H. Haken H.C. Wolf

Atomic and Quantum Physics

An Introduction to the Fundamentals of Experiment and Theory

Translated by W. D. Brewer

Second Enlarged Edition

Springer-Verlag
World Publishing Corporation

H. Haken H.C. Wolf

Atomic and Quantum Physics

An Introduction to the Fundamentals of Experiment and Theory

Translated by W. D. Brewer

Second Enlarged Edition

With 265 Figures

Springer-Verlag
World Publishing Corporation

Professor Dr. Dr. h.c. *Hermann Haken*

Institut für Theoretische Physik, Universität Stuttgart, Pfaffenwaldring 57,
D-7000 Stuttgart 80, Fed. Rep. of Germany

Professor Dr. *Hans Christoph Wolf*

Physikalisches Institut, Universität Stuttgart, Pfaffenwaldring 57,
D-7000 Stuttgart 80, Fed. Rep. of Germany

Translator:

Professor Dr. *William D. Brewer*

Freie Universität Berlin, Fachbereich Physik, Arnimallee 14,
D-1000 Berlin 33

Reprinted by World Publishing Corporation, Beijing, 1989
for distribution and sale in The People's Republic of China only

---

Title of the german original edition:

H. Haken, H.C. Wolf: *Atom- und Quantenphysik.*
Eine Einführung in die experimentellen und theoretischen Grundlagen.
(Dritte, überarbeitete und erweiterte Auflage)

ISBN 3-540-17921-6 3. Auflage Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-17921-6 3rd edition Springer-Verlag New York Berlin Heidelberg

---

ISBN 3-540-17702-7 2. Auflage Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-17702-7 2nd edition Springer-Verlag New York Berlin Heidelberg
ISBN 7-5062-0301-4
ISBN 3-540-13137-X 1. Auflage Springer-Verlag Berlin Heidelberg New York Tokyo
ISBN 0-387-13137-X 1st edition Springer-Verlag New York Berlin Heidelberg Tokyo

Library of Congress Cataloging-in-Publication Data. Haken, H. Atomic and quantum physics Translation of Atom- und Quantenphysik. Bibliography: p. Includes index. 1. Atoms. 2. Quantum theory. I. Wolf, H C (Hans Christoph), 1929- II. Title. QC173.H17513 1987 539.7 87-9450

Typesetting: K+V Fotosatz, 6124 Beerfelden
Offset printing: Druckhaus Beltz. 6944 Hemsbach. Bookbinding: J. Schäffer GmbH & Co. KG, 6718 Grunstadt
2153/3150-543210

Fundamental Constants of Atomic Physics in the International System of Units (SI)

From: Kohlrausch, Praktische Physik, 23. Aufl.
(Teubner, Stuttgart 1986)

Permeability Constant of Vacuum	μ_0	$= 4\pi \cdot 10^{-7}\,\mathrm{Vs\,A^{-1}\,m^{-1}}$
		$= 1.256637 \cdot 10^{-6}\,\mathrm{Vs\,A^{-1}\,m^{-1}}$
Permittivity Constant of Vacuum	ε_0	$= (\mu_0 c^2)^{-1}$
		$= 8.854187 \cdot 10^{-12}\,\mathrm{As\,V^{-1}\,m^{-1}}$
Velocity of Light in Vacuum	c	$= 2.99792458 \cdot 10^{8}\,\mathrm{ms^{-1}}$
Boltzmann's Constant	k	$= 1.380662 \cdot 10^{-23}\,\mathrm{JK^{-1}}$
Faraday Constant	F	$= 9.648455 \cdot 10^{4}\,\mathrm{C\,mol^{-1}}$
Elementary Charge	e	$= 1.6021892 \cdot 10^{-19}\,\mathrm{C}$
Rest Mass of the Electron	m_0	$= 9.109534 \cdot 10^{-31}\,\mathrm{kg}$
Specific Charge of the Electron	e/m_0	$= 1.7588047 \cdot 10^{11}\,\mathrm{C\,kg^{-1}}$
Rest Mass of the Proton	m_p	$= 1.6726485 \cdot 10^{-27}\,\mathrm{kg}$
Planck's Constant	h	$= 6.626176 \cdot 10^{-34}\,\mathrm{Js}$
	$\hbar$	$= h/2\pi = 1.0545887 \cdot 10^{-34}\,\mathrm{Js}$
Rydberg Constant	R_∞	$= 1.097373177 \cdot 10^{7}\,\mathrm{m^{-1}}$
Bohr Radius	a_0	$= 0.52917706 \cdot 10^{-10}\,\mathrm{m}$
Bohr Magneton	μ_B	$= 9.274078 \cdot 10^{-24}\,\mathrm{Am^2}\ (= \mathrm{J/T})$
Nuclear Magneton	μ_N	$= 5.050824 \cdot 10^{-27}\,\mathrm{Am^2}$
Compton Wavelength of the Electron	λ_c	$= 2.4263089 \cdot 10^{-12}\,\mathrm{m}$
Fine Structure Constant	α	$= 7.2973506 \cdot 10^{-3}$
Avogadro's number	N_A	$= 6.022045 \cdot 10^{23}\,\mathrm{mol}$.

Energy Conversion Table see inside back cover

Energy Conversion Table

	J	eV	cm^{-1}	K
1 Joule (J) =	1	$6.24146 \cdot 10^{18}$	$5.03404 \cdot 10^{22}$	$7.24290 \cdot 10^{22}$
1 eVolt (eV) =	$1.60219 \cdot 10^{-19}$	1	$8.06548 \cdot 10^{3}$	$1.16045 \cdot 10^{4}$
1 cm^{-1} =	$1.98648 \cdot 10^{-23}$	$1.23985 \cdot 10^{-4}$	1	1.43879
1 K =	$1.38066 \cdot 10^{-23}$	$8.61735 \cdot 10^{-5}$	$6.95030 \cdot 10^{-1}$	1

Explanation

The energy E is quoted in Joule (J) or watt-seconds (Ws)

$$1\ \mathrm{J} = 1\ \mathrm{Ws}.$$

In spectroscopy, one frequently quotes the term values in wavenumbers $\tilde{\nu} = E/hc$.

The conversion factor is

$$E/\tilde{\nu} = hc = 1.98648 \cdot 10^{-23}\ \mathrm{J/cm^{-1}}$$

Another energy unit, especially in collision experiments, is the electron volt (eVolt, eV). The voltage V is given in volts, and the energy conversion factor is obtained from $E = eV$:

$$E/V = e = 1.60219 \cdot 10^{-19}\ \mathrm{J/V}$$

In t[redacted] kT, the absolute tem- per[redacted]e obtain the conversion fac[redacted]

Preface to the Second Edition

The excellent critique and very positive response to the first edition of this book have encouraged us to prepare this second edition, in which we have tried to make improvements wherever possible. We have profited much from the suggestions of professors and students as well as from our own experience in teaching atomic and quantum physics at our university.

Following a widespread request, we have now included the solutions to the exercises and present these at the end of the book. Among the major new sections to be found in this second edition are the following:

We now include the derivation of the relativistic Klein-Gordon equation and of the Dirac equation because the latter, in particular, appears in atomic physics whenever relativistic effects must be taken into account. Our derivation of the Schrödinger equation allowed us to present this extension in a straightforward manner.

The high precision methods of modern spectroscopy allow the atomic physicist to measure extremely small but important shifts of the atomic lines. A very important effect of this kind is the Lamb shift, for which a detailed theoretical derivation is given in a new section. In order to put this in an adequate framework, the basic ideas of the quantization of the electromagnetic field as used in quantum electrodynamics are given. Again it turned out that all the concepts and methods needed to discuss these seemingly advanced theories had already been presented in previous chapters so that again the reader may easily follow these theoretical explanations.

The section on photoelectron spectroscopy has been enlarged and revised. Furthermore, the two-electron problem has been made more explicit by treating the difference between triplet and singlet states in detail. Finally, our previous presentation of nuclear spin resonance has been considerably enlarged because this method is finding widespread and very important applications, not only in chemistry but also in medicine, for instance in NMR tomography, which is an important new tool in medical diagnostics. This is only one example of the widespread and quite often unanticipated application of atomic and quantum physics in modern science and technology.

It goes without saying that we have not only corrected a number of misprints but have also tried to include the most recent developments in each area. This second English edition corresponds to the third German edition, which is published at about the same time. We wish to thank R. Seyfang, J. U. von Schütz and V. Weberruss for their help in preparing the second edition. It is again a pleasure for us to thank Springer-Verlag, in particular Dr. H. Lotsch and C.-D. Bachem for their always excellent cooperation.

Stuttgart, March 1987 *H. Haken H. C. Wolf*

Preface to the First Edition

A thorough knowledge of the physics of atoms and quanta is clearly a must for every student of physics but also for students of neighbouring disciplines such as chemistry and electrical engineering. What these students especially need is a coherent presentation of both the experimental and the theoretical aspects of atomic and quantum physics. Indeed, this field could evolve only through the intimate interaction between ingenious experiments and an equally ingenious development of bold new ideas.

It is well known that the study of the microworld of atoms caused a revolution of physical thought, and fundamental ideas of classical physics, such as those on measurability, had to be abandoned. But atomic and quantum physics is not only a fascinating field with respect to the development of far-reaching new physical ideas. It is also of enormous importance as a basis for other fields. For instance, it provides chemistry with a conceptual basis through the quantum theory of chemical bonding. Modern solid-state physics, with its numerous applications in communication and computer technology, rests on the fundamental concepts first developed in atomic and quantum physics. Among the many other important technical applications we mention just the laser, a now widely used light source which produces light whose physical nature is quite different from that of conventional lamps.

In this book we have tried to convey to the reader some of the fascination which atomic and quantum physics still gives a physicist studying this field. We have tried to elaborate on the fundamental facts and basic theoretical methods, leaving aside all superfluous material. The text emerged from lectures which the authors, an experimentalist and a theoretician, have given at the University of Stuttgart for many years. These lectures were matched with respect to their experimental and theoretical contents.

We have occasionally included in the text some more difficult theoretical sections, in order to give a student who wants to penetrate thoroughly into this field a self-contained presentation. The chapters which are more difficult to read are marked by an asterisk. They can be skipped on a first reading of this book. We have included chapters important for chemistry, such as the chapter on the quantum theory of the chemical bond, which may also serve as a starting point for studying solid-state physics. We have further included chapters on spin resonance. Though we explicitly deal with electron spins, similar ideas apply to nuclear spins. The methods of spin resonance play a fundamental role in modern physical, chemical and biological investigations as well as in medical diagnostics (nuclear spin tomography). Recent developments in atomic physics, such as studies on Rydberg atoms, are taken into account, and we elaborate the basic features of laser light and nonlinear spectroscopy. We hope that readers will find atomic and quantum physics just as fascinating as did the students of our lectures.

The present text is a translation of the second German edition *Atom- und Quantenphysik*. We wish to thank Prof. W. D. Brewer for the excellent translation and the most valuable suggestions he made for the improvement of the book. Our thanks also go to

Dr. J. v. Schütz and Mr. K. Zeile for the critical reading of the manuscript, to Ms. S. Schmiech and Dr. H. Ohno for the drawings, and to Mr. G. Haubs for the careful proof-reading. We would like to thank Mrs. U. Funke for her precious help in typing new chapters. Last, but not least, we wish to thank Springer-Verlag, and in particular H. Lotsch and G. M. Hayes, for their excellent cooperation.

Stuttgart, February 1984 *H. Haken H. C. Wolf*

List of the Most Important Symbols Used

The numbers of the equations in which the symbols are defined are given in parentheses; the numbers in square brackets refer to the section of the book. The Greek svmbols are at the end of the list.

$\mathbf{A}$	Vector potential
A	Amplitude or constant
A	Mass number (2.2) or area
a	Interval factor or fine structure constant (12.28) and hyperfine splitting (20.10)
a_0	Bohr radius of the H atom in its ground state (8.8)
$\mathbf{B}$	Magnetic induction
b^+, b	Creation and annihilation operators for the harmonic oscillator
b	Constant, impact parameter
C	Constant
c	Velocity of light, series expansion coefficient
c.c.	Complex conjugate
$\mathbf{D}$	Dipole moment
d	Constant
dV	Infinitesimal volume element
$\mathbf{E}$	Electric field strength
E	Energy, total energy, energy eigenvalue
E_{kin}	Kinetic energy
E_{pot}	Potential energy
E_{tot}	Total energy
e	Proton charge
$-e$	Electron charge
e	Exponential function
F	Electric field strength (14.1)
$\mathbf{F}, F$	Total angular momentum of an atom, including nuclear angular momentum and corresponding quantum number (20.6)
F	Amplitude of the magnetic induction [14.4, 14.5]
f	Spring constant
g	Landé g factor (12.10, 16, 21, 13.18, 20.13)
$\mathscr{H}$	Hamilton function, Hamiltonian operator
H_n	Hermite polynomial
h	Planck's constant
$\hbar$	$= h/2\pi$
$\mathbf{I}, I$	Nuclear angular momentum and corresponding quantum number (20.1)
I	Abbreviation for integrals [16.13] or intensity
i	Imaginary unit ($\mathrm{i} = \sqrt{-1}$)
$\mathbf{J}, J$	Total angular momentum of an electron shell and corresponding quantum number (17.5)
$\mathbf{j}, j$	Total angular momentum of an electron and corresponding quantum number [12.7]
$\hat{j}$	Operator for the total angular momentum
k	Boltzmann's constant, force constant
$\mathbf{k}$	Wavevector
$\mathbf{L}, L$	Resultant orbital angular momentum and corresponding quantum number (17.3)
L_n	Laguerre polynomial (10.81)
$\mathbf{l}, l$	Orbital angular momentum of an electron and corresponding quantum number
$\hat{l}$	Angular momentum operator
m, m_0	Mass
m	Magnetic quantum number
m_l	– for angular momentum
m_s	– for spin
m_j	Magnetic quantum number for total angular momentum
m_0	Rest mass, especially that of the electron

N, n Particle number, particle number density
N Normalisation factor
n Principal quantum number or number of photons or an integer
P Spectral radiation flux density (5.2) or probability
P_l^0 Legendre polynomial
P_l^m ($m \neq 0$) Associated Legendre function
$p, \bar{p}$ Momentum, expectation value of momentum
Q Nuclear quadrupole moment (20.20)
Q, q Charge
$R(r)$ Radial part of the hydrogen wavefunction
$\boldsymbol{r}$ Position coordinate (three-dimensional vector)
r Distance
$\boldsymbol{S}$ Resultant spin (17.4)
S Symbol for orbital angular momentum $L = 0$
$\boldsymbol{s}, s$ Electron spin and corresponding quantum number (12.15)
$\hat{\boldsymbol{s}}$ Spin operator $= (\hat{s}_x, \hat{s}_y, \hat{s}_z)$
T Absolute temperature
T_1 Longitudinal relaxation time
T_2 Transverse relaxation time
t Time
u Spectral energy density (5.2), atomic mass unit [2.2]
V Volume, potential, electric voltage
$\bar{V}$ Expectation value of the potential energy
v Velocity, particle velocity
x Particle coordinate (one-dimensional)
$\bar{x}$ Expectation value of position
$Y_{l,m}(\theta, \phi)$ Spherical harmonic functions (10.10, 48 – 50)
Z Nuclear charge
α Fine structure constant [8.10] or absorption coefficient (2.22)
β Constant
Γ Decay constant
γ Decay constant or linewidth gyromagnetic ratio (12.12)
∇^2 Laplace operator $= \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$
ΔE Energy uncertainty
Δk Wavenumber uncertainty
Δp Momentum uncertainty
Δt Time uncertainty (= finite measurement time)
ΔV Finite volume element
$\Delta\omega$ Uncertainty in the angular frequency
Δx Position uncertainty
$\delta(x)$ Dirac delta function (see mathematics appendix)
$\delta_{\mu,\nu}$ Kronecker delta symbol: $\delta_{\mu,\nu} = 1$ for $\mu = \nu$, $\delta_{\mu,\nu} = 0$ for $\mu \neq \nu$
ε Dimensionless energy (9.83)
$\varepsilon^{(n)}$ Energy contributions to perturbation theory
ε_0 Permittivity constant of vacuum
θ Angle coordinate (10.2)
κ Defined in (10.54)
λ Wavelength (exception: expansion parameter in [15.2.1, 2])
$\boldsymbol{\mu}, \mu$ Magnetic moment (12.1)
μ Reduced mass (8.15)
μ_B Bohr magneton (12.8)
μ_N Nuclear magneton (20.3)
ν Frequency [8.1]
$\bar{\nu}$ Wavenumber [8.1]
ξ Dimensionless coordinate (9.83)
ϱ Charge density, density of states, mass density; or dimensionless distance
σ Scattering coefficient, interaction cross section (2.16)
τ Torque (12.2)
Φ Phase
ϕ Phase angle, angle coordinate
$\phi(x)$ Wavefunction of a particle
$\phi_\uparrow, \phi_\downarrow, \phi$ Spin wavefunctions
ψ Wavefunction
Ψ Wavefunction of several electrons
$\hat{\Omega}$ Generalised quantum mechanical operator
Ω Frequency [14.4, 14.5, 15 3]
ω Angular frequency $2\pi\nu$, or eigenvalue [9 3.6]
$\triangleq$ means "corresponds to"

Contents

1. Introduction

1.1 Classical Physics and Quantum Mechanics

Atomic and quantum physics, which are introduced in this book, are essentially products of the first third of this century. The division of classical physics into branches such as mechanics, acoustics, thermodynamics, electricity, and optics had to be enlarged when – as a consequence of the increasing knowledge of the structure of matter – atoms and quanta became the objects of physical research. Thus, in the twentieth century, classical physics has been complemented by atomic physics and the physics of light or energy quanta. The goal of atomic physics is an understanding of the structure of atoms and their interactions with one another and with electric and magnetic fields. Atoms are made up of positively charged nuclei and negatively charged electrons. The electromagnetic forces through which these particles interact are well known in classical physics.

The physics of atomic nuclei cannot be understood on the basis of these forces alone. A new force – the nuclear or strong force – determines the structures of nuclei, and the typical binding energies are orders of magnitude larger than those of the electrons in atoms. The study of nuclei, of elementary particles, and the whole of high energy physics thus form their own branches of physics. They will not be treated in this book.

1.2 Short Historical Review

The word *atom* comes from the Greek and means "the indivisible", the smallest component of matter, which cannot be further divided. This concept was introduced in the 5th and 4th centuries B.C. by Greek natural philosophers. The first atomic theories of the structure of matter were those of *Democrites* (460 – 370 B.C.), *Plato* (429 – 348), and *Aristotle* (384 – 322). It required more than two millenia until this speculative atomism grew into an exact atomic physics in the modern sense.

The meaning of the word *atom* becomes less subject to misinterpretation if it is translated into Latin: an *individuum* is the smallest unit of a large set which possesses all the essential characteristics of the set. In this sense, an atom is in fact indivisible. One can, to be sure, split a hydrogen atom into a proton and an electron, but the hydrogen is destroyed in the process. For example, one can no longer observe the spectral lines characteristic of hydrogen in its optical spectrum.

Atomism as understood by modern science was first discovered for *matter*, then for *electricity*, and finally for *energy*.

The *atomism of matter*, the recognition of the fact that all the chemical elements are composed of atoms, followed from chemical investigations. The laws of constant and