CONTROL OF NONLINEAR MECHANICAL SYSTEMS

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PREFACE

A modern mechanical structure must work at high speed and with high precision in space and time, in cooperation with other machines and systems. All this requires accurate dynamic modelling, for instance, recognizing Coriolis and centrifugal forces, strong coupling effects, flexibility of links, large angles articulation. This leads to a motion equation which must be highly nonlinear to describe the reality. Moreover, work on the manufacturing floor requires coordination between machines, between each machine and a conveyor, and demands robustness of the controllers against uncertainty in payload, gravity, external perturbations etc. This requires adaptive controllers and system coordination, and perhaps a self organizing structure. The machines become complex, strongly nonlinear and strongly coupled mechanical systems with many degrees of freedom, controlled by sophisticated mathematical programs. The design of such systems needs basic research in Control and System Dynamics, as well as in Decision Making Theory (Dynamic Games), not only in the use of these disciplines, but in their adjustment to the present demand. This in turn generates the need to prepare engineering students for the job by the teaching of more sophisticated techniques in Control and Mechanics than those contained in previous curricula.

On the other hand, all that was mentioned above regarding the design of machines applies equally well to other presently designed and used mechanical structures or systems. We have the same fundamental problems in active control of flexible large space structures (LSS) and high rise building structures, as well as in the flight control of air or spacecraft, including air traffic control and air combat games.

Working on basic methodology in all these directions makes one realize how valuable the interface between the applications may be to each of them separately. Techniques used to design flexible links in manipulators and LSS are presently developed jointly. It is perhaps still not sufficiently realized that coordination control of robotic manipulators may be obtained by methods used in air combat games, and that such games may be used in robot decision making. The investigation of such an interface may result in the means to overcome many problems in present design practice. The book attempts to give the fundamental background for such investigation.

As mentioned, the dynamical models in all of the above applications, in order to be realistic, must recognize untruncated nonlinearity of the acting forces and be robust against uncertainties hidden in modelling and/or external perturbations. The study of the interface makes it obvious that in order to handle such models, one must seek for methods entirely different from those used in classical Control Theory, which approximates reality with linearized models. Although it might not be immediately visible from behind the Laplace transformation, Control Theory had been born out of Mechanics, particularly Nonlinear Mechanics. The latter has been developing quite rapidly for the last twenty or thirty years, but this was somehow unnoticed by the control theorists. Now, with the applications mentioned, Nonlinear Mechanics may no longer be ignored in Control Dynamics, and the demand for it is growing rapidly. There is no applied text available which would deal with control of fully nonlinear, uncertain mechanical systems. This book has been written to fill the gap.

The thirty years of research work which this author has devoted to the subject gives him the advantage of knowing what is needed, but also the disadvantage of habitually favoring some of the topics. This bias has proved useful, considering the space limitations which must be imposed on any text. I hope, however, that the book is a healthy compromise between the needs and the bias.

The first chapter outlines the models of mechanical systems used, the second and third introduce the reader to the energy relations and the Liapunov design technique applied later. Chapter Four specifies the objectives of control and the types of controllers used in the three basic directions of study: robotics, spacecraft structures and air games. Sufficient conditions, control algorithms and case studies in these three directions are covered by Chapters 5, 6 and 7, in terms of control (collision, avoidance and tracking, respectively), while Chapter 8 deals with the same problems but subject to conflict.

The text has grown up from lecture notes for junior graduate and senior undergraduate courses taught at Mechanical Engineering, University of Southern California, Los Angeles, in Advanced Mechanics, Analytic Methods of Robotics, Control of Robotic Systems, and at University of Queensland, Australia, in Control Theory, Systems Dynamics and Robot Theory. Apart from natural use in such courses, the book may serve as reference for the design of control algorithms for nonlinear systems.

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LOS ANGELES, JANUARY 1989

J.M. SKOWRONSKI

CONTENTS

Chapter 1

MECHANICAL SYSTEMS

1.1 SIMPLE MECHANICAL SYSTEMS

It seems both convenient and illustrative to introduce some of our later defined notions on simple but typical examples of mechanical systems. Perhaps the simplest and, at the same time, most typical mechanical system is a single link mathematical (idealized) pendulum discussed in the following example.

EXAMPLE 1.1.1. Consider the simple pendulum shown in Fig. 1.1, swinging about the base point 0 in the Cartesian plane Oxy, by the angle $\theta(t)$, for all $t \geq t_0$, where $t_0 \in \mathbb{R}$ is an initial time instant. The plane is a part of the *Cartesian* physical *coordinates space* $Oxyz$ where the position of the point-mass m is specified by the current values of $x(t)$, $y(t)$, $z(t)$ subject to obvious *constraints*: $z = \text{const}$, $x^2 + y^2 = \ell^2$. Under such constraints only one variable can be independent, and thus we say that the system has a single *degree of freedom* (DOF). It is more convenient to choose $\theta(t)$ as the *generalized* (lagrangian) *coordinate* describing such DOF, rather than $x(t)$ or $y(t)$, although the choice of either of these two is obviously possible. We thus define $q(t) \triangleq \theta(t)$, $t \geq t_0$. The point-mass m is considered an *object* in a *point-mass model* of a mechanical system. In our simple case the model consists of the object concerned with the single DOF specified by $q(t)$.

The generalized variable $q(t)$ is free of the Cartesian constraints mentioned but it has its own work limitations in some interval Δ of its

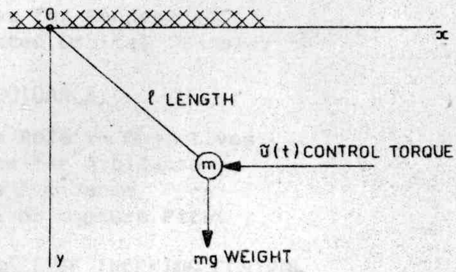

Fig. 1.1

values. For instance, when the pendulum is suspended from a ceiling which is non-penetrable, we must impose $\Delta : -\pi \le q(t) \le \pi$.

In a symbolic way, we represent the system as a single mass cube railed to move in one direction only, generally subject to gravity $\tilde{G}$, spring or elastic (in link) forces $\tilde{K}$ and damping force $\tilde{D}$, as well as to an external input - control force (torque) $\tilde{u}$, see Fig. 1.2. This representation is well known as the *schematic diagram* of the system structure. The arrows crossing the symbols of elastic and damping connections indicate that the corresponding forces may be represented by nonlinear functions. The damping symbol is shown as a damper open from above if the damping is positive, and from below it if is negative.

Fig. 1.2

The point mass subject to weight $\tilde{G}$ is restricted to the vertical motion only as shown in Fig. 1.2, modelling the single degree of freedom.

For simplicity of exposition, we ignore the elastic force in the link. The damping force $\tilde{D}$ is made dependent upon the velocity $\dot{q}(t) \triangleq dq(t)/dt$ and is specified by the function $\tilde{d}|\dot{q}|\dot{q}$, $\tilde{d} > 0$ to be positive damping. Since we ignored elastic forces, the potential force acting upon the mass m reduces to gravity, specified by its component $mg \sin\theta$, where g is the earth acceleration. Then the Lagrange equation of motion gives

$$m\ell^2\ddot{q} + \tilde{d}|\dot{q}|\dot{q} + mg\ell \sin q = \tilde{u} . \tag{1.1.1}$$

With m, ℓ constant and measured, it is convenient to rewrite (1.1.1) in terms of the forces per coefficients of inertia which we call *characteristics* of the forces involved. We obtain

$$\ddot{q} + D(\dot{q}) + \Pi(q) = u \tag{1.1.2}$$

where $D(\dot{q}) \triangleq \tilde{d}|\dot{q}|\dot{q}/m\ell^2$, $\Pi(q) \triangleq (g/\ell)\sin q$ and $u \triangleq \tilde{u}/m\ell^2$. Introducing the force characteristics frees the acceleration term in (1.1.1) from the inertia coefficient. For multidimensional systems, such a procedure is connected with decoupling the equations inertially (dividing by the matrix of inertia), which then makes it possible to apply the results of control theory, usually formalized in terms of the normal form of differential equations, see later examples.

The potential energy of the pendulum is then expressed by

$$V(q) = V^0 + \int_{q(t)} \Pi(q)dq \tag{1.1.3}$$

with the initial storage of energy $V^0 = V(q^0)$, where $q^0 = q(t_0)$, $t_0 = 0$ being the initial instant of time. The equilibria of the pendulum occur at rest positions $\dot{q} = 0$ coinciding with the extrema of the function $V(\cdot)$, i.e. $\sin q = 0$ or $q^e = n\pi$, $n = 0, \pm 1, \pm 2, \ldots$. As is well known from elementary mechanics, the minima correspond to the Dirichlet stable equilibria occurring at the downward positions of the pendulum, after each full rotation by 2π. The maxima correspond to Dirichlet unstable equilibria occurring at the upwards positions obtained on every half-turn by π from the preceding stable equilibrium. The maximal values of $V(\cdot)$ at these positions form the energy thresholds which have to be passed before another stable equilibrium is attained, i.e. before the pendulum realizes a rotation.

Obviously the gravity characteristic $\Pi(q) = (g/\ell)\sin q$ is a highly nonlinear function. It can, however, be expanded as a Taylor or power series

$$\Pi(q) = (g/\ell)\left[q - \frac{q^3}{3!} + \frac{q^5}{5!} - \cdots\right]. \qquad (1.1.4)$$

Since q is bounded for physical reasons, the equilibria which are zeros of (1.1.4) become zeros of some polynomial with a number of terms related to the number of rotations performed, see Fig. 1.3.

As there are many cases in engineering design where truncation of the series is necessary, if only for computational reasons, it is of interest to see when and to which extent such an operation is physically justified.

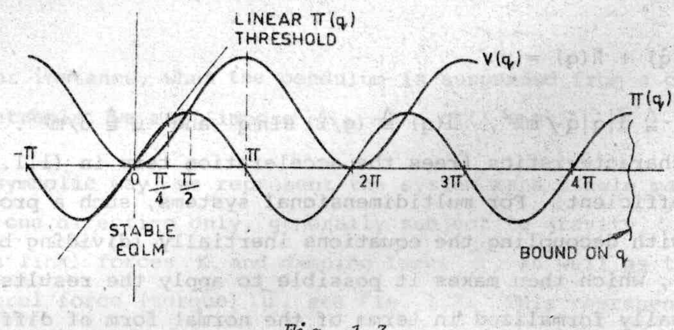

Fig. 1.3

For $q \in [-(\pi/4), (\pi/4)]$, the linear approximation $\Pi(q) = (g/\ell)q$ may be satisfactory, see Fig. 1.3. However, for larger swing angles, we need to include the nonlinear terms in (1.1.4) (the more of them the larger the swing angle). For the pendulum turning upwards, but not falling down again: $q = \pm\pi$, we need at least

$$\Pi(q) = (g/\ell)(q - \tfrac{1}{6}q^3), \qquad (1.1.5)$$

for the pendulum falling down again: $q = \pm 2\pi$, we need

$$\Pi(q) = (g/\ell)(q - \tfrac{1}{6}q^3 + \tfrac{1}{120}q^5), \quad \ldots \text{ etc.}$$

Consequently the equation of motion (1.1.2) becomes nonlinear and must be treated as such, if we do not want our model to disagree principally with the physical reality of the rotating pendulum. The nonlinear terms of the force characteristics cannot be truncated. This also means that we must recognize the existence of several stable equilibria separated by thresholds. With positive damping, these equilibria will attract motion trajectories in the *phase-space* (state space) $0q\dot{q}$ from specific regions of attraction, see Fig. 1.4, and will in fact be in competition as to their attracting role. Each attracting equilibrium (attractor) will have its own (winning) region of attraction. Given the same controller, depending upon where the trajectory starts (initial conditions) it will land in a corresponding attractor. By truncating the nonlinearities, we ignore all the equilibria except the single basic equilibrium at $q = 0$ and we may be led into a false sense of security, assuming that trajectories from everywhere will land in that equilibrium. Such a conclusion may be true only in some neighbourhood of the basic equilibrium (below the thresholds) but a slight change of initial conditions beyond this neighbourhood, i.e. beyond a threshold, may produce unstable trajectories tending somewhere else than intended. Then we may need a power expensive controller to rectify the situation. Moreover, the further from the threshold we are, the more costly such a controller becomes, possibly beyond its saturation value.

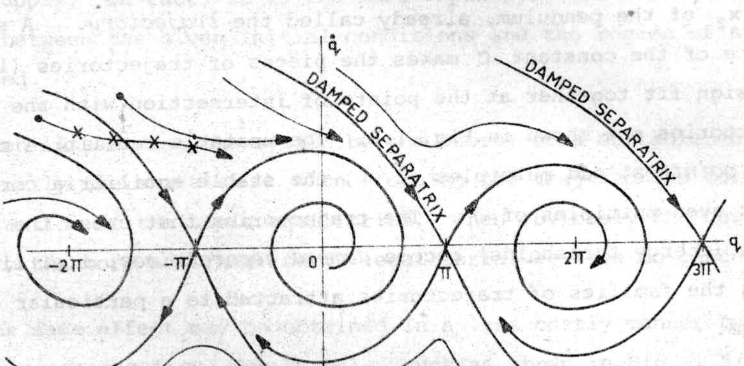

Fig. 1.4

Let us have a closer look at the trajectories. In our case it will be possible to do it directly, as the equation (1.1.2) with $u \equiv 0$ is integrable in closed form. We choose the state variables x_1, x_2 by substituting $x_1 \triangleq q$, $x_2 \triangleq \dot{q}$ and rewrite (1.1.2) with $d = \tilde{d}/m\ell^2$ as

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -(g/\ell)\sin x_1 - d|x_2|x_2 + u , \end{aligned}\right\} \qquad (1.1.6)$$

or in terms of the directional field in the phase-plane (state-plane) $0x_1x_2$:

$$\frac{dx_2}{dx_1} = \frac{-(g/\ell)\sin x_1 - d|x_2|x_2 + u}{x_2} . \qquad (1.1.7)$$

For the moment let us free the system from control, i.e. assume $u(t)$ to be a given function of time, in particular $u(t) \equiv 0$. Since $x_2 = |x_2| \operatorname{sign} x_2$, we can rewrite (1.1.7) as

$$\frac{dx_2^2}{dx_1} \pm 2dx_2^2 = -2(g/\ell)\sin x_1 \tag{1.1.8}$$

where the plus sign is used for $x_2 > 0$, and the minus whenever $x_2 < 0$. Except for these changes in sign, (1.1.8) is a linear equation in x_2^2 with x_1 as an independent variable. Hence the solutions of (1.1.8) are elementary:

$$x_2^2 = Ce^{\pm 2dx_1} + \frac{2g\cos x_1}{\ell(1+4d^2)} \pm \frac{4gd\sin x_1}{\ell(1+4d^2)} \tag{1.1.9}$$

where C is a constant of integration and where the signs ± are interpreted as in (1.1.8).

The above first integral of (1.1.8) is a curve $x_2(x_1)$ in the phase-plane Ox_1x_2 of the pendulum, already called the *trajectory*. A suitable choice of the constant C makes the pieces of trajectories (1.1.9) due to the ± sign fit together at the points of intersection with the x_1-axis. The trajectories are shown in Fig. 1.4. The unstable equilibria correspond to *saddle points* at odd multiples of π, the stable equilibria correspond to *foci* at even multiples of π. The trajectories that cross the unstable equilibria (energy thresholds) become *damped separatrices*, i.e. lines separating the families of trajectories attracted to a particular stable equilibrium.

The number of full rotations exhibited by the pendulum depends upon the initial magnitude of the velocity x_2. The greater this initial speed, the greater the number of full rotations, provided the system is free. The region between the separatrices enclosing the corresponding stable equilibrium is the region of attraction to this equilibrium considered the attractor. As mentioned, for the free system, the trajectories from outside this region will be attracted to some other attractor, and we may never be able to attain the target of a trajectory unless we start from a suitable region of attraction. The trajectories from beyond the threshold can also become entirely unstable and unbounded.

Let us now consider the controlled system, i.e. when u(t), generally non-zero, has been determined by a specified control program which is a function of the state x_1, x_2:

$$u(t) = P(x_1(t), x_2(t)), \quad t \geq t_0 . \tag{1.1.10}$$

Between the separatrices, there is little need for a controller to produce any inputs in order to lead the pendulum to the corresponding stable equilibrium. However, if the initial values x_1^0, x_2^0 lie between separatrices other than these bounding the desired stable equilibrium (see trajectory denoted by crosses in Fig. 1.4), i.e. outside the region of attraction to the desired equilibrium, an additional force is needed in order for a trajectory to pass over the energy threshold, which corresponds to the separatrix to be crossed. In terms of the equation (1.1.2) it means to produce the control program for u(t) which cancels some (one or more) nonlinear terms of the polynomial by which the gravity force is represented, say for instance,

$$u(t) = -(g/\ell)(\tfrac{1}{6}\theta^3 - \tfrac{1}{120}\theta^5 + \cdots) . \qquad (1.1.11)$$

It cuts off the thresholds or, in physical terms, forces the rotation back to the basic region of $\theta \in [-\pi,\pi]$. It may prove expensive in terms of power supply. In fact, it is the more expensive, the more thresholds must be cut between the given initial conditions and the region of attraction attempted.

When linearizing the system by the choice of a suitable control, we have exactly the above case when forcing the trajectories to the region of attraction of the basic equilibrium. Then obviously the cost is the greater, the more nonlinear terms (equilibria) we have to cancel.

The same effect may be obtained in a less costly manner by a *gravity* or *spring compensation*, for example, such as shown in Fig. 1.5(a) below. A counterweight Mg on the radius L adds to $M(\theta) = (g/\ell)\sin\theta$ an additional $M'(\theta) = (ML/m\ell)\sin(-\pi)$ which, with a suitable choice of M and L, holds the link swinging about its upward position instead of downward. Moreover, the above is done not by control torque but by structural means. This reduces the control torque needed to that generating a small swing about the new equilibrium. A very similar effect could have been obtained by inserting a spring about the suspension point of the equilibrium, the spring supporting the link in the desired position, see Fig. 1.5(b). For the linearisation effect, the spring characteristic may be specified, for instance, by $K(\theta) = -(kg/\ell)\theta - (g/6\ell)\theta^3 + \dots$, $k < 1$, again depending upon the number of nonlinear terms we wish to cancel.

The control or structural cancellation of the nonlinear terms is however not possible when the *nonlinearity is a target of our design* - as for instance if we want to produce a system that will aim at achieving a specified equilibrium, say second or third from the basic, i.e. will aim at work after a specific number of rotations. The latter case often applies in engineering. □

The gravity or spring compensation mentioned in the above example apply generally, even for a system of much higher dimensions. In some cases the designer has also an option of nonpotential compensation. He may use extra damping - positive by inserting fluid dampers or negative by designing self-oscillatory devices.

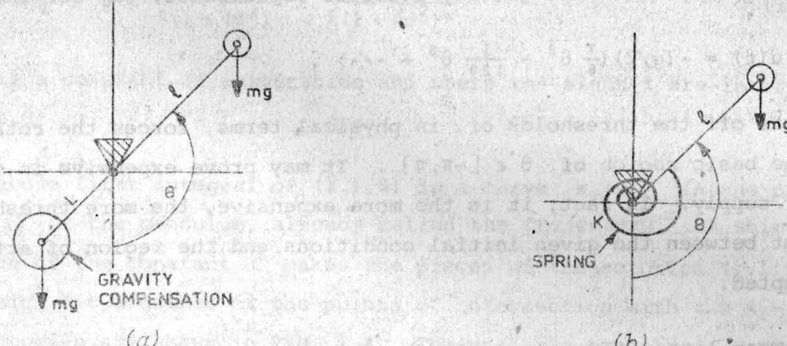

Fig. 1.5

As mentioned, the model of Example 1.1.1 was a special simple version of the class of mechanical models called *point-mass representation*, where the mass or more generally the inertia of the system is reduced to a finite number of material points, each with at most three degrees of freedom (DOF) - translations in the three dimensional Cartesian space $Oxyz$ of physical coordinates. Such masses are considered reduced objects of the system. In Example 1.1.1 we had a single object moving with a single DOF. Let us consider now some cases with two DOF.

EXAMPLE 1.1.2. Two simple pendula of Example 1.1.1 are coupled by a spring at the given distance from their joint suspension base, see Fig. 1.6. Their basic equilibria are attained in the vertical downwards positions of the pendula. The system now has two objects, point-masses m_1, m_2, with positions specified by coordinates x^i, y^i, z^i, $i = 1,2$ measured from their corresponding basic equilibria in $Oxyz$. As the pendula move in the vertical plane, we have constraints $z^i = \text{const}$, $(x^i)^2 + (y^i)^2 = \ell^2$, and moreover the coupling generated constraint $y^i > 0$, $i = 1,2$, as full rotation is not possible. Again generally, the two point-mass modelled object will have six DOF $(3+3)$, but the constraint relations reduce the DOF of the system to two. Similarly as in Example 1.1.1, it is more convenient to choose the