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Authors

S.J.L. van Eijndhoven
J. de Graaf
Eindhoven University of Technology
Den Dolech 2, P.O. Box 513
5700 MB Eindhoven, The Netherlands

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continued on page 275

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CONTENTS

PROLOGUE

The introduction of generalized functions has considerably advanced mathematical analysis, in particular harmonic analysis and the theory of partial differential equations. In a non rigorous way, electrical engineers and physicists have been using generalized functions for almost a century. Still they employ generalized functions such as the Heaviside step function and the Dirac delta function as if they were genuine C^∞-functions. But it took some time before mathematical justification has been taken up.

The first mathematical concepts which started up a theory of generalized functions were the finite parts of divergent integrals used by Hadamard and the Riemann-Liouville integrals due to Riesz. Later Sobolev defined generalized derivatives by means of integration by parts, and Bochner developed the theory of the Fourier transform for functions increasing as some power of their argument. Many of these results were unified by Schwartz in his monograph: Théorie des Distributions. Here the unifying concept is the notion of dual system of locally convex topological vector spaces. Generalized functions (distributions) can be regarded as continuous linear functionals on such spaces of well behaved functions.
Later on, also Gelfand and Shilov defined many classes of generalized functions. They introduced the notion of countable Hilbert space (a functional analytic generalization of the theory of tempered distributions) and the notion of Gelfand triple. But, as importantly, they showed how to use generalized functions in mathematical analysis. It turned out that generalized functions connect many aspects of classical analysis, of functional analysis, of the theory of partial differential equations and of the representation theory of locally compact Lie groups.
Thus, generalized functions have gained wide popularity among mathematicians.

The theories of Schwartz and of Gelfand-Shilov can roughly be described as follows. One starts with a vector space S of 'good' functions. Here we can take the set $\mathcal{D}$ of infinitely differentiable functions with compact support

or the set S of infinitely differentiable functions of rapid decrease. The vector space S is called the test space. It carries a suitable Hausdorff topology which makes S a locally convex topological vector space. The choice of the topology is not arbitrary; an extra condition will be imposed. The topological dual of S is denoted by S'. In order to consider S' or a representation of S' as a space of generalized functions we construct a space S^* that can be identified with S' and that contains S. Therefore, let X be a Hilbert space (e.g. $L_2(\mathbb{R})$ or a Sobolev space) such that S is a dense subspace of X and such that the embedding of S in X is continuous. We observe that the existence of X is equivalent to the existence in S of a positive non-degenerate continuous sesquilinear form. By means of the inner product, the dense subspace S of X induces the weak Hausdorff topology $\sigma(X,S)$ on X. Next, one considers the sequential completion S^* of X with respect to this topology. The mentioned condition one has to impose on the topology for S is the following: each member of S' can be represented by an element of S^* by means of the canonical pairing of S and S^*. Thus S' and S^* can be identified. Since $S \subset X \subset S^*$ and since the members of S are functions, S^*, and hence S' can be regarded as a space of improper functions. In this way, for instance $\mathcal{D}'$ can be interpreted as a space of improper functions which are derivatives of some order of continuous functions on the real line.
Also, the Lighthill-Temple approach of generalized functions can be described in the above functional analytic setup. They consider so-called regular sequences in S which converge in a weak sense. It turns out that a sequence is regular if it is $\sigma(X,S)$-convergent. Two regular sequences are equivalent if the difference of these sequences is a null sequence. A generalized function in the sense of Lighthill is just an equivalence class of regular sequences. So the theory based on the triplet $S \subset X \subset S^*$ and the theory based on regular sequences are equivalent.

In this book we present a new theory of generalized functions in a completely functional analytic setting. So our test space and our distribution space are not described by means of classical analysis but in the language of functional analysis. In fact, we introduce a model for a particular type of distribution theories.
The first inspiring ideas which have led to the construction of this model can be found in a paper by De Bruijn, [Br]. In the paper [Br], De Bruijn proposed a new distribution theory based on complex analysis. In this theory for the test space the space $S_{\frac{1}{2}}^{\frac{1}{2}}$ of Gelfand-Shilov is taken. Three

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kinds of functions occur: smooth functions, smoothed functions and generalized functions. A function is said to be smooth if it belongs to the space $S_{\frac{1}{2}}^{\frac{1}{2}}$ which consists of a specific growth class of entire functions. A smoothed function f is derived from a smooth function g through application to g of an operator from a set of smoothing operators. This set of smoothing operators is a one-parameter semigroup denoted by $(N_\alpha)_{\alpha>0}$. De Bruijn proved that each smooth function is smoothed and that each smoothed function is smooth, i.e.,

$$\bigcup_{\alpha>0} N_\alpha(S_{\frac{1}{2}}^{\frac{1}{2}}) = S_{\frac{1}{2}}^{\frac{1}{2}} .$$

Now, a generalized function is a mapping F from $(0,\infty)$ into the set of smooth functions which satisfies $N_\alpha F(\beta) = F(\alpha+\beta)$ for all positive α and β. De Bruijn established a pairing between the spaces of smoothed functions and of generalized functions. However, no topologies are introduced for these spaces and questions about duality and continuity of linear mappings can be linked to sequential convergence only.

In [G], one of the authors of the present monograph has generalized De Bruijn's theory considerably by replacing the semigroup $(N_\alpha)_{\alpha>0}$ by an arbitrary one-parameter semigroup $(e^{-tA})_{t>0}$ where A denotes a nonnegative self-adjoint operator in a separable Hilbert space X. The series of papers [G] contain the fundamental concepts on which the contents of our book are based. These papers describe the topological spaces $S_{X,A}$ and $T_{X,A}$ which establish a dual system. The space $S_{X,A}$ is an inductive limit of Hilbert spaces. This inductive limit is not strict! So the theory on inductive limits, in which always strictness is assumed, cannot be applied. The main feature in [G] is the introduction of the set of Borel functions $B(\mathbb{R})$. Because of the Spectral Theorem each element of $B(\mathbb{R})$ gives rise to a normal operator in X. As a major result it has been proved that $S_{X,A}$ is the maximal common dense domain of the operators $\varphi(A)$, $\varphi \in B(\mathbb{R})$ and that the inductive limit topology for $S_{X,A}$ is the coarsest topology for which all operators $\varphi(A)\colon S_{X,A} \to X$, $\varphi \in B(\mathbb{R})$, are continuous. These observations have led to complete characterizations of null sequences, of bounded subsets and of compact subsets of $S_{X,A}$, just as for strict inductive limits. Furthermore, the full strength of Hilbert space theory became available. In [G] this has led already to a detailed exposition of continuous linear mappings, of topological tensor products and of so called Kernel theorems. Considerations of this type are not current in classical distribution theory.

In the thesis [E1] we continued the description of the theory of the spaces $S_{X,A}$ and $T_{X,A}$. A major source of inspiration for this thesis was the functional analytic approach in [G] to continuous linear mappings. It became clear that our theory resembles Hilbert space theory in many aspects. This reflects e.g. in the treatment of algebras of continuous linear mappings as presented in this work.

Also in [E1] we have started our mathematical interpretation of Dirac's formalism, searching both for a justification of the bold way in which Dirac treats the (generalized) eigenfunctions corresponding to the continuous spectrum of a self-adjoint operator and for a mathematical apparatus which does full justice to Dirac's bracket formalism. Our interpretation goes further than any other interpretation of Dirac's formalism that is known to us.

The present treatise is a coupling and further elaboration of the contents both of [G] and of [E1]. Further, we devote a great deal of attention to the description or discovery of spaces of analytic functions which can be typified as a space of type $S_{X,A}$ for a well chosen A and X.

The second part of this prologue is devoted to a short survey of each chapter of this book.

For a nonnegative unbounded self-adjoint operator A in a separable Hilbert space X the analyticity space $S_{X,A}$ is the dense subspace of X defined by

$$S_{X,A} = \bigcup_{t>0} e^{-tA}(X) .$$

On $S_{X,A}$ a (non-strict) inductive limit topology is imposed. The space $S_{X,A}$ contains all analytic vectors of the operator A, i.e.,

$$w \in S_{X,A} \Leftrightarrow \exists_{a>0}\, \exists_{b>0}\, \forall_{n\in\mathbb{N}}: w \in \mathcal{D}(A^n) \wedge \|A^n w\| \leq n!\, a^n b .$$

We call $S_{X,A}$ the analyticity space. The elements of the space $T_{X,A}$ are mappings $F: (0,\infty) \to X$ which satisfy

$$\forall_{t>0}\, \forall_{\tau>0}: F(t+\tau) = e^{-\tau A} F(t) .$$

They are called trajectories and the space $T_{X,A}$ is called the trajectory space. Examples of such trajectories are $t \mapsto A^m e^{-tA} x$ with $x \in X$ and $m \geq 0$. By a suitable choice of seminorms the space $T_{X,A}$ becomes a Frechet space.

The Hilbert space X can be embedded in $T_{X,A}$ by means of the embedding emb: $X \to T_{X,A}$ defined by

$$\text{emb}(x): t \mapsto e^{-tA} x , \qquad t > 0, \; x \in X .$$

Thus we obtain the triplet $S_{X,A} \subset X \subset T_{X,A}$.
For each $w \in S_{X,A}$, there exists $\tau > 0$ such that $w \in \mathcal{D}(e^{\tau A})$. So it makes sense to define a pairing between $S_{X,A}$ and $T_{X,A}$ in the following way:

$$\langle w, G \rangle = (e^{\tau A} w, G(\tau)) , \qquad w \in S_{X,A} , \quad G \in T_{X,A} .$$

Here $(\cdot,\cdot)$ denotes the inner product of X. Due to the trajectory property of the elements of $T_{X,A}$, the definition of $\langle \cdot,\cdot \rangle$ does not depend on the choice of $\tau > 0$. With this pairing the spaces $S_{X,A}$ and $T_{X,A}$ can be regarded as each other's strong duals.

Chapter I contains detailed discussions of several topological features of analyticity spaces and trajectory spaces, and of their duality. Furthermore, this chapter contains precise characterizations of five types of continuous linear mappings. One of these types is established by the so called extendible mappings, i.e., all continuous mappings from $S_{X,A}$ into $S_{X,A}$ which have an extension as a continuous mapping from $T_{X,A}$ into $T_{X,A}$.

The theory of analyticity spaces yields a functional analytic description for a new type of distribution theories. If $X = L_2(M,\mu)$ with M some measure space, then loosely speaking we could say that $T_{X,A}$ consists of improper functions on M.
In Chapter II we discuss a number of classical distribution theories which can be obtained by a suitable choice of A and X. This chapter is based on our papers [EG 1-4] in which we have characterized certain analyticity spaces in terms of classical analysis. The following types are discussed:

- Analyticity spaces based on the diffusion equation $\frac{\partial u}{\partial t} = -\Delta^{\nu} u$, $\nu \geq \frac{1}{2}$. We will restrict ourselves to $\mathbb{R}$, $(0,\infty)$, $[-\pi,\pi]$ and $[0,\pi]$, and for $\nu = \frac{1}{2}$ to the unit sphere S^q in $\mathbb{R}^{q+1}$. So we consider $X = L_2(\mathbb{R})$, $L_2((0,\infty))$, $L_2([-\pi,\pi])$, $L_2([0,\pi])$ and $L_2(S^q)$, and $A = \Delta^{\nu}$ where Δ denotes the Laplacian. We give precise characterizations of the analytic functions corresponding to the elements of the respective analyticity spaces $S_{X,A}$. Moreover, a number of examples of continuous linear mappings on these

spaces is discussed. These examples are based on simple geometric and analytical considerations.

- Analyticity spaces connected to the Gelfand-Shilov spaces S_α^β. For a number of spaces S_α^β we give an operator $A_{\alpha,\beta}$ in the Hilbert space $L_2(\mathbb{R})$ such that $S_\alpha^\beta = S_{L_2(\mathbb{R}),A_{\alpha,\beta}}$.
- Analyticity spaces based on classical polynomials. The Hermite, Laguerre and Jacobi polynomials give rise to orthonormal bases in the Hilbert spaces $L_2(\mathbb{R})$, $L_2(\mathbb{R},x^{2\alpha+1}dx)$, $\alpha > -1$, and $L_2([-1,1],(1+x)^\alpha(1-x)^\beta dx)$, $\alpha,\beta > -1$, respectively. Each orthonormal basis consists of the normalized eigenfunctions of an appropriate self-adjoint operator. The elements of the associated analyticity spaces can be described by series expansions with respect to the corresponding basis. It leads to conditions on the order of growth of expansion coefficients. We characterize spaces of analytic functions with precisely these kinds of expansion coefficients.

Further, in this chapter we discuss certain relations between the representation theory of Lie groups and our theory.

In Chapter III we continue the description of the functional analytic theory An important device in distribution theory is the existence of Kernel theorems. In classical distribution theory there are considered two Kernel theorems: One for the continuous linear mappings from $\mathcal{S}$ into $\mathcal{S}'$ and one for the continuous linear mappings from $\mathcal{S}'$ into $\mathcal{S}$. Here we present a Kernel theorem for each of the considered five types of continuous linear mappings. In our view, a Kernel theorem gives conditions such that all continuous linear mappings of a specific type arise from the elements (kernels) out of a suitable topological tensor product.
The natural description of a complete topological product of two analyticity spaces or two trajectory spaces is again an analyticity space or a trajectory space. However, in order to describe a complete topological tensor product of an analyticity space and a trajectory space, we have to introduce two new types of spaces. We have called them analyticity-trajectory spaces and trajectory-analyticity spaces. These spaces can be regarded as generalizations of analyticity and trajectory spaces. For these compound spaces we study the usual topological features. We introduce a pairing between an analyticity-trajectory and a trajectory-analyticity space, and we show their

duality. Moreover, we describe certain intersections and algebraic sums. In a separate section we present an inclusion diagram which contains all relevant spaces. The last section of Chapter III contains the five announced Kernel theorems and their proofs.

In Chapter IV we study the following operator algebras: the algebra $\mathbf{L}(S_{X,A})$ of continuous linear mappings from $S_{X,A}$ into itself, the algebra $\mathbf{L}(T_{X,A})$ of continuous linear mappings from $T_{X,A}$ into itself and the algebra $\mathbf{E}(S_{X,A})$ of extendible linear mappings. The algebra $\mathbf{E}(S_{X,A})$ can be regarded as a $*$-algebra of unbounded operators with common dense domain. We use this algebra in a mathematical model for a description of quantum statistical mechanics. We proceed our research with respect to the introduced algebras along the lines of operator theory for Hilbert spaces.
In this chapter we devote also some attention to infinite matrices. If $S_{X,A}$ is a nuclear space, then to every element of $\mathbf{L}(S_{X,A})$, $\mathbf{L}(T_{X,A})$ and $\mathbf{E}(S_{X,A})$, respectively, there can be associated an infinite matrix. We derive simple characterizations of these matrices. As a class of examples we discuss the continuous linear mappings which have a matrix with only one nonzero co-diagonal. These mappings are usually called weighted shifts. In fact, weighted shifts and their finite combinations frequently appear in applied mathematics and in special function theory.
Last but not least we apply the mathematical calculus in the construction of nuclear analyticity spaces on which the operators out of a given set act continuously.

Chapter V is the self-contained part of this monograph in which we present a mathematical interpretation of Dirac's formalism. It consists of two parts. In the first part we present a theory of generalized eigenfunctions based on our Gelfand triple $S_{X,A} \subset X \subset T_{X,A}$. To this end we introduce the notion of Dirac basis. It is the continuous analogue of the usual notion of orthonormal basis. In a well-specified way any element of $T_{X,A}$ can be expanded towards a Dirac basis. We prove that to any self-adjoint operator P in X there can be associated a Dirac basis. If P can be extended to a closed mapping in $T_{X,A}$, then this Dirac basis consists of (generalized) eigenvectors. A main role in the proof of this result is played by a so called measure theoretical Sobolev lemma. Also we insert commutative multiplicity theory for self-adjoint operators.

In the second part of this chapter we present a mathematical apparatus which can cope with the formal computation with bras and kets in Dirac's formalism. A reinterpretation of Dirac's bracket notion leads to a mathematical theory which involves a pairing between any bra and any ket, Fourier expansion of kets, orthogonality of complete sets of eigenkets and matrices of unbounded linear mappings, all in the spirit of Dirac.

In the epilogue we indicate related results which have been achieved in the past five years.

We conclude this introduction with a summary of advantages of the functional analytic approach employed in this monograph and of possibilities for applications.

- Generalized functions are represented by trajectories, a concept which is very close to the physical intuition of what a generalized function should be.
- Test function spaces can be constructed that are invariant under a set of given operators. We can always do this in an abstract way. However, the characterization of thus obtained test spaces in terms of classical analysis may be a hard job. For results in this direction see Chapter II.
- Many of the test spaces of Gelfand-Shilov are special examples of analyticity spaces. So our general theory applies to them. See Chapter II.
- Spaces of real analytic functions on compact analytic manifolds as considered by Hashizume et al. are special examples of analyticity spaces. See [HMO].
- Hyperfunctions of fixed bounded support can be represented by trajectories. So our general theory also applies here. See Chapter II.
- Generalized functions (and δ-functions in particular) can be introduced on geometrical measure spaces. See Chapter II.
- Our functional analytic approach makes it possible to transfer large pieces of Hilbert space theory to distribution theory. This has led to a detailed exposition of continuous linear mappings, of topological tensor products and of five kernel theorems. Systematic considerations on continuous linear mappings and operator algebras are not current in distribution theory! See Chapter III, IV.
- For solving the generalized eigenvalue problem for any arbitrary self adjoint operator an adapted theory of generalized "functions" is needed. Our approach has all flexibility required in such cases. The corresponding

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