INTRODUCTION TO SIMULATION:
Programming Techniques and Methods of Analysis

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Programming Techniques and Methods of Analysis

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INTRODUCTION TO SIMULATION

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PREFACE

Simulation studies represent a widespread and increasingly important area of application for digital computers. As such, simulation is an important topic for students in the computer science discipline. It is also an important topic for students and practitioners in many other disciplines, including engineering, business administration, and the quantitative sciences. Usually the most crucial factor required in applying simulation successfully in a specific problem area is knowledge of that problem area. Thus, the most appropriate person to develop a simulation study is the person most familiar with the system to be studied. However, the development of effective and efficient computer programs can also be critical in these projects. Often this requires the participation of programming specialists as well as subject specialists.

The objective of this textbook is to provide an introduction to the use of digital computers in simulation studies. It is aimed toward upper-division undergraduate students who have completed an introductory programming course and a basic lower-division sequence of mathematics courses, including an introduction to statistics. The topics covered include methods for programming discrete-event model simulations and methods for the analysis of simulation results.

The approach used in this book is to emphasize programming aspects of the simulation process and to cover a few specific examples in detail. Previous experience in teaching simulation-related courses has indicated that many students do not have an intuitive feeling for processes which involve dynamic interaction. Therefore, the approach of going from concrete examples to the more general concepts, rather than vice versa, enables a significantly larger portion of the students to gain a reasonable understanding of the primary ideas which underlie the simulation process.

The selection of topics and the level of presentation are oriented toward making this material accessible to a wide variety of readers and toward providing a basis for further study in this subject area. Thus, more introductory and elementary material in both the programming and the analysis areas are included than will be needed by many readers. This has also led to treating some topics in a somewhat cursory manner, although their importance to and relationship with the subject area have been indicated.

A simulation programming system for discrete event models is developed in Chapter 4. This is intended to demonstrate the basic requirements of such a system as well as to provide a means by which the student can develop nontrivial example programs within the time limits of a single course. I believe this is the most appropriate method for introducing simulation programming, since it provides the proper basis for understanding and learning special-purpose simulation programming languages. This type of extension is considered in Chapter 5. Moreover, there are circumstances for which a general-purpose language is preferred for simulation programming. PL/I is the language primarily used in this book; however, this material has been successfully covered by students with programming experience limited to FORTRAN.

In presenting the analysis methods, I have made an attempt, in each case, to relate their application to the example problems. The methods included are only a small portion of those that have been applied in simulation studies, but they do illustrate most of the "functions" of the analysis required in this process.

The examples for the most part include more details and repetition than are necessary for the reader with a good background in statistics. But often students with only an introductory course in statistics or those who have not used statistics recently need this type of review.

The example programs included in this text are an essential part of the material. In teaching courses, I have made these programs available to the students for use in several of the exercises. One method for doing this with an IBM 360/370 System is outlined in Appendix B. This could also be done with other equipment and with other programming languages, but it would require more effort by the instructor.

This is an introduction to simulation, not a survey of the entire subject area. The programming and mathematical skills common to many undergraduate students are used as the basis for presenting the concepts and procedures used in simulation studies. This book, I hope, will serve the interested reader as a starting point and guide toward the effective use of this very powerful tool.

James A. Payne

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CHAPTER

ONE

INTRODUCTION AND OVERVIEW

In the movie *The Man with the Golden Gun*, secret agent James Bond escaped one set of pursuers by driving his car up the twisted approach section of a collapsed bridge and jumping it over a canal. After the car left this approach section, it did a complete roll in midair, landed upright on the other side of the canal, and sped off. The screenplay credited this feat to Bond's vast knowledge, skill, and daring, but in reality this stunt was made possible by computer simulation.

The idea of using a curved takeoff ramp to achieve a roll-over jump had occurred to a number of stunt drivers, but the detailed plan for the first successful jump of this type was developed by analysts at the Calspan Corporation. This group derived a set of mathematical equations that describe the motion of an automobile in this type of situation and programmed a digital computer to solve these equations. The program was then used to determine the resulting trajectory, using various shapes for the takeoff ramp and different initial velocities for the car. In this manner, a ramp design was specified and an acceptable range for initial speed was determined. When this scheme was attempted the first time, it worked.

The roll-over jump became a part of the *American Thrill Show* and was later adapted for the film segment in *The Man with the Golden Gun*. A report on this work was presented at a Society of Automotive Engineers meeting in 1976 [7]. As discussed in the report, this work was carried out as part of a long-term project related to safety aspects of highway and roadside design.

The design of this stunt is a novel example of a procedure which has been widely used and is becoming increasingly beneficial in many areas of application. This process is to represent a given situation by mathematical and logical relationships embodied in a computer program and then to study the situation under various conditions by observing the behavior of the computer model. This simulation approach to problem solving has been used with many different types of problems, using a great variety of techniques for implementation.

The range of application areas and procedures used with this approach is too great to cover in any detail in one book. Furthermore, simulation techniques are, to some extent, specialized to specific types of problems or application areas. However, there are some general characteristics of these procedures which are common to and are useful for a wide variety of practical problems.

This book is an introduction to computer methodology for carrying out simulation studies. The emphasis is on programming techniques used with certain studies of this type and on methods required to analyze results obtained from the associated simulations. While computer methodology is only a part of the work required in simulations, these types of projects have given rise to some of the largest and most logically complex computer programs ever developed. This is a major area of computer application. Thus, simulation is an important area of computer science, but it is also a topic that illustrates how computers can interact with many fields of study.

This first chapter includes a discussion of the nature of the simulation process, indicates how it is used, and outlines the topics covered in the book. The objective is to provide a preview of the book's contents and to indicate how this material relates to the practice of applying the simulation approach to problem solving.

1.1 THE SIMULATION PROCESS

Simulation is a method used to study the dynamics of systems. The term *systems* is used here to mean a group of units which operate in some interrelated manner. Often the purpose of system studies is to gain an understanding of the overall operation when a group of well-understood units are connected. Simulation provides a description of system behavior as it evolves over a period of time.

The following definition is given by Shannon [14]:

> Simulation is the process of designing a model of a real system and conducting experiments with this model for the purpose either of understanding the behavior of the system or of evaluating various strategies (within the limits imposed by a criterion or set of criteria) for the operation of the system.

This is typical of the many formal definitions of this term and includes many types of activities not directly covered by the topics in this book.

Here we are primarily concerned with techniques for using general-purpose digital computers in simulations. We choose these topics not because they are the most important or interesting topics in the field of simulation but rather because they are a good starting point for the study of this subject. Our viewpoint is that a certain amount of experience in the practical details of carrying out simulation procedures is necessary in order to properly appreciate the more abstract concepts of a general discussion.

Simulation does not necessarily involve computers. But the availability of these devices has been the impetus to extend the application of simulation to many new areas. This process began with the development of analog computers, which were primarily applied to engineering design problems involving continuous models. Later the development of digital computers led to increased use of simulation in the areas of business and economics where discrete models were prevalent. As digital computing has decreased in cost and become more available, it has become the dominant mode for the simulation of both discrete and continuous models. These factors have led to the increasing use of computer simulation in almost all disciplines of study. In view of anticipated advances in digital computer technology, indications are that this trend will continue.

The simulation approach can be used to study almost any problem. However, it is a reasonable approach only under certain conditions. It requires that a model be constructed that represents system behavior in terms of mathematical and logical relationships between variables. This model must adequately represent the primary effects which relate to the problem being studied. Until such a model is available, simulation cannot be used.

After a computer model of the system is available, simulation is used to investigate the performance of the system. Each simulation run is essentially an experiment on the system. The advantage of simulation is that these experiments can be completely controlled and completely observed.

If the system model can be analyzed using mathematical techniques, this method is generally superior to the simulation method. Analytical solutions are more accurate, provide more information, and are usually more easily obtained than simulation results. Simulation should not be used to replace mathematical analysis. Instead, mathematical analysis should be used to replace simulation to as great a degree as possible.

The primary reason for using simulation is that many models cannot be adequately analyzed by standard mathematical techniques. This is usually the case when the interactions between variables are nonlinear or when random effects are inherent in the system. Systems with waiting lines, or queues, are examples of this situation. Although there are mathematical methods to solve queuing-type problems, the solutions are available only for limited cases and do not provide complete information about system behavior.

Simulation is a very powerful method for solving system problems because of its wide applicability and because it provides a laboratory to study systems without the costs of building or modifying the real systems. The basic method of simulation is easily understood and is simple to apply. However, it is very difficult to draw accurate conclusions from simulation studies. The difficulties are due to problems associated with both developing an effective simulation model and drawing meaningful conclusions from simulation results.

1.2 USES OF SIMULATION

In order to indicate the potential advantages that are offered by simulation, we will discuss a few examples of the type of applications where this method has been useful. This will also help to partially classify the many kinds of problems studied using this approach.

The simulation project for the automobile roll-over jump is an example where the model of the system was represented by a set of differential equations. The technique for describing the motion of a vehicle of this type is provided by the theories or laws of classical mechanics. The proper application of these laws can produce a mathematical model which very accurately and reliably represents the vehicle motion as a function of the external conditions acting upon the vehicle. The advantages of using simulation with this example include:

1. The ability to study the system without the expense and risk of experiments with the real system
2. The ability to determine the ramp shape and driving conditions required to achieve a complete roll before landing
3. The ability to determine the effect of variations from the specified driving conditions on the landing conditions

This is an example of engineering design where the objective is to produce a system that meets certain specifications. There are many areas of engineering where procedures similar to this may be used, and indeed much of the early development of simulation techniques was in these areas. However, the primary characteristics of engineering design models, and hence of their simulation requirements, are determined by the nature of the theories used to describe the physical processes. Thus many of the simulation techniques used in these areas are specialized to certain forms of equations.

Another application area of simulation is in production scheduling. This type of problem may be illustrated by the situation where a fixed number of parts must be produced, which requires the use of machine A and then machine B. Each part requires a specified length of time on each machine type. These times are known but may be different for each part. The problem is to select a sequence in which the jobs are to be run on the machines in order to minimize the time between the start of the first job on machine A and the completion of the last job on machine B. The model required to represent this system in a simulation program consists of the logical operations of selecting the next part to be processed and determining the time at which the processing is completed. The changes in system representation occur only when a part starts or completes processing on a machine; thus, the changes are discrete. A simulation program for this problem would use some rule for specifying the sequence in which parts are processed. The advantages of this approach would include:

1. The ability to compare a large number of parts selection patterns with little human effort

2. The ability to determine the best selection pattern before producing the parts
3. The ability to observe the effects of changes in the selection pattern, which may lead to general rules for specifying parts selection

A production-scheduling problem is typically more complex than this example, involving many operations and more involved relationships between operations. However, the modeling for such problems would be similar to that required for this example. In a production-scheduling problem the model development does not require any theories or special mathematical notation. Only simple arithmetic and logical relationships are needed to represent the system and its operation.

Another production-related example can be used to illustrate a significant aspect of most simulation problems—unpredictable behavior. Consider the case of a shop where 20 tire-building machines are operated by production workers. These machines malfunction from time to time and must be serviced by a repairperson. There are four repairpeople on duty at all times, and the machines are such that only one repairperson can work effectively on them at a time. There is a daily quota of tire production, so any down time above a fixed amount makes overtime work necessary.

Sometimes more than four machines are down, so all the repairpeople are busy and machines are waiting for service. At other times all machines are operating properly and all repairpeople are idle. More often, two or three repairpeople are not occupied. This raises the question of how many repairpeople should there be.

It has been observed that there is one chance in ten that a machine will break down in any given hour. The time required to repair also appears to vary in a random manner but with the following distribution:

40 percent require $\frac{1}{2}$ hour to fix
30 percent require 1 hour to fix
20 percent require $1\frac{1}{2}$ hours to fix
10 percent require 2 hours to fix

In order to simulate the behavior of this system some source of values to represent the chance occurrences is required. If a ten-sided fair die was available, with integers 1 through 10 marking the sides, a series of numbers could be generated by throwing the die repeatedly. This series could then be used to determine if a specific machine failed in a given period and also to determine the time required to repair the machines which did fail.

Using this series of random numbers and a logical model of the shop procedures, a 1-day period of operation could be simulated. The simulation could also be modified for different numbers of repair people, and in each case the cost of overtime operation could be calculated. Such a simulation would represent only a hypothetical day of operation. It would represent what might happen but would not predict the operation for any specific day.

However, if the daily operation were simulated repeatedly, with different