Introductory Signals and Circuits

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INTRODUCTORY SIGNALS AND CIRCUITS

Preface

The rapid rate of progress and the increasing number of new applications in circuit theory should call for frequent examination of the content and approach used in teaching the subject at the introductory level. In considering this examination, we must note that progress in integrated circuit fabrication requires that additional models be incorporated into the study. The widespread availability of digital computers for the solution of equations makes a more general formulation meaningful and necessary. The increased emphasis on the electric network as a signal processor requires that more attention be given to properties of signals. Further, there is a need to utilize a concrete vehicle for introducing topics in systems engineering; experience indicates that the electric circuit is the most successful such vehicle.

In this book, we present our attempt to adapt the body of knowledge known as electric circuit theory to the developments cited above. The book is intended to provide foundation concepts which are believed to have lasting significance and broad implications. We assume that the student who studies from this book will pursue the subject further in one or more advanced courses.

In keeping with this plan, the topics we have selected have been treated in sufficient detail to arouse student interest, but not in such depth that he will lose sight of the end objectives. We believe this kind of treatment will give the student an accurate picture of the objectives and methods of circuit theory. With this background, he will be more effective both in subsequent studies of circuit theory *per se* and also in many applications of circuit theory to the topics within the discipline of electrical engineering.

A number of the authors' convictions concerning teaching methods will be evident throughout the book:

(1) We believe that the fundamental principles should be expressed in their most general forms, to encompass applications to nonlinear, time-varying, or distributed systems, or for computer solution, but that at this point we can best illustrate principles for the linear, lumped, time-invariant case. Appreciation for the limitation and consequences of the linearity assumption is heightened by introducing occasional examples of nonlinear circuits and systems.

(2) We believe that the student should study simple ways for solving simple problems first. Most beginning engineering students are unable to comprehend the beauty of a completely general solution!

(3) Students must have extensive drill at this point in their careers, and this drill must make it possible to first understand the principle, and also to extend it to the solution of complex problems. Thus there is a need for both easy and more difficult problems.

In implementing these goals, we have provided numerous examples, drill exercises with answers for the student to test his understanding, and problems, some quite difficult, for the student to apply his knowledge and thus establish a learning situation. We have also postponed the general method of analysis of networks to the later parts of the book.

Given the philosophy we have just outlined, there remains the problem of selecting material to be covered. Which subjects should we treat, and which can be delayed for later study? To explain our choice of topics, we state that our experience over years of teaching is that for complete understanding to be achieved *frequency-domain ideas are more difficult for beginning students than time-domain ideas.* And of the frequency-domain ideas, one of the most difficult is that of the frequency spectrum. While the Fourier series and transform are excellent artifices for unifying the understanding of the subject, and indispensable in understanding signals in communication systems, these subjects best come later in the student's program of study. Having made this decision for the Fourier transform, we have decided to exclude the other transform methods also.

In stressing concept over specific facts, we have chosen to delay the study of methods for the efficient solution of simultaneous algebraic equations until Chapter 15. There will be no serious difficulty, of course, if this material is covered earlier, even before Chapter 1. Each instructor should decide the point at which this information has maximum usefulness. His decision will be influenced by the degree to which the computer will be used as an adjunct to the course.

We believe that the student should not be taught three-phase circuits as a *special topic*, for he may then tend to regard them as different from other circuits, which is certainly not the case. While three-phase problems are treated in the book, they are distributed throughout rather than being lumped in a single chapter.

The material in this book can be covered in about seventy class hours at the sophomore level. At the junior level, progress may be somewhat faster. There is enough material for a first course and perhaps part of a second course. It is assumed that the later courses will include transform methods.

We are indebted to many people who have assisted in the writing of the book, and most important of these are the students in our classes who provided the motivation for writing and who tested the product. We are deeply indebted to the community of scholars engaged in the study of circuits and systems at the University of Illinois who over the past years have provided a stimulating and congenial milieu in which this book could be developed. These include William R. Perkins, Donald A. Calahan, Leon O. Chua, Franklin F. Kuo, S. Louis Hakimi, Wataru Mayeda, James A. Resh, Ronald A. Rohrer, the late Sundaram Seshu, Manoel Sobral, Jr., Timothy N. Trick, Nelson Wax, and James R. Young. We have benefited from discussions with fellow authors Charles A. Desoer, Ernest S. Kuh, Benjamin J. Leon, and Leon O. Chua concerning conventions and symbols that should be used. We express special appreciation to Franklin F. Kuo and Jack Bourquin who read the complete manuscript and made numerous suggestions for improving the presentation. Finally, we are indebted to Mrs. Divona Keel, who typed the various drafts of the book with efficiency and spirit.

J. B. Cruz, Jr.
M. E. Van Valkenburg

January 1967

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Signal Sources and Signal Processing 1

1.1 Reference conventions for signal entities

We use the word *signal* to mean a time history of voltage or current in an electric network. The voltage or current variation with time may represent a message, music, a television picture, etc. In our elementary study of signals, we will include the sinusoidal waveform used to transmit energy. We exclude signals which must be described by statistical properties, leaving this important subject for later study. All signals we study may be described by real numbers which may be either positive or negative (and zero). It is important that we first understand the meaning of the positive or negative sign identified with the signal.

Consider a battery, an electric-energy source, which has its own exclusive symbol, that of Figure 1.1(a). The terminal of the battery with an excess of positive charge is called the *anode*, that with an excess of negative charge the *cathode*. It is conventional that the anode be distinguished by a plus sign and the cathode by a minus sign, as shown in the figure.

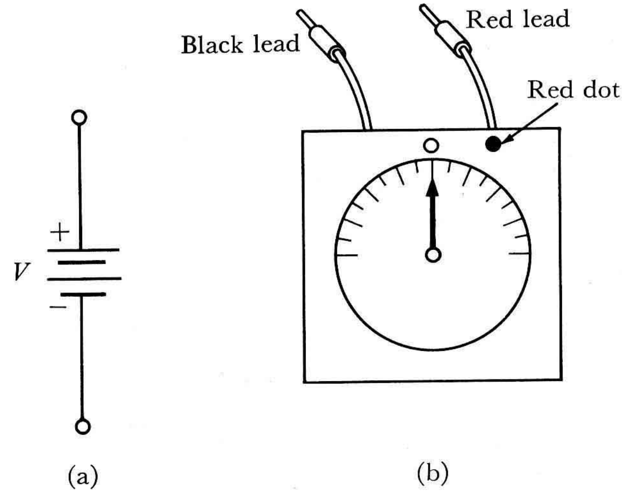

Figure 1.1 (a) Symbol for a battery. (b) Representation of an ideal voltmeter.

The battery voltage may be measured by a voltmeter. An idealized voltmeter or galvanometer suited to measurements of our interest measures instantaneous voltage, has a zero center scale, and is capable of deflecting to both the left and the right as in Figure 1.1(b). Reversal of voltmeter leads reverses the direction of deflection. It is usual to call one direction of deflection positive and the other negative. This is the *second* use to which we have put the words positive and negative, but this need not be a source of confusion if we make clear which use we intend.

Consider the experiment in which the two leads of the voltmeter are connected to the battery and the deflection is positive. The lead connected to the plus terminal or anode is designated as the positive reference lead (red); the other is the negative reference lead (black).† With these identifications defining the voltmeter deflections, we consider a general electric source. This source may reverse polarity with the passage of time. How can we describe this polarity variation?

We first postulate that the output terminals of the source have reference marks painted on them, one a plus and one a minus, these assignments having been made *arbitrarily* at the time the source left the factory. To this source we connect our calibrated voltmeter with the red lead of the voltmeter connected to the plus terminal of the source, and the black lead to the minus terminal. With this connection, a positive deflection of the voltmeter (or its counterpart in the form of an oscillograph) implies that the plus terminal of the source has an excess of positive charge and the

† In the laboratory, our red lead will often be called the "live" or "hot" lead, and the black one the "ground" lead.

source polarity is like that of the battery. Similarly, a negative deflection implies a source polarity opposite to that of the battery. Thus we see that the plus mark on the general source has a different meaning than the plus mark on the battery. In the case of a battery, the plus mark implies positive polarity; for the source, the plus mark is a *reference* in terms of which the polarity variation with time may be described. For this reason, the plus mark on the general source could just as well be a sign of the Zodiac. We can make the identifications "like the battery" and "opposite to the battery" only in terms of the reference marks on the source. Plots like those in Figure 1.2(b) have meaning only in terms of the reference marks.

For example, if the equation of a recorded voltage is $v(t) = V \sin t$, where V is a positive number, then the plus terminal of the source has a positive polarity from $t=0$ to $t=\pi$, a negative polarity from $t=\pi$ to $t=2\pi$, etc. A number of schemes are used instead of the plus and minus we have used; three of the most common alternate designations are shown in Figure 1.2(a).

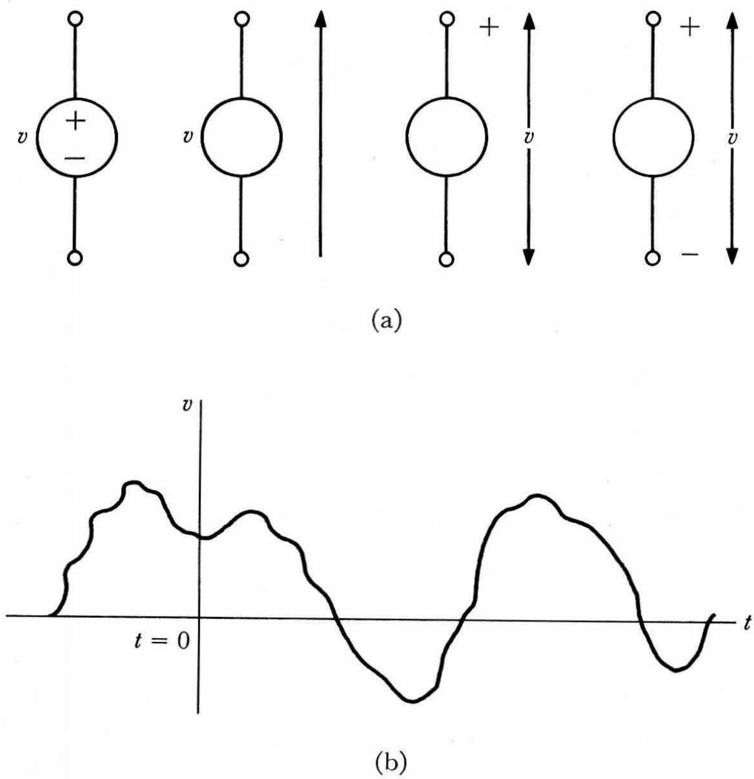

Figure 1.2 (a) Several alternate representations of voltages and their associated reference marks. (b) Plots of *v* versus time.

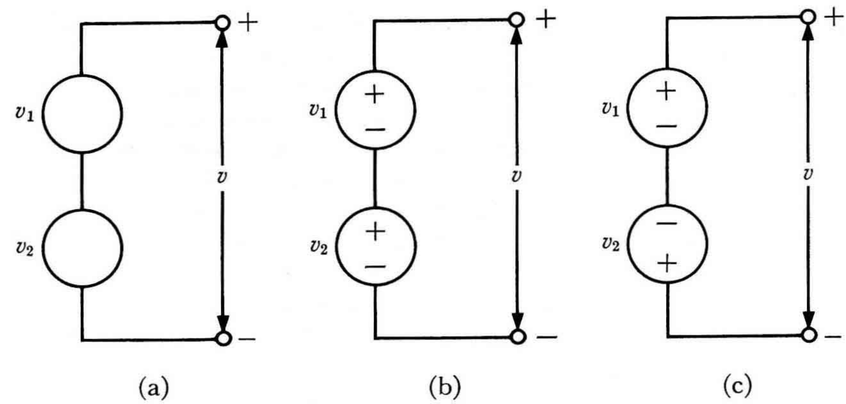

Figure 1.3 Two voltages in series, illustrating the need for reference marks in describing the voltage of the combination.

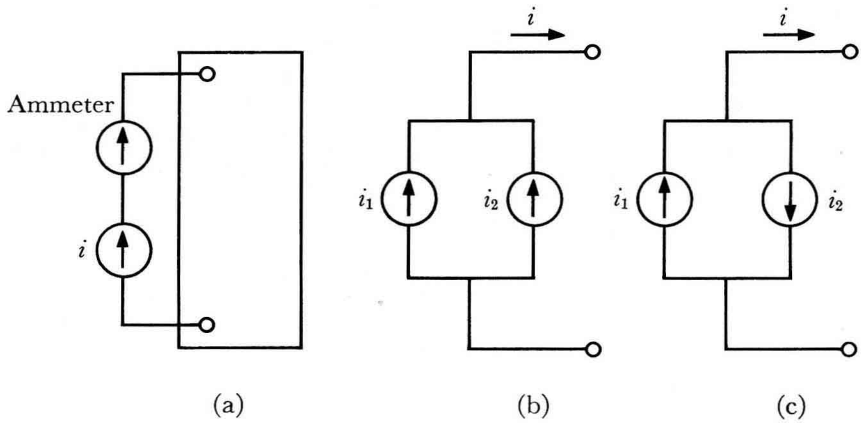

Figure 1.4 Conventional reference for currents.

The need for reference marks is further illustrated by the connection of two sources shown in Figure 1.3(a). Without reference marks on the sources, we do not know the relationship of v_1, v_2, and v. With the reference marks of Figure 1.3(b), we see that $v = v_1 + v_2$; with those of Figure 1.3(c), we have $v = v_1 - v_2$.

Statements similar to those given for the voltage reference apply to the current reference. The symbol used to indicate reference direction† is shown in Figure 1.4(a). The ammeter deflects in a positive direction when current is in the reference direction, negative when in the opposite of the

† Ammeters found in the laboratory are marked with plus and minus signs. The plus goes with the tail of the arrow, the minus with the head.

reference direction. The marks of Figure 1.4(b) imply that $i = i_1 + i_2$, while those of Figure 1.4(c) imply that $i = i_1 - i_2$.

In our discussion of source notation, we have used a single-subscript symbol like v_1 together with reference marks (plus and minus) to describe sources. Another notation commonly used in circuit theory is known as the *double-subscript notation.* If we use this notation, the voltage v_{jk} is *defined* to be the voltage between j and k with the plus mark implied at j. Thus we read v_{12} as the potential at 1 with respect to 2, meaning the voltmeter reading with the red lead connected to 1 and the black lead to 2.

In describing current sources by double subscripts, we see that i_{jk} is the current in the path from j to k with the reference arrow from j to k. Thus i_{12} is a current with the reference direction such that the tail of the arrow is at 1, the head at 2.

The equivalences for the two forms of notation for voltage and current are illustrated in Figure 1.5. Observe that for both cases, the reversal of subscripts implies a change of sign; thus we have

$$v_{12} = -v_{21}, \qquad i_{ab} = -i_{ba}, \qquad \text{etc.}$$

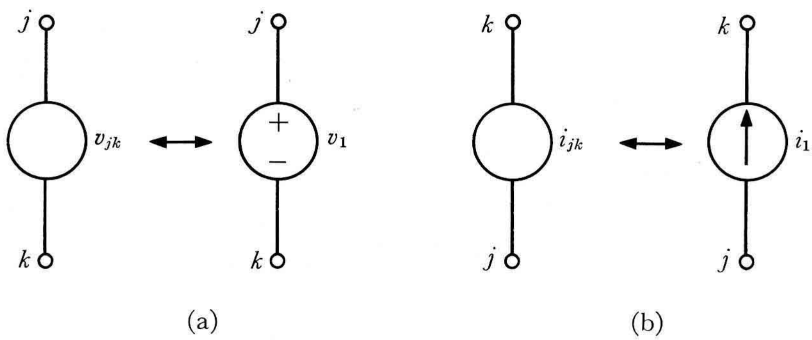

Figure 1.5 Equivalent representations, using double-subscript and single-subscript notation.

● EXERCISES

1.1-1 The voltage signal from a source is described by the equation $v_1(t) = 5 \sin(2t + \pi/6)$ volts. When $t = \pi/2$ sec, determine the terminal voltage and its polarity with respect to the reference marks of the source.

1.1-2 The current in a circuit is $i_{12} = 5 \sin(2t + \pi/6)$ amps. At $t = \pi/2$ sec, what is the current direction? The direction of electron flow? Repeat for $t = 0$.

1.1-3 Two identical ideal voltmeters are connected across a time-varying source. The red lead of one meter and the black lead of the other are connected to the same terminal of the source. The remaining leads are connected to the other terminal. What is the algebraic sum of the readings of the meters at all instants of time? How do the readings of the two meters compare?

1.2 Models of signal sources and notation

The devices associated with electric networks are commonplace: coils, capacitors, resistors, transistors, vacuum tubes. In studying circuit theory or network theory, we are not concerned with these devices per se but with abstractions or idealizations of them known as *models*. Good models are required to be simple and yet must accurately represent the device (or system) under specified conditions. In general, the choice of a model represents a compromise between simplicity and accuracy requirements.

Devices which are important sources of electric energy include the battery, electromechanical generators, and electronic generators. These and other electric energy sources may be represented by two models: the voltage source and the current source.†

Let a source of electric energy be connected to an arbitrary network as in Figure 1.6(a) and let the resulting current be denoted by $i_1(t)$ and the voltage across the terminal pair by $v_1(t)$. Suppose that instead of using Network No. 1, we had used some other arbitrary network as in Figure 1.6(b). In general, the resulting $i_2(t)$ would be different from $i_1(t)$ and $v_2(t)$ would be different from $v_1(t)$. However, if $v_1(t) = v_2(t)$ *no matter what arbitrary network we use*, then we define the source as an *ideal voltage source* or simply a *voltage source*. The voltage-source model is a source of electric energy with a prescribed voltage across its terminals. The resulting current depends not only on $v(t)$ but also the nature of the connected network.

If in Figure 1.6, $i_1(t) = i_2(t)$ *no matter what arbitrary network we use*, then we define the source as an *ideal current source* or simply a *current source*. This time, the source has a prescribed current through its terminals. Both

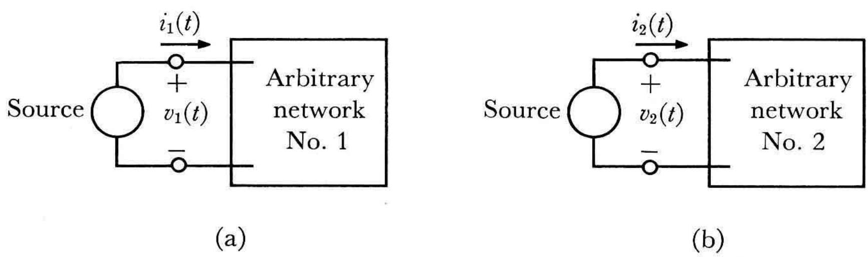

Figure 1.6 Arbitrary networks connected to identical sources of electrical energy. If the sources are voltage sources, $v_1(t) = v_2(t)$. If the sources are current sources, $i_1(t) = i_2(t)$.

† More elaborate source models will be introduced later.