# Microphysical Reality and Quantum Formalism

## Volume 1

Fundamental Theories of Physics

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## Volume 1

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## PREFACE

Quantum mechanics has reached maturity as an awesome scientific theory, and undeniably no experiment has so far produced any result conflicting with its predictions. Nevertheless, an increasing number of scholars are seriously questioning the limits of this discipline's validity, a fact that is eloquently attested to by the four international conferences devoted to the foundations of quantum theory which were held in 1987 alone - in Joensuu, Vienna, Gdansk, and Delphi, respectively. There is an increasing awareness that the founding fathers of quantum mechanics have left behind a theory which, though spectacularly successful in its applications, severely limits our intuitive understanding of the microworld, and that their reasons for doing so were at least partly arbitrary and open to question.

The problem of the relationship between the existing quantum theory and objective reality at the atomic and subatomic levels can be tackled in essentially two ways:

(i) One may focus attention on the formalism of the theory and attempt to deduce from it a coherent description of our measuring processes and a deeper understanding of the microworld.

(ii) Alternatively, one may start from the experimental evidence and/or from models of the objective reality compatible with it and go on to investigate whether or not formalization of this knowledge can be accomodated within the broad confines of existing quantum theory.

The thirty-eight papers collected in the present book, the first of a two-volume set, approach the forementioned problem mostly from the first point of view. The large majority of them was presented at the International Conference on Microphysical Reality and Quantum Formalism which was held in Urbino (Italy) from September 25 through October 3 of 1985. In more than one way, this meeting was a unique event - because of the large number of participants (about two hundred physicists and philosophers of science from all parts of the world), because of a feeling of liberation and achievement that prevailed among them, and because of the exceptionally high quality of many of the papers that were read.

Remarkable findings are communicated in the present volume; from among the most interesting ones, we choose to mention:

1. The discovery of a new and typically quantum-mechanical phenomenon, that of "haunted measurements."
2. The discovery of a severe limitation on Heisenberg's inequalities and an organic attempt to overcome it.
3. Entirely new formulations of quantum theory, in particular one that involves a novel use of the correspondence principle.
4. Highly interesting attempts at surmounting the well-known difficulties that plague the quantum mechanical theory of measurement.

5. Developments of the idea that the microworld is perfectly time-symmetric and that irreversibility belongs uniquely to the macroscopic domain.
6. Critical re-examinations of the meaning and of the very existence of the Einstein-Podolsky-Rosen paradox.

The unifying idea behind researches such as these is the realization that new and important discoveries lie within the reach of scholars working on the foundations of modern physics. On the other hand, it becomes clear on inspection that very significant differences in perspective, programs, and priorities exist between the workers involved. But such a diversity of approaches was of course to be expected in a field of activity that has started to flourish only in recent years and that seeks to fathom the full extent of revolutionary ideas originated by profound thinkers who, in addition to a successful formalism, also left us in possession of a badly divided set of conceptions concerning the relationship of this formalism to microphysical reality.

The editors wish to thank the organizers of the Urbino conference and everyone else who contributed to the success of this impressive meeting. Particularly deserving of our appreciation are the members of the International Advisory Committee: David Bohm (London), Max Jammer (Ramat-Gan), Trevor Marshall (Manchester), Oreste Piccioni (La Jolla), Karl Popper (London), Ilya Prigogine (Brussels), Emilio Santos (Santander), Roman Sexl (Vienna), John Wheeler (Austin), and Eugene Wigner (Princeton). We also owe a debt of gratitude for financial support to the Italian Consiglio Nazionale delle Ricerche, to the Istituto Nazionale di Fisica Nucleare, and to the Provincia di Pesaro e Urbino.

*Franco Selleri*
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# QUANTUM MECHANICS OF MACROSCOPIC SYSTEMS AND MEASUREMENT PROCESSES

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ABSTRACT

It is explicitly shown that the wave packet reduction by measurement takes place as a sort of phase transition even in the case of the negative-result measurement, provided that the macroscopic nature of measuring apparatus (especially of its local system) is mathematically formulated by means of a continuous direct sum of many Hilbert spaces. The unitarity of the S matrix of elementary interaction processes between object and apparatus systems is maintained. Not only the general theory but also a few solvable models of measuring apparatus are presented to elucidate the mechanism of wave packet reduction. A critical review of debates on measurement problems is first given for the purpose of clarifying our points.

## 1. INTRODUCTION

The theory of measurement in quantum mechanics must cover a wide class of fundamental problems, including not only theoretical analyses of physical measurement processes but also serious questions against the Copenhagen interpretation itself, as we have seen at this conference. In this talk, however, we restrict ourselves to the rather narrow problem of whether the reduction of the wave packet in the measurement of an observable can be described by quantum mechanics itself. Among many theories, we know the famous debates between the von Neumann-Wigner theory (negative answer) and the ergodic-amplification theory (affirmative answer). Our theory will lead us to the conclusion that quantum mechanics can give an affirmative answer to the problem--by showing the wave packet reduction in an explicit form.[1]

In §2 we set up the measurement problem in a definite form, examing the above debates, in order to remove confusions in the theory of measurement and make its points clear. Undoubtedly, one of the essential problems is how to mathematically formulate the macroscopic nature of the measuring apparatus. In §3 we first point out that an object system does not always interact with the whole apparatus system



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but with its local system, which is microscopically large but macroscopically small. In other words, one collision of the object system with such a local system is enough to yield the wave packet reduction by measurement. There it is also stressed that the local system is (i) still macroscopic and has (ii) a finite size on a microscopic scale. The macroscopic nature, characterized by the indefiniteness of energy and particle number, can be described within the mathematical framework of a continuous direct sum of many Hilbert spaces. The finite-size effect is reflected in the phase shift, proportional to the size parameter in the S matrix for the collision. There a few solvable models of apparatus systems are given to show such an effect. Based on these discussions, in §4 we derive the reduction of the wave packet in an explicit form. Section 5 is devoted to concluding remarks, in which we discuss recent experiments on neutron interference from the viewpoint of the theory of measurement.

## 2. WHAT IS THE PROBLEM?

Consider a measurement of an observable F in a quantum-mechanical object system Q in a state

$$\psi^Q = \Sigma_i\, c_i u_i, \quad c_i = (u_i, \psi^Q), \tag{1}$$

where $u_i$ is the eigenstate i of F. Following von Neumann, many authors have often expressed wave packet reduction by measurement (of the first kind) as

$$\rho^Q = |\psi^Q\rangle\langle\psi^Q| \to \bar{\rho} = \Sigma_i\, |c_i|^2 \xi(u_i), \tag{2}$$

in terms only of Q states, where $\xi(u_i) = |u_i\rangle\langle u_i|$. However, this expression for wave packet reduction is not complete, for the following reason: If we take a special case with only two eigenstates and $c_1 = c_2 = 1/\sqrt{2}$, then we obtain

$$\bar{\rho} = \frac{1}{2}[\xi(u_1)+\xi(u_2)]$$
$$= \frac{1}{2}[\xi(u_+)+\xi(u_-)],$$

where $u_\pm = (u_1 \pm u_2)/\sqrt{2}$. In this case, Eq.(2) must simultaneously describe different experiments of two observables, $F = \lambda_1\xi(u_1)+\lambda_2\xi(u_2)$ and $G = \lambda_+\xi(u_+)+\lambda_-\xi(u_-)$, which, in general, do not commute with each other. This is a contradiction.[(2)] Consequently, one cannot avoid bringing apparatus states into the mathematical expression of the reduction of wave packet in the following way:

$$\Xi^{tot} = \rho^Q \times \sigma^A \to \bar{\Xi}^{tot} = \Sigma_i\, |c_i|^2 \xi(u_i)\times\sigma^A_{F(i)}, \tag{3}$$

where $\Xi^{tot}$ and $\sigma^A$ stand for the initial-state statistical operators of the total system and of system A, respectively, $\bar{\Xi}^{tot}$ for the final-state statistical operator of the total system, and $\sigma^A_{F(i)}$ for the final-state statistical operator corresponding to the observation of the i-th eigenvalue of F. Therefore, our theory has to derive Eq.(3) by applying quantum mechanics to the total system composed of the object Q and the apparatus A. Needless to say, the central problem is to erase the phase correlations among $u_i$'s in the initial object state $\psi^Q$ through the measuement process. Here note that the measurement process takes a very long time on the microscopic time scale, even though its duration looks like a point on the macroscopic time scale.

It should also be remarked that Eq.(3) describes a sort of irreversible process but not the usual thermally irreversible process terminating in thermal equilibrium, and that the same equation expresses not only a transition of the whole ensemble but also one transition $\psi^Q \rightarrow u_i$ (with probability $|c_i|^2$) in a single measurement on one system, because the right-hand side is a sum of probabilities corresponding to exclusive events.

Here we briefly survey the debates between the von Neumann-Wigner theory and the ergodic amplification theory. According to the former, the measurement process should be written as

$$\Psi \equiv \psi^Q \times \Phi^A = \Sigma_i \, c_i u_i \times \Phi^A \rightarrow \Sigma_i \, c_i u_i \times \Phi_i \equiv \tilde{\Psi}, \tag{4}$$

where $\Phi^A$ is the initial state of the apparatus system and $\Phi_i$ stands for the i-th eigenstate of an appropriate observable (of system A), designed so as to have a one-to-one correspondence to $u_i$. Equation (4) was derived by strict application of the superposition principle to the measurement process, but it never gives wave packet reduction, because $\Psi$ is still a pure state while the right-hand side of Eq.(3) is in a mixed state. This was later generalized to the Wigner theorem,[3] stating that the total system can never attain wave packet reduction via a unitary time evolution starting from an initial state in which system Q is in a superposed state and system A in a mixed state. For later discussions we have to point out here that this theorem is proved by assuming A to be in a mixed state in *one* Hilbert space. Anyway, at every step of a chain of measurements, leading from the object to the observer, we can never have wave packet reduction as a physical process within the framework of their theory. Their answer to the measurement problem is "no." Eventually they were led to the introduction of the so-called "abstract ego" or "consciousness," as is well known.

By contrast, the ergodic amplification theory attemped to give the answer "yes" by identifying wave packet reduction with thermally irreversible processes, such as discharge phenomena in counters.[4] One of the essential ideas of this theory is to erase the phase correlations by destroying the unitarity of the time evolution in thermally irreversible processes. But Wigner categorically refuses to destroy the unitarity. Consequently, the two theories have become surrounded by a serious debate.

Wigner criticized the ergodic amplification theory on the following basis[5]: (i) the above Wigner theorem and (ii) the negative-result measurement. We briefly explain the latter in the case of the Stern-Gerlach experiment. This is a typically quantum-mechanical measurement that is divided into two steps, spectral decomposition and detection. In the first step, a stationary beam of particles (with spin one-half) in a superposed state is spatially decomposed by a magnetic field into components moving in the upward and downward directions, respectively, in such a way that

$$\psi_0^Q = (c_a u_a + c_b u_b)\phi \rightarrow \psi^Q = c_a u_a \phi_a + c_b u_b \phi_b \tag{5}$$

where $c_a$ abd $c_b$ are constants, $u_a$ and $u_b$ represent spin-up and spin-down eigenfunctions, respectively, and $\phi$, $\phi_a$, and $\phi_b$ stand for position wave functions. In the standard experiment, $\phi_a$ and $\phi_b$ are, respectively, wave packets moving towards different detectors, say counters $D_a$ and $D_b$. Detection by $D_a$ ($D_b$) means that the particle has an up-spin (down-spin), so that one may act as if the very measurement, i.e., the wave packet reduction, could be achieved by the triggering in counters.

In the case of negative-result measurement, $D_b$ is removed, and another counter, say $D_0$, is located in front of the magnetic field. An anti-coincidence experiment of $D_0$ and $D_a$ gives us the so-called negative-result measurement, signifying that the particle went downward and hence has a down-spin. This means that the down-spin measurement is completed, and consequently the wave packet reduction takes place, without resort to the "real triggering" of $D_a$ (the triggering of $D_0$ is connected only with formation of the wave packet $\phi$ and thus is irrelevant to the spin measurement). In this case the "no triggering" of $D_a$ never means that the particle wave function did not interact with detector $D_a$. Both the triggering and its absence are only sort of displays to show results of the measurement; for details, see ref. 1.

From the negative-result-measurement paradox we have learned the important fact that the wave packet reduction is to be distinguished from thermally irreversible processes such as real triggering in counters. In addition to this, Machida and the present author[1] have also pointed out, by means of a gedanken experiment on a simple model, that any energy supply or amplification is not essential for wave packet reduction. Consequently, our theory of measurement must get past the Wigner theorem and derive explicitly the wave packet reduction even in the case of the negative-result measurement.

Before embarking on the main part of our theory, we have to mention a misleading discussion often referred to in many papers. Some authors have considered process (5) to be a measuring process, by formally identifying it with Eq.(4), and then concluded that the particle position itself plays the role of the apparatus system and the wave packet reduction never occurs in the Stern-Gerlach experiment. However, process (5) is merely a spectral decomposition but does not contain any detection. Note that we cannot perform any experiment without resort to detection.

## 3. WHAT IS THE MACROSCOPIC NATURE OF APPARATUS SYSTEMS?

As was mentioned in §1, an object system interacts with one of the local systems of a measuring apparatus but not necessarily with the whole apparatus system. The macroscopic nature of a measuring apparatus is that such a local system is still (i) macroscopic and has (ii) a finite size on the microscopic scale (or even on macroscopic scale in some cases).

(i) ***A macroscopic system has no definite energy and particle number.*** Quantum mechanics tells us that it will take a longer time than $\tau = \hbar/\delta\varepsilon$ to measure sharply energy of a dynamical system with accuracy smaller than the level spacing $\delta\varepsilon$. For a macroscopic system with a huge number of degrees of freedom, however, $\delta\varepsilon$ is so small that $\tau$ exceeds very much even the age of the universe. Thus we have to mention that a macroscopic system is not in a stationary state with a definite energy, even if the system were isolated and even if we spent the longest time available to us for measurement. A usual measuring apparatus, in particular, has a local system that is open, so that its energy is much more indefinite. Particle number is also in the same situation. Consequently, it is natural to represent a quantum mechanical state of the above local system in terms of the following statistical operator:

$$\rho^A = \sum_{N'\varepsilon I(N:\Delta N)} W_{N'}\rho^A_{N'}, \tag{6a}$$

$$\rho^A_{N'} = \Sigma_n |\Phi^{N'}_n\rangle w^{N'}_n \langle\Phi^{N'}_n|, \tag{6b}$$

where $I(N:\Delta N)$ denotes an interval of width $\Delta N$ around N, $W_{N'}$ a positive weight factor normalized according to $\Sigma_{N'} W_{N'} = 1$, $\Phi^{N'}_n$ the nth eigenstate of Hamiltonian $H_{N'}$ for N' particle system, and $w^N_n$ the Boltzmann factor. Needless to say, the macroscopic nature also requires N and $\Delta N$ to be very large but $\Delta N/N$ very small. In the infinite-N limit we can replace the discrete sum in Eq.(6a) by the following integral

$$\sigma^A = \int d\ell\ W(\ell)\rho^A(\ell) \equiv \mathcal{W}\cdot\rho^A(\ell), \tag{7}$$

where $\ell$ (= $\lim_{N'\to\infty, a\to 0} aN'$, a being a characteristic length of the order of atomic size) is now a continuous size parameter of the local system and $W(\ell)$ a continuous positive function distributed around L (= lim aN) with width $\Delta L$ (= lim $a\Delta N$) subject to the normalization condition $\int W(\ell)d\ell = 1$. Here $\rho^A(\ell)$ is the statistical operator of a local system with a sharp size $\ell$; $\mathcal{W}$ symbolically represents the averaging procedure with weight function $W(\ell)$; and $W(\ell)$, or especially $\Delta L$ is a representative of the structure of the local system. Based on a one-dimensional detector model, we can roughly estimate by $(\Delta L)^2 = N(\Delta a)^2+(\Delta N)^2a^2$, $\Delta a$ being the width of the spacing distribution around the mean value a: $\Delta L \simeq 10^{-4}$ cm for $\Delta N \simeq \sqrt{N} \simeq 10^4$ (note that $N \simeq N_A^{1/3}$ with $N_A \simeq 10^{24}$) and $a \simeq \Delta a \simeq 10^{-8}$ cm.

Equation (7) for the statistical operator belonging to a dynamical system with a huge number of particles can also be justified from a mathematical point of view.[6] It is known that von Neumann rings describing physical quantities can be represented in a "large" Hilbert space given by the following direct sum of small Hilbert spaces:

$$\mathcal{H} = \mathcal{H}_1+\mathcal{H}_2+\cdots\cdots+\int d\mu(\zeta)\mathcal{H}(\zeta), \tag{8}$$

where $\mu(\zeta)$ is a smooth function of a continuous parameter $\zeta$. The discrete part of Eq.(8) can be identified with the (discrete) super-selection-rule space in which $\mathcal{H}_1$, $\mathcal{H}_2$, etc. describe, respectively, one particle states, two particle states, etc. In other words, the Along this line of thought we can consider the continuous part of Eq.(8) to be the continuous super-selection-rule space, in which $\zeta$ is the corresponding (continuous) super-selection-rule charge. Furthermore we have the mathematical theorem that

$$\lim_{N\to\infty} \mathcal{H}_N \subset \int d\mu(\zeta)\mathcal{H}(\zeta) \quad \text{or} \quad \lim_{N\to\infty} \sum_{N'\in I(N:\Delta N)} \mathcal{H}_{N'} \subset \int d\mu(\zeta)\mathcal{H}(\zeta). \tag{9}$$

Therefore, we know that the statistical operator of Eq.(7) describing macroscopic systems belongs to the continuous super-selection-rule space, where the size parameter $\ell$ is nothing but the super-selection-rule charge. Mathematics also tells us that the continuous part has a "center" composed of commutable quantities, i.e., classical or macroscopic observables. Actually we will soon see those quantum observables which fall into the "center" at the infinite-N limit. Summarizing, the measuring apparatus as a macroscopic system can be described by the statistical operator (7) belonging to *a continuous direct sum* of many Hilbert spaces but not to a single Hilbert space.

(ii) *The finite size of the local system is reflected in a phase shift proportional to its size.* According to the theory of nuclear reactions,[7] we can decompose the S matrix for collisions of a particle with a finite-size system as follows:

$$S = e^{i\delta}(1+iK)(1+iK)^{-1}e^{i\delta}, \tag{10}$$

where $\delta$ is diagonal and K off-diagonal in the channel representation. The finite-size effect of the target system appears in diagonal elements of $\delta$ equal to $-\bar{p}\ell/2\hbar$, where $\bar{p}$ is generally a sort of effective momentum and $\ell$ the linear size of the compound system in the relevant channel. In the perfect-mirror model given by a previous paper,[1] $\bar{p}$ and $\ell$ are equal, respectively, to the particle momentum and the size of the mirror-wagon system. Here we present the other simple models of a one-dimensional detector, the Dirac comb model and the one-dimensional emulsion model.

The Dirac comb model[8] is given by the fixed potential

$$V(x) = \Omega \sum_{n=0}^{N-1} \delta(x-x_n), \tag{11}$$

with $x_0 = 0$ and $x_{n+1} = x_n+a$, where $\Omega$ is a constant and a the spacing of the lattice points. We can exactly solve the scattering problem for a transmission process, and we write down its S matrix element as

$$S = (z_+-z_-)[z_+^N(z_+-b^*)-z_-^N(z_--b^*)]^{-1}e^{2i\delta}; \tag{12a}$$

$$\delta = -k\ell/2, \quad \ell = Na, \tag{12b}$$

where $k = \surd(2mE/\hbar)$, m and E are mass and energy of the incoming particle, respectively, and