QUANTUM FIELD THEORY

LEWIS H. RYDER

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University of Kent at Canterbury

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FOR DANIEL

Yet nature is made better by no mean
But nature makes that mean: so, over that art
Which you say adds to nature, is an art
That nature makes.
William Shakespeare

Omnia disce, videbis postea nihil
esse superfluum
(Learn everything, you will find
nothing superfluous.)
Hugh of St Victor

Preface

This book is designed for those students of elementary particle physics who have no previous knowledge of quantum field theory. It assumes a knowledge of quantum mechanics and special relativity, and so could be read by beginning graduate students, and even advanced third year undergraduates in theoretical physics.

I have tried to keep the treatment as simple as the subject allows, showing most calculations in explicit detail. Reflecting current trends and beliefs, functional methods are used almost throughout the book (though there is a chapter on canonical quantisation), and several chapters are devoted to the study of gauge theories, which at present play such a crucial role in our understanding of elementary particles. While I felt it important to make contact with particle physics, I have avoided straying into particle physics proper. The book is pedagogic rather than encyclopaedic, and many topics are not treated; for example current algebra and PCAC, discrete symmetries, and supersymmetry. Important as these topics are, I felt their omission to be justifiable in an introductory text.

I acknowledge my indebtedness to many people. Professors P.W. Higgs, FRS, and J.C. Taylor, FRS, offered me much valuable advice on early drafts of some chapters, and I have benefited (though doubtless insufficiently) from their deep understanding of field theory. I was lucky to have the opportunity of attending Professor J. Wess's lectures on field theory in 1974, and I thank him and the Deutscher Akademischer Austauschdienst for making that visit to Karlsruhe possible. I am also very grateful to Dr I.T. Drummond, Dr I.J.R. Aitchison, Professor G. Rickayzen and Dr W.A.B. Evans for reading various sections and making helpful suggestions. I wish to thank Miss Mary Watts for making a difficult and unattractive manuscript a handsome typescript, and this with constant good humour and cheerfulness. I also thank Mr Bernard Doolin for drawing the diagrams with such great care. I am grateful to the late Fr Eric Doyle, OFM, for unearthing the quotation by Hugh of St. Victor. I am grateful to the organisers of the first and second UK Theory Institutes in High Energy Physics for the opportunity they provided for stimulating discussions. I wish to record my special thanks to Dr Simon Mitton of Cambridge University Press for his constant encouragement and his indulgence over my failure to meet deadlines. Finally, and most of all, my thanks go to my wife, for her unfailing encouragement and support over a long period.

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Canterbury, Kent
August, 1984

Lewis Ryder

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Introduction: synopsis of particle physics

1.1 Quantum field theory

Quantum field theory has traditionally been a pursuit of particle physicists. In recent years, some condensed matter physicists have also succumbed to its charms, but the rationale adopted in this book is the *traditional one: that the reason for studying field theory lies in the hope that* it will shed light on the fundamental particles of matter and their interactions. Surely (the argument goes), a structure that incorporates quantum theory – which was so amazingly successful in resolving the many problems of atomic physics in the early part of this century – and field theory – the language in which was cast the equally amazing picture of reality uncovered by Faraday, Maxwell and Hertz – surely, a structure built on these twin foundations should provide some insight into the fundamental nature of matter.

And indeed it has done. Quantum electrodynamics, the first child of this marriage, predicted (to name only one of its successes) the anomalous magnetic moment of the electron correctly to six decimal places; what more could one want of a physical theory? Quantum electrodynamics was formulated in about 1950, many years after quantum mechanics. Planck's original quantum hypothesis (1901), however, was indeed that the electromagnetic *field* be quantised; the quanta we call photons. In the years leading up to 1925, *the quantum idea was applied to the mechanics* of atomic motion, and this resulted in particle–wave duality and the Schrödinger wave equation for electrons. It was only after this that a proper, systematic treatment of the quantised electromagnetic field was devised, thus coming, as it were, full circle back to Planck and completing *the quantisation of a major area of classical physics.*

Now, in a sense, quantisation blurs the distinction between particles and fields; 'point' particles become fuzzy and subject to a wave equation, and the (electromagnetic) field, classically represented as a continuum, takes on a particlelike nature (the photon). It may then very well be asked: if we have charged particles (electrons, say) interacting with each other through the electromagnetic field, then in view of quantisation, which renders the particle and the field rather similar, is there an *essential* distinction between them? The answer to this question takes us into elementary particle physics. The salient point is that photons are the quanta of the field *which describes the interaction* between the particles of matter. The electrons 'happen to be there' and because they interact (if they did not we would not know they were there!) the electromagnetic field and, therefore, photons *become compulsory*! But this is not all. Muons and protons and all sorts of other charged particles also happen to exist, and to interact in the same way, through the electromagnetic field. The reason for the existence of all these particles is so far unknown, but we may summarise by saying that we have a *spectrum of particle states* ($e, \mu, p, \Sigma, \Omega$, etc.) and a *field through which these particles interact* – an interaction, in short. This treatment and mode of comprehension of electrodynamics provides the paradigm, we believe, for a complete understanding of particle interactions. The idea is simply to apply the same methods and concepts to the other interactions known in nature. The only other interaction known in classical physics is the gravitational one, so let us first consider that.

1.2 Gravitation

The gravitational field is described by the general theory of relativity. It turns out, however, that the quantisation of this theory is beset by great problems. First, there are mathematical ones. Einstein's field equations are much more complicated than Maxwell's equations, and in fact are non-linear, so consistency with the superposition principle, the mathematical expression of wave–particle duality, which requires the existence of a linear vector space, would seem to be threatened. Second, there are conceptual problems. In Einstein's theory the gravitational field is manifested as a curvature of space–time. In electrodynamics, the field is, as it were, an actor on the space–time stage, whereas in gravity the actor becomes the space–time stage itself. In some sense, then, we are faced with quantisation of space–time; what is the meaning of this? Finally, there are practical problems. Maxwell's equations predict electromagnetic radiation, and this was first observed by Hertz. Quantisation of the field results in the possibility of observing individual photons; these were first seen in the photoelectric effect, in Einstein's classic analysis. Similarly, Einstein's equations for the gravitational field predict gravitational radiation, so there should, in principle, be a possibility of observing individual *gravitons*, quanta of the field. However, although some claims have been made that gravitational radiation has been observed, these are not unanimously accepted, and the observation of *individual gravitons*, a much more difficult enterprise, must be a next-generation problem! The basic reason for this is that gravity is so much weaker than the other forces in nature. In view of this, the particle physicist is justified in ignoring it; and, because of the difficulties mentioned above, is happy to! On the other hand, the methods that have been recently developed for the quantisation of non-abelian gauge fields, relevant for an understanding of the strong and weak nuclear forces, do seem to have relevance to gravity, and these will be briefly described in the book, where appropriate.

1.3 Strong interactions

After electromagnetism and gravity, the remaining interactions in nature are nuclear; the so-called strong and weak nuclear forces. The question is, can these forces be described by a *field*? Yukawa surmised that the strong force between protons and neutrons in a nucleus could, but that the field quantum had to have (unlike the photon) a *finite mass*; this is because the nuclear force has a finite range. Fig. 1.1 shows a Feynman diagram (explained in chapter 6) for the exchange of a virtual field quantum (π^+) between a proton and neutron. The uncertainty principle allows this to happen provided

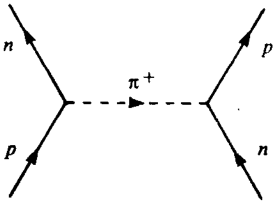
$$\Delta E \Delta t = (m_\pi c^2)\Delta t \sim \hbar$$

where m_π is the mass of the field quantum (pion) and Δt is the time for which it exists. If the range of the force is r, then we may put $r = c(\Delta t)$, giving, with $r \approx 10^{-15}$ m,

$$m_\pi c^2 \approx \frac{\hbar c}{r} \approx 200 \text{ MeV}.$$

When the π^+ was discovered (in 1947) with a mass of 140 MeV/c^2 and possessing strong nuclear interactions, this was considered a triumph for Yukawa's theory. The view that the pion was the quantum of the strong field, however, began to run into difficulties:

Fig. 1.1. Exchange of a quantum of the strong field (pion) between the proton and neutron.



1. At high energies, the proton–neutron force was not well described by pion exchange.
2. The interaction between pions themselves could not (because of parity) be described by pion exchange.
3. With the discovery of strange particles (1950s and early 1960s), and the classification of particles by the group $SU(3)$, pions were found to be only three members of a supermultiplet of eight, the others being K and η – 'ordinary' elementary particles of 'matter'. If field quanta are in essence different from 'matter' quanta, they should surely not appear together in the same supermultiplet.
4. In the quark model (1964), pions are bound states of a quark–antiquark pair, just like all other mesons – their privileged role disappeared completely! (The photon, for example, is most definitely not made of quarks!)

In the sections below we review briefly the elementary particle spectrum and the quark model. But here we note that the quark model, while providing the death-blow to the pion as candidate for quantum of the strong field, also gives a clue to its successor; for the 'real' strong interaction is not that between nucleons, but that between quarks. What provides the interaction between quarks?

There is evidence (see §1.11 below) that the quark possesses a quantum number rather like electric charge except that (i) it has three degrees of freedom – three 'types' of charge, (ii) it is unobservable in the free state – this is to say that individual quarks are not observed, and the reason may be that systems carrying (non-zero values of) this quantum number are forbidden in the free state. The quantum number is known as colour, the degrees of freedom being chosen variously as red, white and blue, red, green and blue, etc. It is believed that colour, like electric charge, gives rise to a quantised field, massless and of spin one, like the photon. By means of this field, quarks interact. The quanta are known as gluons, and the dynamics of the quark–gluon system, quantum chromodynamics (QCD), to mirror quantum electrodynamics (QED). QCD is responsible, for example, for the binding of three quarks into protons and neutrons. No wonder the proton–neutron force is only approximately describable by pion exchange – it must in reality be a rather complicated force, a sort of 'residual' force between the quarks!

1.4 Weak interactions

To complete this preliminary account, we must mention the fourth known interaction in nature; the weak nuclear interaction, responsible for beta decay. In Fermi's original theory, this interaction was a *point* interaction between the four participating particles (in neutron decay $n \rightarrow p + e^- + \bar{\nu}_e$). In other words, there was no need for a field, because there is no effect to be propagated between one point and another. (It will be recalled that the notion of 'field' in physics was introduced partly to give a more satisfactory account of 'action-at-a-distance'; if there is no action at a distance, there is clearly no need for a field.) Fermi's theory worked extremely well; in fact, with a modification for parity violation, there was for several decades no direct evidence that it was wrong. It was, nevertheless, believed to be wrong, principally because it is non-renormalisable (see chapter 9). One of the recent triumphs in particle physics has been the appearance of a worthy successor to Fermi's theory, which was worked out by Glashow, Weinberg and Salam in the 1960s. In this theory, electromagnetism and the weak force are combined in a non-trivial way. The weak field acquires *quanta* – the W and Z bosons – which are more than 80 times as massive as the proton. In addition, *neutral current reactions* such as $\nu + p \rightarrow \nu + p +$ (neutral hadrons) are predicted, as well as a fourth (charmed) species of quark. Observation of all these things has established the Weinberg–Salam theory as the correct theory of weak interactions. Not the least important aspect of this theory is the fact that it is a *unified* theory – an 'electroweak' theory, in the new jargon. Weak and electromagnetic interactions are now unified. A putative 'grand unified theory' (GUT), which unifies the electroweak interaction with the strong interaction (QCD) would seem to be the next obvious thing to look for, but at the time of writing (1983) there is no hard evidence that the forces of nature are grand-unified.

These, then, are the ingredients of contemporary theories of fundamental particles and interactions. In the next sections of this chapter they will be described in greater detail, for the benefit of readers to whom the current scene in high energy physics is not familiar. The aim is to describe, in rather simple terms, the physical considerations which have led up to, and which follow from, the introduction of the quark model. This will provide a motivation for the study of quantum fields, as well as a chance to explain some of the concepts which will be used in the application of field theory to particle physics. It offers no pretence to completeness, but references to more detailed literature are given at the end of the chapter. Readers who know about particle physics are advised to skip to chapter 2. In the remainder of this chapter I shall use one or two concepts and techniques without adequate explanation. This applies particularly to Feynman diagrams. I ask the reader's indulgence to make the best he can of these sections until he meets the explanations later in the book.

Table 1.1. *Classification of fundamental particles*

Hadrons	Baryons	$p, n, \Lambda, \Sigma, \Omega, \Lambda_c, \ldots$, etc.	(hundreds)
	Mesons	$\pi, K, \rho, \psi/J, \Upsilon, \ldots$, etc.	(hundreds)
Leptons		$e^-, \nu_e, \mu^-, \nu_\mu, \tau^-, [\nu_\tau], \ldots$	(six only?)
Field quanta		Photon, γ Weak bosons, $W^\pm, Z^0$ [Gluons]	

Particles in square brackets have not yet been discovered

1.5 Leptonic quantum numbers

There is a basic classification of fundamental particles into those which experience the strong interaction – called *hadrons*, those which do not, called *leptons*, and, thirdly, the quanta of the interaction fields (see above). The hadrons are subdivided into *baryons* which have half-odd integral spin, and *mesons*, with integral spin (in units of $\hbar$ – spin is a purely quantum phenomenon). Table 1.1 gives a summary. Hundreds of baryons and mesons have been found. Five leptons have been found, and for aesthetic reasons it is suspected that the τ lepton, like e and μ, is accompanied by a neutrino ν_τ. Baryons and mesons come with all sorts of spins – as high as $\frac{7}{2}$ for baryons, and 3 for mesons, have been measured, but the leptons all have spin $\frac{1}{2}$. The field quanta all have spin 1. It is believed that the known hadrons are composite states of six types of quark (see below) so it would be nice if there were also six leptons.

In order for the division of spin $\frac{1}{2}$ particles into baryons and leptons to have any significance, transitions between these different types should not occur. For example, a proton should not decay into a positron (the antiparticle of the electron):

$$p \nrightarrow e^+ + \gamma; \tag{1.1}$$

and indeed this decay has not been observed. What could forbid it? Charge would be conserved, and so would mass-energy and angular momentum. There must be a *conserved quantum*[‡] *number* – call it *baryon number B* – such that baryons have $B = 1$ and other particles have $B = 0$. Conservation of baryon number then clearly forbids the decay (1.1).[‡‡] The reader might

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[‡] It is common practice to call conserved quantities 'quantum numbers', but it is not at all obvious that baryon number has anything to do with the quantum theory. If it had, matter would become unstable by (1.1) as $\hbar \to 0$.

[‡‡] It is characteristic of grand unified theories that baryon number is not absolutely conserved. They therefore predict proton instability. Observation of proton decay would count as important evidence that a GUT may be correct.