been carried out in the UK once every 10 years since 1801 (with the exception of 1941 when rather more urgent matters were being dealt with). While this type of exercise should give highly detailed information and reflect data from all parts of the relevant population, it does take a long time to analyse the data and is very costly. A census is of limited use to the majority of business, social or economic applications, unless the identified population is small. For example, a census of all homes would be an expensive way of estimating the population with TV sets. In contrast, if you were representing a manufacturer who sold only to a small number of wholesalers and wanted their views on a new credit-ordering system, then a census would be a suitable method to use.

1.2.2 Selection: random and non-random

Where the relevant identified population is too large for a cost-effective census to be conducted a **sample** of that population must be selected, and individual responses generalized to represent the facts about, or the views of, the entire population. However, the method of selection will have implications for the

validity of this generalization procedure: if you were to ask the next five people you see how they would vote at a general election, it is very unlikely that the answers given would be a guide to a general election result. Sampling procedures can be divided into two broad categories: those where individuals are selected by some random method prior to the collection stage, and those where the individuals are non-randomly selected at the collection stage.

Random

Random does not mean haphazard selection, but means that each member of the population has some calculable chance of being selected. There is no one in the identified population who **could not** be selected when the sample is set up. A simple random sample gives every individual an equal chance of selection. To select a random sample a list or sampling frame is required, where each member is given a number and a series of random numbers (usually generated on a computer) are used to select the individuals to take part in the survey. There is thus no human interference in the selection of the sample, and samples selected in this way will, in the long run, be representative of the population. This is the simplest form of random sampling (see section 1.24 for more complex designs).

Non-random

Non-random is a catch-all for other methods of selecting the sample, where there is some judgement made in the selection procedure, and this may lead to some sections of the population being excluded from the sample, for good or bad reasons. For example, if the interviewer is asked to select who will take part in the survey and he or she has a particular aversion to men with beards, then this group may be excluded. If, then, men with beards have different views on the subject of the survey from everyone else, this view will not be represented in the results of the survey. However, a well-conducted non-random survey will produce results more quickly, and at a lower cost, than a random sample; for this reason it is often preferred for market research surveys and political opinion polls.

The most usual form of non-random sampling is the selection of a **quota sample**. In this case various characteristics of the population are noted, for example the divisions on sex, age and job type; and the sample aims to include similar proportions of people with these characteristics. This suggests that if people are representative in terms of known identifiable characteristics they will also be representative in terms of the information being sought by the survey. Having identified the proportions of each type to be included in the sample, each interviewer is then given a set number, or quota, of people with these characteristics to contact. The final selection of the individuals is left up to the interviewer. Interviewers you may have seen or met in shopping precincts are carrying out quota sample surveys.

Exercise

Apart from groups specifically avoided by a poor interviewer, which groups in the population would be excluded from such a survey?

Setting up a quota survey with a few quotas is relatively simple. The results from the Census of Population will give the proportions of men and women in the population, and also their age distribution.



Table 1.1

(i) Age distribution of population over 15 years

Age (years)	Percentage
15-20	19
20-30	25
30-50	26
50 +	30

(iii) Population (percentages)

Age (years)	Men	Women
15-20	10	9
20-30	12	13
30-50	12	14
50 +	12	18

(ii) Sex distribution of population over 15 years

Sex	Percentage
Men	46
Women	54

(iv) Number for sample of 1000

Men	Women
100	90
120	130
120	140
120	180

From the component tables of Table 1.1 we see that the general population over 15 years old consists of 46% men and 54% women, and thus our sample should also exhibit this division of sex. We can obtain similar information on the age distribution of the whole population, but this must be further analysed to show the separate age distributions of the sexes (part iii) so that we do not get the situation that all of the women in the sample are under 50 years old. Having combined the information from parts i and ii of Table 1.1 we can take each percentage of the sample size using part iii, here 1000, to find the number of each type of person to contact.

For a quota sample, it is important that the characteristics on which the quotas are based are easily identified (or at least estimated) by the interviewer, or else a lot of time will be wasted trying to identify the people who will be eligible to take part in the survey. If the number of quotas is large, some of the groups will be very small, even with an overall sample size of 1000.

1.2.3 Numbers

So far we have suggested that a census will be **too costly for most** subjects of business surveys, but that just asking five people **would be unlikely** to give a full representation of the views of the general public. **The variability** of the population will influence the sample size required; in the **extreme case** where everyone held exactly the same opinion, then it would **only be necessary** to ask one person. (This is a highly unlikely situation!) If **everyone in the population** held distinct views, then a census would be the only way to elicit the full range of views. (Again, very, very unlikely!) Sample size will also be related to how precise the results required from the survey are to be; and if the proportions are to be based on some subgroup of the sample, it is the size of these subgroups that must first be determined. Since most surveys do not aim to find out a single piece of information, but the answers to a whole range of questions, the determination of sample size can become extremely complex. It has been found that samples of about 1000 give results that are acceptable when surveying the general population. (See section 12.2.2 for the calculation of sample sizes.)