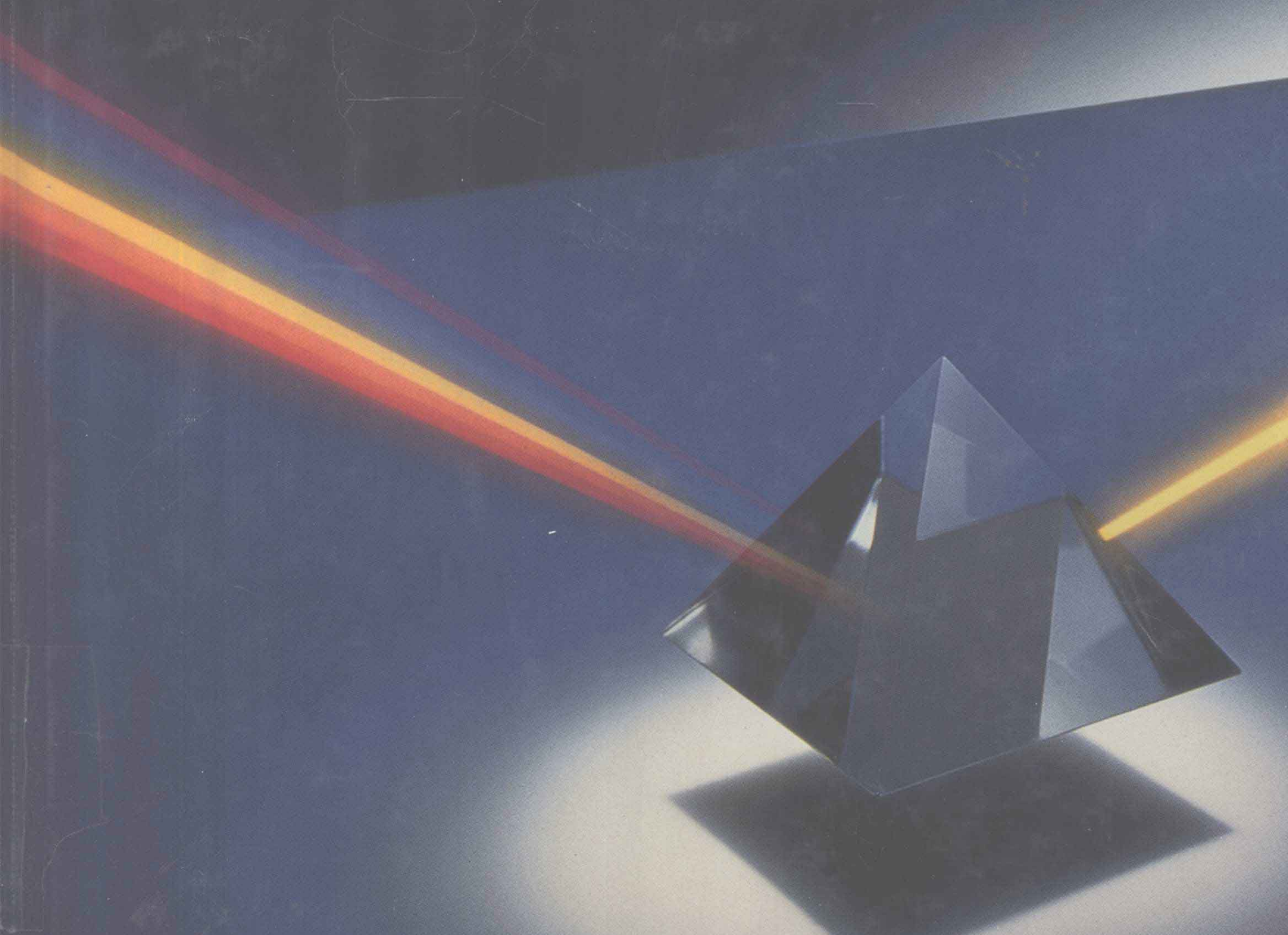
CALCULUS AND ANALYTIC GEOMETRY

Sherman K. Stein
Anthony Barcellos



FIFTH EDITION

CALCULUS

AND ANALYTIC GEOMETRY

FIFTH EDITION

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American River College

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ABOUT THE AUTHORS

Sherman K. Stein is Professor of Mathematics at the University of California, Davis, where he has been on the faculty since 1953. He has degrees in mathematics from the California Institute of Technology and Columbia University. In 1975 he received an honorary doctor of letters degree from Marietta College in recognition of his textbooks. Stein was chosen as one of the Distinguished Teachers on the UC Davis campus in 1974. He was selected in 1975 for the Mathematical Association of America's Lester R. Ford Award for expository writing.

In addition to his calculus book, Stein has written a series of high-school algebra and geometry books in collaboration with G. D. Chakerian and C. D. Crabill, as well as *Mathematics: The Man-made Universe,* a text for "mathematical ideas" courses. Stein's principal research field is abstract algebra, with a special emphasis on tiling and packing problems.

Stein has lectured and written on the teaching of mathematics. His talk, "Gresham's Law: Algorithm Drives Out Thought," delivered at the 1987 meeting of the American Mathematical Society in San Antonio, was published in *For the Learning of Mathematics,* 1987, and in the *Journal of Mathematical Behavior,* 1988.

Anthony Barcellos has been a member of the mathematics faculty of American River College in Sacramento, California, since 1987. He holds degrees in mathematics from Porterville College, the California Institute of Technology, and California State University, Fresno. Barcellos began his student teaching at UC Davis under the supervision of Alan H. Schoenfeld in 1975 and later became Sherman Stein's teaching assistant. While working at Davis, he received both the departmental and the campus citations for distinguished teaching by a teaching assistant. His textbook experience began with the *Student's Solutions Manual* to the second edition of Stein's calculus book in 1977; he has participated in each subsequent edition.

Barcellos worked as a science journalist for the *Albuquerque Journal* in 1978 under the sponsorship of the American Association for the Advancement of Science's Mass Media Fellowship Program. In 1979 he joined the staff of the California State Senate as a Senate Fellow and worked thereafter for the Commission on State Finance, California's principal econometric forecasting agency.

Barcellos has served on the board of editors of the Mathematical Association of America's *College Mathematics Journal.* He received both the George Pólya Award (1985) and Merten M. Hasse Prize (1987) for expository writing for an article on Mandelbrot's fractal geometry. His interviews of Benoit Mandelbrot, Martin Gardner, and Stanislaw Ulam were published in *Mathematical People* (Birkhäuser Boston). From 1985 to 1990, Barcellos was editor of *Sacra Blue,* the monthly magazine of the Sacramento Personal Computer Users Group.

To
Joshua,
Rebecca,
and
Susanna

To
My
Parents

The great body of physics, a great deal of the essential fact of financial science, and endless social and political problems are only accessible and only thinkable to those who have had a sound training in mathematical analysis, and the time may not be very remote when it will be understood that for complete initiation as an efficient citizen of one of the new great complex worldwide states that are now developing, it is as necessary to be able to compute, to think in averages and maxima and minima, as it is now to be able to read and write.

H. G. Wells
Mankind in the Making, p. 192, Scribner's, New York, 1904.

. . . at about the age of sixteen, I was offered a choice which, in retrospect, I can see that I was not mature enough, at the time, to make wisely. The choice was between starting on the calculus and, alternatively, giving up mathematics altogether and spending the time saved from it on reading Latin and Greek literature more widely. I chose to give up mathematics, and I have lived to regret this keenly after it has become too late to repair my mistake. The calculus, even a taste of it, would have given me an important and illuminating additional outlook on the Universe, whereas, by the time at which the choice was presented to me, I had already got far enough in Latin and Greek to have been able to go farther with them unaided. So the choice that I made was the wrong one, yet it was natural that I should choose as I did. I was not good at mathematics; I did not like the stuff. . . . Looking back, I feel sure that I ought not to have been offered the choice; the rudiments, at least, of the calculus ought to have been compulsory for me. One ought, after all, to be initiated into the life of the world in which one is going to have to live. I was going to have to live in the Western World . . . and the calculus, like the full-rigged sailing ship, is . . . one of the characteristic expressions of the modern Western genius.

Arnold Toynbee
Experiences, pp. 12–13, Oxford University Press, London, 1969.

TO THE INSTRUCTOR

We live in a fascinating time in which to teach calculus. Today, the way calculus is taught has become the focus of much discussion if not of a real revolution. Instructors have been urged to use calculators and computers, add meaningful applications for motivation, encourage students to express themselves in writing, and—above all—emphasize conceptual learning.

While the calculus debate encourages authors to depart from tradition, it also tempts them to follow what might be a passing fancy, or to hear only the most strident voices. In this fifth edition, we have taken advantage of the increased freedom but resisted the lure of change for its own sake. We have adhered to one guiding principle: *Make concepts as clear to the student as we can.* That is a theme common to all the reformers, and we have tried to implement it in every area of the text: organization, examples, proofs, applications, exercises, and illustrations.

Student-Oriented Text. This book has been designed to be read and used. Students entering a calculus course differ widely in background, interest, and study habits. We have kept this in mind and have written this text to make the concepts accessible to a broad spectrum of students. To help achieve our goals we brought students into the editorial process even more than before. We surveyed over one hundred past and present student users of the fourth edition, soliciting their comments on that edition and on drafts of this one. Thus we have had not only the suggestions of professors, which publishers usually gather and colleagues happily volunteer, but the advice of those who will buy the book and read it. (Most students *do* read the text, not just its examples and exercises.)

Sometimes the students and the professors agreed; often they did not. The professor said, ''Make the book shorter.'' The student said, ''Add more explanations, more motivation. Tell us what we will do and why. Then tell us the core of what we have done.'' The professor said, ''Get rid of wide margins; save trees.'' The student said, ''We need the margins. We want more white space. Then it's easier to read and we have space to write in.'' The professor said, ''String the steps of a calculation on a line; save space.'' The student said, ''Display them below each other so they're easier to follow.'' The professor said, ''Cut the summaries. They're redundant.'' The student said, ''The summaries put it all together.'' In these cases, *the students prevailed.*

Also in response to student suggestions, we have added many illustrations (''for those of us who think geometrically, not linearly''), expanded chapter and section introductions, and included many new summaries for most of the sections. An example of a newly written chapter introduction is the opening section of Chapter 10, which previews the two chapters on series and shows why series are important.

The chapter summaries in this book are the most extensive of any calculus text. Although instructors may use them only as another source of exercises, students find these overviews very helpful. Taken together with the concise section summaries, they comprise a built-in study guide.

In order to enhance readability, we did not clutter the exposition with too many digressions. The students have enough to read as it is. Some of the illustrative material and applications are set off in boxes, where they make their point without interrupting the exposition. (See, for instance, the conversation about half-life in Section 6.7, the relation of k^n to chain reactions in Section 10.2, and the role of complex numbers in alternating currents in Section 11.6.)

In the sense that students can read this text, it might be termed ''easy.'' However, because of its ample supply of both routine and challenging exercises, the instructor can choose the level of the course. (Of course, that level is also influenced by other factors, such as the preparation of the student, the speed at which material is covered, and whether certain topics—such as ϵ, δ—are included. These options are discussed in the *Instructor's Manual*.)

Organization. The text remains fairly standard, yet it permits a couple of variations by the instructor. As before, each section focuses on one main idea and generally corresponds to one lecture. Except for the section on natural growth (6.7), Chapter 6 can be covered before Chapter 5. That permits a full treatment of differential calculus that includes *all* the elementary functions quite early. The small portion of Chapter 6 that uses Chapter 5 is clearly indicated. Also, Chapter 8, Applications of the Definite Integral, can precede Chapter 7, Computing Antiderivatives. Exercises in Chapter 8 that use material from Chapter 7 are so indicated.

There are several changes in organization in this fifth edition. The opening chapter is now a survey of calculus, introducing informally the derivative, the definite integral, and series. It may be covered in one to three lectures or merely suggested as reading at the start of the term. Methods of estimating an integral, including the trapezoidal method and Simpson's method, now come much earlier (5.4), preceding the fundamental theorem (5.7). In this way, the student is encouraged to view the definite integral as a function of its right-hand endpoint and to conjecture what its derivative is. To introduce the fundamental theorem, we added a new section (5.6), ''Background for the Fundamental Theorem of Calculus.'' In it, the student has a chance to become familiar with the idea that ''area is a function of x'' and to carry out experiments with a calculator that suggest the fundamental theorem.

We have added several other new sections. In Section 4.10 we use an analogy with racing cars to show why the second derivative measures the error in using a differential or linear approximation. This is a prelude to the new Section 11.3, which shows why the error in Taylor series is controlled by a higher derivative. Section 12.2, on projections, is also new. The notion of a projection is so important in theory and applications that we have now collected the properties of the various projections in one place, permitting a unified treatment of a topic that recurs throughout subsequent sections.

We have limited the coverage of differential equations in the main body of the text to a brief treatment in Section 6.7 of separable differential equations (needed for studying natural growth); material on linear differential equations with constant coefficients now appears as Appendix M. This is in keeping with prevailing practice, where differential equations constitutes a full sophomore course in its own right. While the same could be said of vector analysis, the integral theorems of vector analysis fit so naturally into multivariate calculus that it is easy to include them in an introductory text. We feel it is better to present vector analysis well, with adequate motivation, than to attempt to cover both vector analysis and differential equations in the perfunctory way that space and time limitations would require. Students would gain little from abbreviated accounts of either. On many campuses the vector analysis course is incorporated in the third semester or fourth quarter of the introductory calculus sequence; this text supports that natural progression.

We have split infinite series into two chapters (10 and 11), corresponding roughly to series with constant terms and power series. We also split into two sections the vector treatment of lines and planes (Sections 12.4 and 12.7) and the chain rule for partial derivatives (Sections 14.6 and 14.11), with Section 14.11 devoted to the case where a variable is both intermediate and terminal. We have emphasized the ∂ notation for partial derivatives in Chapter 14, in keeping with the notation favored by those who use them in applications (and prefer to reserve subscripts for vector components). The subscript notation is retained, however, as an alternative for those who prefer it.

The former chapter on Green's theorem, the divergence theorem, and Stokes' theorem is also now split in two. Chapter 16 covers Green's theorem and concerns mostly vector fields in the plane, where the diagrams, arguments, and calculations are much easier. The student who has mastered this chapter—in particular the relation of Green's theorem and Stokes' theorem in the plane to fluid flow, work, subtended angle, central fields, and conservative fields—is well prepared for vector analysis over surfaces and solids, which constitutes Chapter 17.

The discussion of vector analysis has been extensively revised, mainly at the insistence of colleagues in physics and engineering who wish to emphasize concepts over routine calculations. There are more sections than before (11 versus 7 in the previous edition), but they are now shorter and more focused. The only new concept is "solid angle."

The mention of solid angles reminds us of other topics needed in applications and assumed there, although they are not always covered in a calculus course. Complex numbers is such a topic. Few students have seen them in high school, and, if they did, they seldom acquired any feel for the geometry. That is why we devote a full section (11.6) to them. The following section presents the relation $e^{i\theta} = \cos\theta + i\sin\theta$, which deserves emphasis in freshman calculus for several reasons: the most immediate is its application in sophomore physics and engineering courses.

Proofs. Proofs are as much a part of the course as are examples and exercises. They are not ornaments. They are not included to assure the student that a statement is true. (After all, students seldom express the fear that a theorem that has been around for at least a century might be false.) A proof should be included in an introductory text not to show that something is true, but to show *why* it is true, to reinforce definitions and principles, and to tie the concepts together. Proofs should not appear to the student to be pulled out of a hat, however much the trick may appeal to the instructor. For instance, we have shifted from the usual "neat" proof that absolute convergence implies convergence ["$a_n = (a_n + |a_n|) - |a_n|$"] to a more intuitive, though slightly longer, proof. (See Section 10.7.) Instead of treating $\lim_{x \to 0} (1 - \cos x)/x = 0$ as a sleight-of-hand consequence of $\lim_{x \to 0} (\sin x)/x = 1$, we examine it as a limit in its own right and show the student how similar situations can lead to very different outcomes. (See Section 2.7.)

For the same reasons, we obtain the formula for the dot product in terms of components in a more instructive way than by the law of cosines. The proof depends on the simplest property of a projection. Section 12.2 treats the projection of a line segment on a line or plane, a vector on a vector or plane, and a flat surface on a plane.

At the Tulane conference on "Lean and Lively Calculus" in 1986 we heard the engineers say, "Teach the concepts. We'll take care of the applications." Steve Whitaker, in the engineering department at Davis, advised us, "Emphasize proofs, because the ideas that go into the proofs are often the ideas that go into the applications." Oddly, mathematicians suggest that we emphasize applications, and the applied people suggest that we emphasize concepts. We have tried to strike a reasonable balance that gives the instructor flexibility to move in either direction.

Exercises. We have included more routine exercises for students who need additional drill to hone their skills. But we have also added exercises that require students to provide written explanations, discussions, and conclusions. These are marked with a pencil: . Just as there are both routine and challenging computational problems, there are routine and challenging writing exercises.

Some of the exercises simply ask the student to draw a diagram. These are indicated with a . The ability to sketch a quick and accurate working diagram will serve the student well in calculus and later studies, so we have given many practical drawing tips at appropriate points in the text. Often students cannot solve a problem because their drawings are too sloppy or too small. The drawing exercises are intended to develop skill in sketching useful diagrams. (See, for instance, Section 8.2.) In addition, there are more exercises that exploit the calculator or computer. These are marked with a or .

"Exploration" problems are marked with a compass: . These exercises usually fit the so-called "tri-ex" format: "experiment, extract, explain." The student is encouraged to *experiment* with various cases, *extract* a general principle, and then *explain* why the principle is valid. The student is given more latitude in tri-ex problems than in typical "guided proof" problems (where the prescribed steps inexorably lead the student to the desired conclusion). While we include guided proof problems where they seem useful, the tri-ex problems are intended to develop more self reliance. (Their solutions are in the *Instructor's Manual,* not the *Student's Solutions Manual.*)

As before, a serious effort has been made to rank the exercises. In general, each exercise set begins with paired exercises that students can use for drill and practice. A horizontal color bar marks the end of these exercises. Exercises appearing after the bar occur singly rather than in matched pairs. They may offer an additional perspective, complete a proof, involve longer calculations, illustrate an application, be more theoretical, invite an explanation, and so on. Thus they are not necessarily harder than exercises before the bar, but simply serve a different purpose.

The difficulty level of the exercises tends to rise throughout each exercise set, but sometimes the effort to group similar problems must interrupt such a gradation. Also, we do not think it is a good idea to tell students that certain exercises are hard while others are easy. Students are then discouraged when "easy" exercises turn out to be difficult for them and may give up too easily on exercises that are described as "hard." To assist the instructor in assigning homework problems, the *Instructor's Manual* contains notes on the exercises.

Illustrations and Design. We have improved most figures in the text and added many new ones. We have made a much greater use of color, even in diagrams of some plane figures, where a third or fourth color can highlight a feature. For instance, we use colors to distinguish a curve from a tangent and the tangent from its approximating secants. Many diagrams of lines, vectors, and surfaces in space benefit from the use of several colors. In addition, we have expanded the use of computer graphics, without succumbing to the temptation of adding computer-created function graphs merely to demonstrate the power of modern software. Unnecessarily complex art may easily discourage students from developing their own ability to draw neat, useful diagrams.

Computers and Calculators. These execute algorithms and sketch graphs far more rapidly and accurately than humans can with pencil and paper. We have included exercises that exploit these tools and also describe some of the software available. However, we do not make the text dependent on them. First of all, the software is changing rapidly. Second, the hardware and its accessibility vary from campus to campus. Third, incorporating a computer on a daily basis demands extra labor and careful coordination. Instructors of large classes already have their hands full simply giving their lectures, conducting office hours, and reading papers or coordinating readers and teaching assistants. So, in many cases, we cite some of the related software and suggest exercises, but we leave it to the instructor to decide how to exploit the calculator and computer. For further information, see "Computers and Calculators" on page xxxiii.

Supplements. Calculus texts today are accompanied by ancillaries designed to assist the student and instructor. The *Instructor's Manual* and *Student's Solutions Manual* are principal among these. We have all seen examples where answers in manuals disagree with answers in the text, and where solutions in manuals use techniques not available to the students. We have worked to eliminate such discrepancies.

The *Instructor's Manual* contains our own pedagogical notes to the instructor on alternative approaches to the material in the text, plus solution sketches for even-numbered exercises (and for exploration problems omitted from the *Student's Solutions Manual*).

Also available to instructors adopting the text are a test-item file (in both printed and computerized formats) and an extensive system of color overhead transparencies for use in the classroom. Software packages for numerical and graphical displays are available to adopters by contacting the local McGraw-Hill sales representative.

Related McGraw-Hill titles that will be of interest to users of this book are *Introduction to Math CAD for Scientists and Engineers* by Sol Wieder (1992). *Discovering Calculus with the HP-28 and HP-48* by Robert T. Smith and Roland B. Minton (1992), and *Explorations in Calculus with a Computer Algebra System* by Donald B. Small and John M. Hosack (1991).

Acknowledgments. In conclusion, we wish to acknowledge the assistance of the many people who have helped us prepare this edition. Rodney Cole of the physics department and Stephen Whitaker of the engineering department at the University of California, Davis, made many suggestions, as did Henry Alder, Carlos Borges, Dean Hickerson, Lawrence Marx, and Howard Weiner of the mathematics department. Phil R. Smith of the mathematics department at American River College and Sándor Szabó at the University of the Pacific provided helpful comments.

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At least as important were the comments of students, collected in an anonymous survey.

Assisting with the preparation of the manuscript were undergraduates Michèle Brown and Keith Sollers, and graduates Mallory Austin and Masato Kimura. Patrick Reardon and Philippe Bérard helped to prepare the art manuscript. Duane Kouba, Ali Dad-del, Aaron Klebanoff, Don Johnson, Linda Namikas, Kathy Smith, Tom Schiller, Denise Quigley, and Pat Rhodes checked the problems. Richard and Judith Kinter, John Lynskey, Gordon Nelder-Adams, and Phil R. Smith helped to type the manuscript.

We are indebted to the McGraw-Hill staff. Former mathematics editor, Robert Weinstein, who believes, rightly so, that publishers are teachers—not just producers of books—encouraged us to undertake this substantial revision, and his successor, Richard Wallis, has maintained his high standards.

Sherman K. Stein
Anthony Barcellos

SAMPLE COURSE OUTLINES

The outlines sketch three courses:

Full This includes a review of precalculus material. The pace permits a thorough treatment, including the assignment of conceptual, exploratory, and writing exercises.

Core Here we assume that the students' precalculus background is sound (functions, graphs, trigonometry, and conics) and that the instructor wishes to emphasize only the most essential concepts and de-emphasize computations, which are consigned to calculators or a later numerical analysis course.

Medium This is one of the many possibilities that lie between the two extremes. The syllabi will suggest others, such as the Core at the pace of the Full.

Chapter	*Full:Lectures*	*Medium:Lectures*	*Core:Lectures*
1	3	1	0 (to be read)
2	13 (2 on 2.4, 2.6, 2.8)	9 (2.9 & 2.10 together)	5 (omit 2.2, 2.5, 2.6, 2.9, 2.10)
3	8 (2 on 3.2, 3.4)	6	5 (3.1 & 3.2 together)
4	13 (2 on 4.1, 4.7, 4.9)	9 (omit 4.4)	7 (omit 4.4, 4.6, 4.10)
5	8 (2 on 5.1)	7	5 (omit 5.4, 5.6)
6	12 (2 on 6.3, 6.7, 6.8)	9	4 (6.1 & 6.2 together, omit 6.4, 6.7, 6.8, 6.9)
7	9 (2 on 7.2, 7.7)	7	6 (omit 7.7)
8	9 (2 on 8.8)	8	4 (8.2 to be read, omit 8.5, 8.6, 8.7)
9	7	5 (omit 9.5, 9.7)	4 (omit 9.5, 9.6, 9.7)
10	8 (2 on 10.7)	8 (2 on 10.7)	7
11	8 (2 on 11.6)	7	4 (omit 11.3, 11.6, 11.7)
12	10 (2 on 12.2, 12.3, 12.4)	8 (2 on 12.4)	5 (omit 12.5, 12.7)
13	6 (2 on 13.5)	5	2 (omit 13.3, 13.4, 13.5)
14	14 (2 on 14.6, 14.7, 14.9)	13 (2 on 14.7, 14.9)	7 (14.2, 14.4 to be read, omit 14.10, 14.11)
15	7	7	4 (15.1 & 15.2 together, omit 15.3, 15.6, 15.7
16	10 (2 on 16.2, 16.4, 16.5, 16.6)	9 (2 on 16.2, 16.4, 16.5)	6
17	6 (2 on 17.1, 17.2)	5 (2 on 17.2)	4
Total	151 (3 semesters, 4/week)	123 (4 quarters, 3/week)	79 (1 year, 3/week)

Appendices	*Full:Lectures*	*Medium:Lectures*	*Core:Lectures*
A	1	1	0
B	2	1	0
C	2	1	0
D	1	1	0
E	1	0 (to be read)	0
F	1	0 (to be read)	0
G	4	3 (omit G.4)	0
H	2	1	0
I	2	0	0
J	2	1	0
K	2	1	0
L	3	2	0
M	2	1	0
Total	25	13	0