Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

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Proceedings, Honolulu 1982/83

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Springer-Verlag
Berlin Heidelberg New York Tokyo

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held at the University of Hawaii, Honolulu, USA
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AMS Subject Classifications (1982): 3Cxx, 3Exx, 3Fxx, 12Axx, 12Bxx, 12Fxx, 13Axx,13Cxx, 13Dxx, 13Exx, 13Fxx, 13Gxx, 13Hxx, 13Jxx, 16A46, 18Exx, 18Fxx, 18Gxx, 20-04, 20Kxx, 22Bxx

ISBN 3-540-12335-0 Springer-Verlag Berlin Heidelberg New York Tokyo
ISBN 0-387-12335-0 Springer-Verlag New York Heidelberg Berlin Tokyo

Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.
2146/3140-543210

PREFACE

A conference on Abelian Group Theory was held at the Manoa Campus of the University of Hawaii from December 28, 1982 to January 4, 1983. It was probably the best attended conference on Abelian Group Theory to date with 55 participants from all over the world and the busiest one with 49 talks. A special feature were general interest lectures by Hyman Bass, Columbia University, on "Non-linear Algebra", and by Claus Michael Ringel, Universität Bielefeld, on "Representations of Algebras". The Conference offered surveys by Laszlo Fuchs, Tulane University, on "Torsion Modules over Valuation Rings", Fred Richman, New Mexico State University, on "Mixed Groups", Paul Eklof, University of California at Irvine, on "Set Theory and Structure Theorems", Rüdiger Göbel, Universität Essen on "Endomorphism Rings", and Lee Lady, University of Hawaii, on "Torsion Free Groups of Finite Rank". The research reports attested to lively activity in the traditional and in new areas of inquiry within and around Abelian Group Theory. The best represented groups were those of finite rank without torsion, a field employing increasingly sophisticated tools from ring theory and algebraic number theory. The use of set theoretic methods continues to flourish after the pioneering work of Saharon Shelah in the 1970s, and is delivering impressive results.

This volume contains the papers of the participants of the Conference and a number of additional articles of others who could not or did not come. It reflects faithfully and almost completely the present status of Abelian Group Theory and should be invaluable to those in Abelian Groups and those who want to be informed.

Rüdiger Göbel, Universität Essen
Lee Lady and Adolf Mader, University of Hawaii

ACKNOWLEDGEMENTS

The University of Hawaii hosted the Honolulu Conference and provided facilities, secretarial assistance, and inexpensive accommodations. The University of Hawaii Foundation and the UH Division of Natural Sciences sponsored the general interest speakers, and the East West Center at the UH helped with housing participants. Adolf Mader did most of the organizational work. His wife, Emma Mader, provided moral and other support, and added heart to the meeting of minds particularly by preparing with friends a most enjoyable party on New Years Eve. The final touches and repairs of the manuscript were done by Elisabeth Wrase of the Universität Essen. One of the manuscripts was voluntarily typed by stud.math. Elisabeth Kersten of the UE. To all sincere thanks.

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Contributed papers:

A SEMINAR ON SPLITTING RINGS FOR TORSION FREE MODULES OVER DEDEKIND DOMAINS

E. L. Lady

This seminar is an introduction to the concepts dealt with in [10] through [15]. It is in some sense a "prequel" to those papers, since it provides most of the background material needed to read them. It is based on extensive talks given in a disjointed, disorganized fashion to varying audiences in Honolulu from time to time during the past few years.

In writing papers, one feels that one has fulfilled one's obligation to the reader with respect to preexisting mathematics if one provides precise and accurate citations to the literature. In talking before a live audience, unfortunately, one finds that this procedure is not tolerated. In the process of figuring out how to present this material in a self-contained way, I was struck by how simple many of these matters ultimately turned out to be. I was also struck by how much simpler the proofs I was presenting for my own theorems were than those in my published papers. Furthermore, I became aware that various things which in my papers I had either taken for granted or dismissed with a bit of hand waving were met with considerable puzzlement and sometimes skepticism from live audiences. And in some cases I found myself hard pressed to come up with careful, convincing proofs. In a few cases I found that my assertions were not exactly true.

Since it takes the better part of a semester to deliver this Seminar (at a quite rapid pace), I eventually decided that it would turn out in the end to be less effort to write the whole thing down rather than to continue giving it live whenever another visitor arrived for an extended stay in Honolulu. (Besides, eventually the permanent faculty here would get tired of hearing it.)

The notion of a splitting ring for a torsion free module is somewhat analogous to that of a splitting field for a semi-simple algebra. Namely, when one extends scalars up to the splitting ring I, the structure of the module becomes trivial. The hope is, then, that by looking at those modules which are I-split for a particular splitting ring I (usually with finite rank) one obtains a more manageable class.

The starting point for my study of splitting rings was the Kurosh Matrix Theorem [8], as interpreted by Beaumont and Pierce [3]. Letting $\hat{W}$ be the completion of a discrete valuation ring W, one sees that a finite rank torsion free W-module G is determined up to quasi-equality by $d(\hat{W} \otimes G)$ and the usual map $\sigma: QG \rightarrow \hat{W} \otimes QG$. Namely, G is quasi-equal to $\sigma^{-1}(M \oplus d(\hat{W} \otimes G))$, where QG is the divisible hull of G and M is a free $\hat{W}$-submodule of $\hat{W} \otimes QG$ such that QM is a complementary summand to $d(\hat{W} \otimes G)$ within $\hat{W} \otimes QG$.

The Kurosh Theorem is in bad repute (at least in the West.) Partly this is because Kurosh looked at matters in terms of matrices, which put later generations of mathematicians off. Partly it is because the Theorem really only makes sense within the context of quasi-isomorphism, which Kurosh was unfamiliar with. And partly it is because too much was claimed for it. As a method for constructing torsion free modules, it is probably as good as anything, once one stops thinking in terms of matrices. As a method for getting insight into a particular module, it seems little better than nothing at all.

My starting point was to replace the complete ring $\hat{W}$ by a pure subring I, and look at the class of modules G such that $I \otimes G$ is the direct sum of a free I-module and a divisible one. Later on, some representation theoretic papers I was reading and a remark made to me by Mary Turgi at the Las Cruces conference in 1976 made me realize that it was more useful to stand the Kurosh Construction on its head and look at a reduced module G as the intersection of its divisible hull QG and a free I-module. This was spelled out in [10]. Somewhere along the way it also became important to drop the restriction that W be local. One then takes I to be a pure subring of the W-adic completion of W, although it was only in [12] that I realized that that was what I was doing. (The fact that the conceptual framework keeps changing is one of the things that make [10] - [15] hard to read.)

The first five sections of this Seminar are largely self-contained, provided that the reader knows elementary facts about tensor products, functors, natural transformations, and the like, such as are covered in most graduate algebra courses, as well as the rudiments of abelian group theory. In these sections, citations to the literature are provided only for historical purposes. In Section 6, it was necessary to assume a little more knowledge about commutative rings, such as can be found in most standard textbooks on the subject. Abelian group theory, being the study of modules over dedekind domains, is a very specialized branch of commutative ring theory, and it seems not unreasonable to assume that its practitioners should have some familiarity with the wider theory.

In particular, certainly the reader will know that a dedekind domain is a commutative noetherian ring such that every non-zero prime ideal is maximal and every localization at a prime ideal is a discrete valua-